A Thermally Activated Model for the Plastic Flow and Fracture of Solids

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1. Recent works on fracture mechanics emphasized the role of the crack tip plastic deformation in the fracture initiation process. In the framework of this conception, the temperature dependence of the fracture initiation process will be envinced by considering the plastic strain energy dissipated at the crack tip in a material which plastic flow properties are described by a thermally activated model [1].

2. The thermally activated model for the plastic flow, consints of a set of perfectly elastic and rigid plastic elements in series - parallel connection (figure 1). The notional length of an element - l - and the elastic parameter E_o are constants, while the critical slipping or the frictional stress σ_i^* is a distributed parameter with a probability density function $\varphi(\sigma^*)$. Upon initial loading, if n elements of N have yielded, it is easily shown that the stress strain characteristic of the model is described by :

$$\varepsilon = \sum_{i=1}^{n} \frac{\sigma - \sigma_i^*}{NE_o} \qquad (1)$$

For a large number of elements eq.(1) becomes :

$$\varepsilon = \frac{1}{E_o}\int_0^{\sigma}(\sigma - \sigma^*)\,\varphi(\sigma^*)\,d\sigma^* \qquad (2)$$

and after derivation in eq.(2) a relationship between slope characteristic of, the stress-strain curve and the probability density function is obtained :

$$\frac{d\varepsilon}{d\sigma} = E_o\,\varphi(\sigma^*) \qquad (3)$$

Physically the rigid-plastic elements might be thought as modelling the slip planes in a metal or alloy in which dislocations responsible for plastic flow are mobilised. But, the dislocation movement is a thermally assisted process [2] and thus the variation of the critical slip stress is due to the thermal energy distribution over the elements of the model. For the solid state conditions the general approach of the problem of distribution of the thermal energy over the elements of a system which are considered in a weak thermal interaction is based on Gibbs statistics [2] . According to, the proportion of the elements $q(U)$ with the energy comprised in the interval $(U\,;\,U+dU)$ is :

$$q(U) = \psi(U)dU = A\exp\left(-\frac{U}{kT}\right)dU \tag{4}$$

where k is the Boltzmann's constant ; T - absolute temperature ; A a normalization constant ($A = -1/kT$ for U varying between 0 and ∞). In this thermodynamic acception it results that for a certain stage of plastic flow :

$$\varphi(\sigma^*)d\sigma^* = \psi(U)dU \tag{5}$$

From eq.(3),(4) and (5) it results :

$$\frac{U}{kT} = -\ln\left(E_0\frac{d\varepsilon}{d\sigma}\right) \tag{6}$$

For steels the experimental results evince the following relationship :

$$U = -c\ln(\sigma/\sigma_0) \tag{7}$$

with c and σ_0 experimental constants.

It follows from eqs.(6) and (7) the well known power relationship between stress and strain :

$$\sigma = \sigma_0\left(\frac{E_0\varepsilon}{\sigma_0\lambda_T}\right)^{\lambda_T} \tag{8}$$

where, the strain hardening exponent λ_T is temperature dependent according to :

$$\lambda_T = kT/(c+kT) \tag{9}$$

3. In order to illustrate the temperature influence on the fracture initiation process from pre-existing cracks, it will be assumed - in line with current simplified views - that an energy balance between elastic energy (W_e) release rate and the plastic energy (W_p) dissipation rate at the crack tip is a necessary condition for the onset of the crack extension. Thus :

$$\delta W_e/\delta l = \delta W_p/\delta l \tag{10}$$

The incremental change of the dissipated plastic energy in a δl width strip element adjacent to the crack tip (fig.2a) is :

$$\delta W_p = \int_0^{R_{pf}} w_p(r)\,dr\,\delta l \tag{11}$$

where : $w_p(r)$ is the density of the dissipated plastic energy in a point located in the strip element at the distance r from the crack plane (fig.2a) and R_{pf} is the extension of the plastic enclave in r direction. The variation of $w_p(r)$ will be considered linear as against r (fig.2b) and thus at fracture initiation :

$$\delta W_p = \frac{1}{2}w_f R_{pf}\,\delta l \tag{12}$$

where w_f is the crack tip plastic strain energy density dissipated at fracture. Assuming that a constant stress criterion (σ_F) controls the fracture initiation, then from eq.(8) it results :

$$w_f = \frac{\sigma_0^2}{E_0}\,\frac{\lambda_T}{\lambda_T+1}\left(\frac{\sigma_F}{\sigma_0}\right)^{\frac{\lambda_T+1}{\lambda_T}} \tag{13}$$

Considering further, a proportionality relation $w_f = \beta R_{pf}$ as is schematically illustrated in fig.3, then from eqs. (10),(12),(13) and in line with the current acceptions [3] : $\delta W_e/\delta l = \mathcal{G}$ ($\mathcal{G}$ - the crack extension force), it results finally for the critical stress intensity factor K_{Ic} the following relationship (plane strain conditions):

$$K_{Ic} = \sqrt{\frac{E}{2\beta(1-\nu^2)}}\;\frac{\sigma_0^2}{E_0}\,\frac{\lambda_T}{\lambda_T+1}\left(\frac{\sigma_F}{\sigma_0}\right)^{\frac{\lambda_T+1}{\lambda_T}} \tag{14}$$

where E is Young's modulus and ν is Poisson's ratio. Eq.(14) expressing the variation of K_{Ic} with temperature was used in fig.4 for illustrating in appropiate coordinates, the reasonable concordance between theory and some known experimental data [4] .

When a considerable crack tip plastic enclave develops prior to fracture initiation, then from a similar analysis, the temperature variation of the critical crack opening displacement (δ_c) is evinced by the relationship:

$$\delta_c = \left(\frac{\sigma_0}{E_0}\right) \frac{\sigma_0}{2\beta} \frac{\lambda_T^2}{\lambda_T+1} \left(\frac{\sigma_F}{\sigma_0}\right)^{\frac{\lambda_T+2}{\lambda_T}} \qquad (15)$$

In order to check the validity of eq.(15), experimental data were obtained following two testing methods. According to the first, a single notch specimen was sequentially loaded till fracture by impact bending, the C.O.D evolution being measured under microscop after each sequence. Following the second method, a double notch specimen was loaded in impact bending, the critical C.O.D being measured on the "twin" notch. No significant differences were found between the two testing methods, the experimental data correlating well with theoretical analysis (eq.15).

4. With the view of predicting the temperature dependence of the fracture initiation characteristics K_{IC} and δ_c a thermally activated model for the plastic flow was used in conjunction with the analysis of the crack tip energy balance at the onset of fracture. The theoretical prediction is found to be reasonably supported by the experimental results.

REFERENCES

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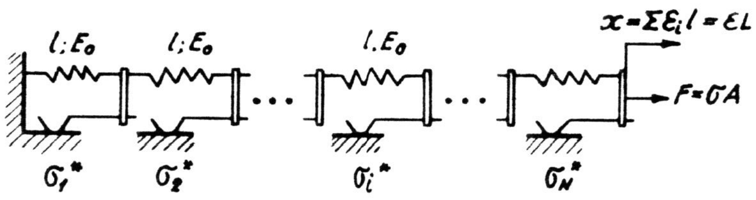

Figure 1

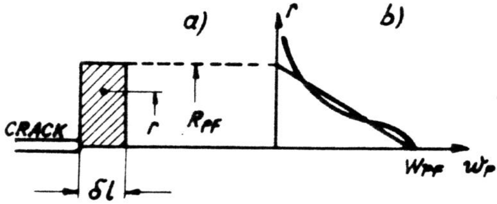

Figure 2

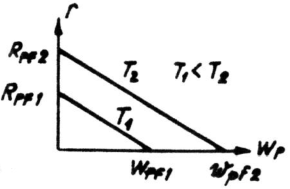

Figure 3

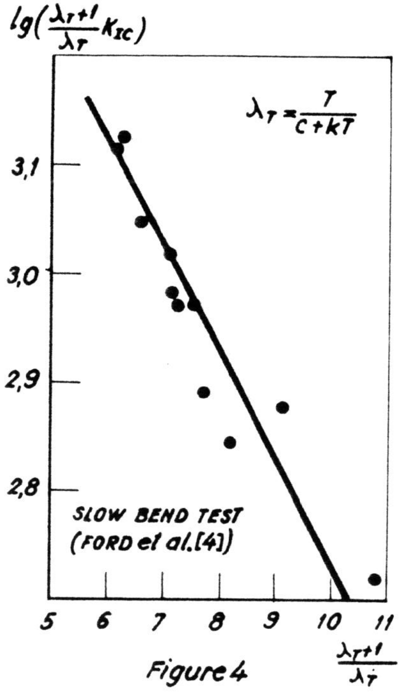

Figure 4

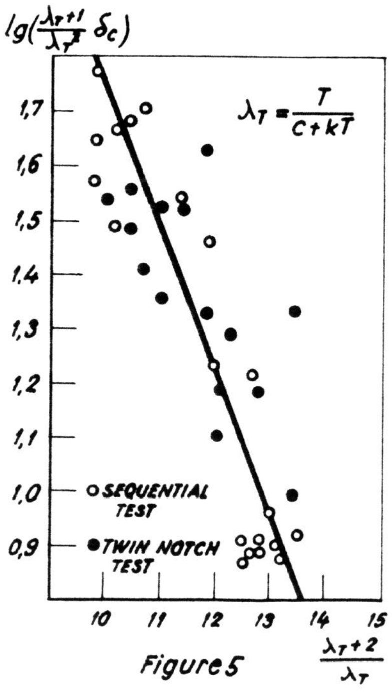

Figure 5