A Rupture Criterion for Ductile Fatigue Crack Advance

P. Neumann
Argonne National Laboratory, Argonne, Illinois, 60439, U. S. A.
Present address: MPI f. Eisenforschung, 4 Düsseldorf, Germany.

In 1949, Orowan[1] introduced double slip rupture as a possible mechanism for ductile fracture. It has been thoroughly reviewed from a continuum mechanics point of view by McClintock.[2] This mechanism was modified and first applied to explain the ductile fatigue crack advance by the author in 1967,[3] and independently by Pelloux in 1969.[4] Recent experimental work[5,6] verified the geometric features of this mechanism in ductile metals. In this paper it will be tried to close the gap between schematic drawings like fig. 1 on the one hand, and continuum mechanics calculations on the other, by deriving a quantitative rupture criterion, which then may be used in continuum mechanics calculations to describe the boundary conditions for the strain fields at the crack tip.

The sequence of events during the opening of a crack by the double slip mechanism is shown in fig. 1. The elementary process of crack advance used in fig. 1 is shown in fig. 2a: A shear displacement d_1 along a single slip plane 1 through the crack tip moves the tip. The corresponding action on the slip system 2 determines the spacing s_1 between the activated slip lines of the system 1 and vice versa. The result is an average strain on both sides of the crack of $\overline{\varepsilon_i}=d_i/s_i$ (i=1,2) and an angle γ at the crack tip that is equal to the angle α between the slip planes of the two slip systems. We can easily go to the limit of a continuous slip distribution by d_i, $s_i \to 0$ keeping $\overline{\varepsilon_i}$ = const. Then we still have $\gamma = \alpha$.

A more general picture of the elementary slip process at the crack tip is shown in fig. 2b. The only modification as compared to fig. 2a is the finite thickness h_1 of the slip band of shear strain $\varepsilon_1 = d_1/h_1$. Fig.2a is contained in fig.2b as a special case for $h_1=0$.

If this more general elementary slip process is used to advance a crack in the manner of fig.1, the geometry becomes much more complicated than in fig.1. By somewhat lengthy though elementary geometrical considerations, the following result can be obtained: If the elementary process of fig.2b is repeatedly applied in the manner of fig.1, the crack advances as well and the crack tip angle γ asymptotically approaches with increasing number of repetitions a stationary value γ_s given by

$$\gamma_s = \alpha + \sum_{i=1}^{2} \text{arc tg}\left[\frac{\sqrt{(2-c_1)(2-c_2)}}{(1-c_i)\sin\alpha}\sqrt{\frac{\overline{\varepsilon_i}}{\overline{\varepsilon_k}}} - \text{ctg}\,\alpha\right];\ k=1,2 \neq i \qquad (1)$$

with

$$c_i = 1 - \frac{h_i}{s_i} \qquad (2)$$

For $h_i = 0$, i.e. sharp slip lines we have $c_i = 1$ and for $h_i = s_i$, i.e. quasihomogeneous slip we have $c_i = 0$. Thus, the c_i describe the coarseness of slip on the two slip systems and are material constants. For the symmetric case $c_1 = c_2 \overset{\text{def}}{=} c$ and $\overline{\varepsilon_1} = \overline{\varepsilon_2}$ (1) reduces to

$$\gamma_s = \alpha + 2\ \text{arc tg}\left[\frac{2-c}{(1-c)\sin\alpha} - \text{ctg}\,\alpha\right] \qquad (3)$$

$\gamma_s(c)$ is plotted according to (3) in fig.3 for different values of α. γ_s varies considerably with c and does not depend on the values of d_i, h_i, s_i, separately. Thus, if we go to the limit $d_i, h_i, s_i \to 0$ keeping c_i, $\overline{\varepsilon_i}$ constant, γ_s is still given by (1) or (3). This means that even in the limit of infinitely close slip bands along the edges of the crack, the crack tip angle γ_s depends on the coarseness of the slip. The reason for this somewhat surprising result lies in the fact that the slip occurring at any instant of time is highly inhomogeneous also in the limit $d_i, h_i, s_i \to 0$. Essentially there are two Lüders bands on two slip systems propagating with the crack.

As previously stated during the discussion of fig. 2a, the amount of slip d_2 on the slip plane 2 determines the spacings of the slip lines on slip planes 1 and vice versa. This means the $\overline{\varepsilon_i}$ cannot be chosen independently. For $\alpha = 90°$ and $h_i = 0$, it can be seen immediately from fig. 2 that

$$d_1 = s_2 \quad \text{and} \quad d_2 = s_1$$

thus

$$\overline{\varepsilon_1} = \frac{d_1}{s_1} = \frac{s_2}{d_2} = \frac{1}{\overline{\varepsilon_2}} \quad \text{or} \quad \sqrt{\overline{\varepsilon_1}\ \overline{\varepsilon_2}} = 1$$

For the general slip process of fig. 2b, it can be shown for the stationary case $(\gamma = \gamma' = \gamma_s)$

$$\sqrt{\overline{\varepsilon_1}\ \overline{\varepsilon_2}} = \frac{1}{|\sin\alpha|}\ \frac{(3-c_1-c_2)}{\sqrt{(2-c_1)(2-c_2)}}\ . \qquad (4)$$

Since the right-hand side is monotonically decreasing in c_i and $0 \leqslant c_i \leqslant 1$, we have

$$\frac{1}{|\sin\alpha|} \leqslant \sqrt{\overline{\varepsilon_1}\ \overline{\varepsilon_2}} \leqslant \frac{3}{2|\sin\alpha|} \qquad (5)$$

for fcc materials $(\cos\alpha = -\frac{1}{3})$ we get

$$\sqrt{\overline{\varepsilon_1}\ \overline{\varepsilon_2}} = \frac{3}{2\sqrt{2}}\ \frac{(3-c_1-c_2)}{\sqrt{(2-c_1)(2-c_2)}} \quad \text{or} \quad \frac{3}{2\sqrt{2}} \leqslant \sqrt{\overline{\varepsilon_1}\ \overline{\varepsilon_2}} \leqslant \frac{9}{4\sqrt{2}} \qquad (6)$$

c_1, c_2, α are material constants and (4) is the only condition that must be fulfilled to keep the sequence indicated in fig.1 going. We may therefore call (4) the coarse slip rupture criterion. It states that there is a critical value of the geometric mean of the two strains on the two slip systems. This critical value ε_c depends slightly on the coarseness of slip [cf. (5)] and lies always between $3/2\sqrt{2} = 1.06$ and $9/4\sqrt{2} = 1.6$ for fcc metals [cf.(6)]. In the symmetrical case $\overline{\varepsilon_1} = \overline{\varepsilon_2}$ we have $\sqrt{\overline{\varepsilon_1}\overline{\varepsilon_2}} = \overline{\varepsilon_1} = \overline{\varepsilon_2}$ and therefore both the strains $\overline{\varepsilon_i}$ must be equal to ε_c. If one strain is smaller than ε_c, then the other must be correspondingly larger according to (5). Note, however, that it is impossible from (5) for one strain to become zero (true double slip criterion). Furthermore, the strains entering the criterion are the local strains occurring during one cycle at the crack tip. They are not accumulated strains. In this respect the criterion deviates from others proposed for fatigue.

As derived, the criterion is applicable to fatigue as well as uni-directional tension. It has to be kept in mind, however, that for increasing crack advances per cycle the internal stresses produced by the large displacements necessary become very large as long as we do not have a fully plastic specimen. In fatigue, only the displacements during one cycle contribute to the internal stresses since they are annihilated during each compression phase again. This explains the strong dependence of crack advance per cycle on the stress amplitude as long as we do not have fully plastic specimens.

A task remaining for further continuum or dislocation calculations is to determine which applied stress, which plastic zone size, and which energy dissipation is necessary to produce the strains at the crack tip of known angle γ_s required by the criterion.

Summary

Examination of the elementary slip processes occurring during the coarse slip rupture process leads to an expression for the stationary crack tip angle γ_S. It depends even in the limit of infinitely close slip bands on the coarseness of slip. Furthermore, a criterion for the necessary strains at the crack tip is derived. This criterion may be used in continuum or dislocation calculations to define the boundary conditions at the crack tip.

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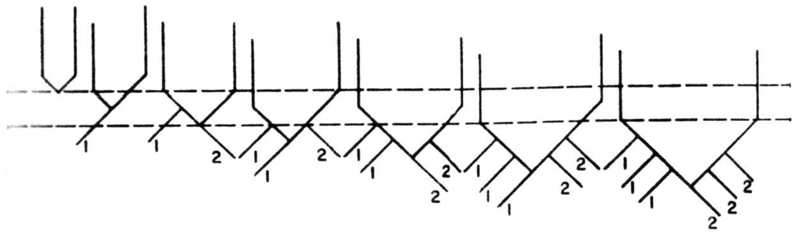

Fig. 1: Sequence of slip processes at the tip of an opening fatigue crack according to the coarse slip model of fatigue.[3]

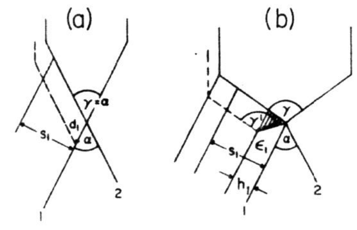

Fig. 2a: Elementary slip process used in fig. 1.

Fig. 2b: More general slip process with finite thickness of slip band.

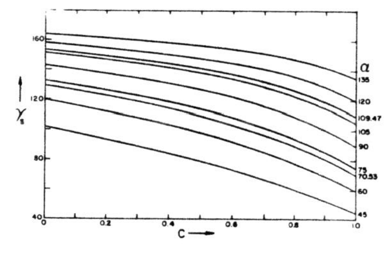

Fig. 3: The stationary crack tip angle γ_S as a function of the coarseness of slip with the angle α between slip planes as parameter. α = 109.47° and 70.53° are the possible slip plane angles in fcc metals.

References

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