A Statistical Theory of Fracture Kinetics of Inhomogeneous Materials

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In previous papers /1-3/ we proposed a theory of thermally activated rupture during creep /4/. We calculated the distribution function $W(\tau)$ of longevity τ of a sample that fractured due to the evolution of an ensemble of cracks generated by means of thermal fluctuations. According to our model the growth of the main crack that determines τ proceeds in two stages. During the first stage microscopic m-cracks pour out in the sample under the applied stress, their characteristic generation time being τ_m. The random fusion of m-cracks results at the end of this stage in the generation of a l-crack - the nucleus of the main crack. During the second stage the l-crack grows by "swallowing" correlated c-cracks that arise in its stress field.

Up to now homogeneous material was considered, i.e. we supposed that the structural features that determine the growth of the cracks (their average dimension $\langle r\rangle$ and τ_m) do not depend on the coordinates and on time. However, the structure of real bodies is inhomogeneous, thus the cracks develop inhomogeneously in space and time due to the polycrystalline and multiphase structure, due to surface effects (the external surface /5/, internal grain boundaries /6/ a.oth.) and due to changes of the defect structure during strain and rupture /7/.

The main kinds of such inhomogeneities were investigated. In this report we present the results we get for the case of randomly distributed space inhomogeneity of generation times τ_m. The following problem was solved. An inhomogeneous sample is divided into elements (homogeneous regions) with volume v_n and characteristic linear dimensions r_n (comparable with the dimensions $\langle r \rangle$ of the m- and c-cracks). To each element corresponds a value $_m$ with the probability density

$$\varphi(\tau_m) = -\frac{d}{d\tau_m} \exp\left[-(\tau_m/\tau_m^*)\right], \qquad (1)$$

τ_m^* being the modal value of τ_m and α the homogeneity exponent: the larger α the more homogeneous is the material. We search for the distribution of longevity of the sample. The scale of the inhomogeneity proves to be essential.

In case of small-scale inhomogeneities $\langle r \rangle \leq r_n \leq R_o$, R_o being the initial dimension of the 1-crack, the whole spectrum φ is represented in the sample and according to /3/ the distribution function $W(\tau)$ proves again to be

$$W(\tau) = \int_0^\tau W_g(t) W_p(\tau - t) dt, \qquad (2)$$

W_p being the distribution function of the time of propagation of the 1-cracks and

$$W_g(t) = 1 - \exp\left[-K_1(t)\right] \qquad (3)$$

the distribution function of thier genetation times; $K(t)$ denotes the number of 1-cracks in the sample at the moment t. We suppose that the generation of 1-cracks proceeds in each homogeneous element in the same way as in homogeneous material. In this case the expression for K_l in /3/ is

easily generalized and becomes

$$K_1(t) = K_o\left[\left(\frac{t}{\tau_m^*}\right)^{\alpha} + \Gamma\left(1+\frac{s'}{\alpha}\right)\left(\frac{t}{\tau_m^*}\right)^{s'}\right]^{s}, \quad K_o = \frac{V}{v_n s!}\left(\frac{v_n}{s'!\langle v \rangle}\right)^{s_o}. \qquad (4)$$

Here V denotes the volume of the sample, $\langle v \rangle$ the average characteristic volume of m-crack generation, $s_o = R_o/\langle r \rangle$, $s' = r_n/\langle r \rangle$, $s = s_o/s'$, Γ is the gamma-function. The first term in the brackets in (4) represents the contribution of "weak" elements that rupture during the time t with certainty, the second term corresponds to "strong" elements that rupture only with some probability. The magnitude of these contributions depend on the homogeneity of the material and on the degree of dispersion. According to these features two extreme cases are possible:

$$K_1(t) = K_o(t/\tau_m^*)^{s_o}, \qquad \alpha \gg s',$$
$$K_1(t) = K_o(t/\tau_m^*)^{s_o \alpha/s'}, \qquad \alpha \ll s'. \qquad (5)$$

In the case $\alpha \gg s'$ the contribution of weak elements is negligible and the inhomogeneous material behaves like homogeneous (cf./3/). In the case $\alpha \ll s'$ weak elements are essential and 1-cracks are generated exclusively in these elements. From (5) we see that the generation of 1-cracks in an essentially inhomogeneousl material is described as the generation in homogeneous material by means of a new exponent $s_n = s_o\alpha/s' = \alpha R_o/r_n \ll s_o$. Thus all the features of W_g and its numerical characteristics deduced in /3/ remain valid. Comparision of these quantities shows that in essentially homogeneous material the generation of 1-cracks proceeds at a higher rate and the dispersion of longevity as well as the size effect are larger. Such effects of inhomogeneities are well known /8/.

While inhomogeneities promote the generation of 1-cracks practically they do not affect the growth rate in the quasi-brittle case /2/: the distribution function W_p of propagation times corresponds to homogeneous material /3/ with $\tau_m = \tau_m^*$. As a result it may happen that the rupture process that was limited in a homogeneous sample by generation of 1-cracks in an essentially inhomogeneous body is limited by their propagation. The numerical characteristics of longevity no longer depend on the volume of the sample - the size effect disappears.

Now let us consider a large-scale inhomogeneity with $r_m \gg R_0$. In this case we have to take into account the incompleteness of the spectrum of values of τ_m represented in the specimen. The probability of the spectrum of φ (1) being cut off at its lower side by a value smaller than τ_m is according to /9/

$$F(\tau_m) = 1 - \exp\left[-(\tau_m/\tau_m^*)^\alpha V/v_n\right] , \qquad (6)$$

the normalization condition being

$$\frac{V}{v_n}\int_{\tau_m}^{\infty}\varphi(x)dx = 1. \qquad (7)$$

If $W(\tau,\tau_m)$ denotes the distribution function of longevity of such a specimen, then the distribution function of an arbitrary specimen is

$$W(\tau) = \int_0^\infty W(\tau,\tau_m)dF(\tau_m). \qquad (8)$$

Since $r_m \gg R_0$ we admit that the developement of the main crack takes place inside a single inhomogeneity. If this inhomogeneity is characterized by a m-crack generation

time x then the corresponding distribution function of longevity $W_0(\tau,x)$ is the same as determined earlier /3/ for a homogeneous sample of volume v_n with $\tau_m = x$. Then we evidently obtain

$$W(\tau) = \frac{V}{v_n}\int_0^\infty\int_{\tau_m}^\infty W_0(\tau,x)\varphi(x)dxdF(\tau_m). \qquad (9)$$

From (9) and (6) follows in particular that the incompleteness of the spectrum $\varphi(\tau_m)$ results in an additional size effect of the numerical characteristics of longevity: in the expression for the average value of longevity arises an additional factor $\Gamma(1+\frac{1}{\alpha})(v_n/V)^{1/\alpha}$. This size effect is due to the random distribution of inhomogeneities in the specimen and is similar to the size effect in the classical weakest link model /9/.

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