The Measurement of G_c under Dynamic Conditions from Crack Propagation Velocity.

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1. - INTRODUCTION

The problem of crack propagation and arrest has been videly investigated in recent years, in particular because of its great importance in applications where fracture initiation is difficult to prevent. The results of these studies have thrown considerable light on the dynamics of propagation as a phenomenon in itself, independently of the characteristics of the material in which it takes place. Relatively little, however, is known about the influence on propagation and arrest of two important factors namely, the loading system and the resistance of the material to propagation, so that the question of which parameter should be taken to characterise the crack propagation resistance of a material, and how it can be measured, has not yet been completely answered.

Some recent studies(1) have pointed out the importance of the loading system in determining both the crack velocity and its length at arrest. However, the assumption was made that the resistance of the material to fracture remained constant during propagation. In order to compare the experimental results with a more realistic theoretical model, in the present publication this assumption is removed, and an attempt is made to find a theoretical relationship between the dynamic G_c and the crack velocity. Furthermore, it is shown it is possible, from the measurement of this velocity, to arrive at the knowledge of the dynamic G_c.

This quantity has already been measured in different ways, by means of strain gauges or load cells, as well as by optical or photoelastic procedures. The present authors consider these different methods to be unsuitable for a correct determination of G_c, the inertial forces in the loading system, as well as in the specimen, being neglected.

2. - DYNAMIC G_c - CRACK VELOCITY RELATIONSHIP

In order to determine the G_c - crack velocity ($\dot{a}$) relationship, we shall refer to the model explained in reference(1) and in Fig. 1. In this model, it was assumed that the specimen-machine (or loading system) complex constituted and adiabatic system, after the unstable onset of propagation. A crack propagation equation was then derived from the principle of energy conservation, making the following assumptions:

1) the machine and the specimen behave as springs, having a concentrated mass M and compliance C_m, and mass $m_s=0$ and compliance $C_s(a)$ respectively.
2) G_c remains constant during propagation, and equal to its initial value.
3) the kinetic energy of the specimen does not contribute to the propagation, even though it is taken into account in the energy balance. In fact, it was assumed that the infinitesimal variation dE_K of this energy was proportional to the fractured surface area $dS = Bda$, with a proportionality constant γ_K, a "kinetic energy per unit of surface by the specimen; $dE_K = B\gamma_K da$.

Removing the assumption (2) and assuming $G_c = G_c(a)$ and $\gamma_K = \gamma_K(a)$, which are some functions, at the moment unknown, of crack length a, the dynamic energy balance becomes

$$Fde = dU_s + BG_c(a)da + B\gamma_K(a)\, da \quad (1)$$

Here Fde is the external work, dU_s the strain energy of the specimen, and $BG_c(a)da$ the dynamic fracture work. If

$$U_m = \frac{1}{2C_m}(e_{to} - e_s)^2, \quad E_{cm} = \tfrac{1}{2} M\dot{e}_m^2 = \tfrac{1}{2} M\dot{e}_s^2$$

are the strain and kinetic energies, respectively, of the machine, and $U_s = \frac{1}{2}F^2C_s = \frac{1}{2}(2B(G_c + \gamma_K + \gamma_p))\, C_s/s$ the strain energy of the specimen, then, since $Fde = -(U_m + E_{cm})$ (1) becomes

$$-\frac{e_{to}-e_s}{C_m}\, de_s + M\ddot{e}_s de_s + B\left[\Gamma_c - \gamma_p\right] d\frac{C_s}{s} + B\frac{C_s}{s}\, d\Gamma_c + B\Gamma_c da = 0 \quad (1')$$

where $\Gamma_c = G_c + \gamma_K = \gamma_s + \gamma_p + \gamma_K$

Assuming $\gamma_p \simeq 0$, since

$$de_s = d(FC_s) = (2B\Gamma_c)^{\frac{1}{2}} \frac{d}{da}(C_s s^{-\frac{1}{2}})da + (2B)^{\frac{1}{2}} s^{-\frac{1}{2}} C_s \frac{d}{da}(\Gamma_c)^{\frac{1}{2}} da \quad (2)$$

it can be shown that the last three terms in (1') are equal to $Fde_s = \frac{e_s}{C_m} de_s$. Equation (1') then gives

$$M\ddot{e}_s + \left(\frac{1}{C_m} + \frac{1}{C_s}\right) e_s = \frac{e_{to}}{C_m} \quad (3)$$

To determine the crack propagation equation from equation (3), let us write (2) as

$$de_s = \Theta(a)\, da \quad (4)$$

where the important function $\Theta(a)$ is expressed through the

differential equation

$$\frac{d}{da}\Gamma_c^{\frac{1}{2}}(2B)^{\frac{1}{2}} s^{-\frac{1}{2}} C_s + (2B\Gamma_c)^{\frac{1}{2}} \frac{d}{da}(C_s s^{-\frac{1}{2}}) = \Theta(a) \quad (5)$$

With $\ddot{e}_s$ evaluated by means of equation (4) and substituting in (3), the equation which is sought is found in terms of a, $\dot{a}$ and $\ddot{a}$ alone:

$$M\Theta(a)\ddot{a} + M\frac{d\Theta}{da}\dot{a}^2 + \left(1+\frac{C_s}{C_m}\right)\left[2B\Gamma_c\right]^{\frac{1}{2}} s^{-\frac{1}{2}} - \left(1+\frac{C_{so}}{C_m}\right)(2B\Gamma_c)^{\frac{1}{2}} s_o^{-\frac{1}{2}} \quad (6)$$

With the auxiliary variables $Y = (2B\Gamma_c)^{\frac{1}{2}}$, $\dot{a}^2 = 2\,z(a)$, $\ddot{a} = \frac{dz}{da}$, (5) and (6) can be written

$$\frac{dY}{da} = \frac{\Theta(a) - \chi(a)\,Y}{s^{-\frac{1}{2}}\,C_s} \quad (7)$$

$$\frac{d\Theta}{da} = \frac{1}{2Mz(a)}\left[\left(1+\frac{C_{so}}{C_m}\right) Y_o\, s_o^{-\frac{1}{2}} - \left(1+\frac{C_s}{C_m}\right) Y\, s^{-\frac{1}{2}} - M\frac{dz}{da}\,\Theta\right]$$

Once the geometry of the specimen and the characteristics of the machine are known, i.e. given the function C_s, s, $\chi(a)$ and C_m, together with the initial conditions $Y_o = Y(a_o)$ and $\Theta_o = \Theta(a_o)$, the experimental evaluation of the function z(a) and $\frac{dz}{da}$ allows the system of differential equations to be integrated. Thus all that is needed to determine the function $Y = (2B(G_c + \gamma_K))^{\frac{1}{2}}$ is the behaviour of crack velocity $\dot{a}$ vs. crack length a.

Figs 1 and 2 show the load-extention diagram for a DCB specimen and the variation of $\dot{a}$ and Γ_c vs. a.

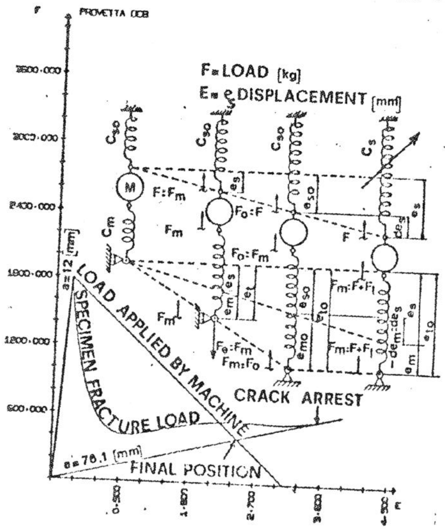

FIG. 1 - Machine-specimen system scheme and load-extention diagram for the case of Fig. 2.

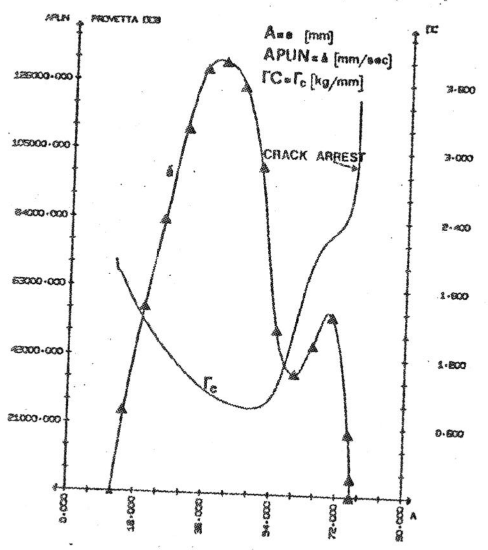

FIG. 2 - Γ_c determination from measured crack velocity $\dot{a}$.

- CONCLUSIONS

The model described leads to a relationship between the speed of a propagation crack and the resistance to fracture propagation Γ_c, based on certain assumptions. The most important of these is that the kinetic energy of the specimen does not contribute to propagation. This assumption implies that, for crack propagation to occur, energy must be supplied by the machine, and this could be verified in high-toughness steels, in which crack propagation speed is not very high.

- REFERENCES

1. - MIRABILE M., S. VENZI - "Theoretical and experimental predictions of influence of the machine and initial crack lenght on fracture propagation and arrest" - First Inter. Conf. on Crack Propagation - Lehigh Univ. Bethleem Pa. 1972.

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- NOTATION

a	Crack length
a_o	Initial crack length
a_f	Final crack length
$\dot{a}$	Crack speed
$\ddot{a}$	Crack acceleration
B	Specimen thickness
C_m	Test machine compliance
$C_s(a)$	Specimen compliance as function of a
C_{so}	Initial specimen compliance
E	Elastic modulus
E_{cm}	Kinetic energy of the machine
E_K	Kinetic energy of the specimen
e	Total specimen deformation
e_s	Elastic part of e
e_p	Plastic part of e
e_{pf}	Final value of e_p
e_{to}	Total specimen + machine deformation
F	Load applied to the specimen
G_c	Plastic + surface energy for a unit surface
h	Beam height in a DCB specimen
Γ_c	Elastic + plastic + kinetic energy for a unit surface
$\gamma_s/2$	Surface energy
$\gamma_p/2$	Plastic energy for a unit surface
$\gamma_k/2$	Kinetic energy released by the unit surface of the crack by the specimen
M	Machine mass
s	$\frac{dC_s}{da}$
s_o	Initial value of s
U_m	Strain energy of the machine
U_s	Strain energy of the specimen
W	Fracturing work
$x(a)$	$\frac{d}{da}(C_s s^{-\frac{1}{2}})$
$\xi(a)$	$F_s + C_s \frac{dF}{da} + \frac{\gamma_p}{F}$
ω^2	$\frac{1}{M}(\frac{1}{C_a} + \frac{1}{C_m})$
ω_o^2	$\frac{1}{C_m M}$