Strain and Stress Distribution in Fully Plastic Notched Bars and the Criterion for Ductile Rupture Initiation

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I. Introduction

Crack initiation generally takes place in the mid-section of a sharply notched bar subjected to tensile load or bending moment. This is mainly due to the triaxial state of stress that exists in the plane-strain region at the mid-section.(1) In order to study the mechanics of crack initiation, accurate determination of the stresses and strains ahead of the notch is required. However, there is not a general technique that is available to obtain both the local stress and strain distributions, especially in fully plastic specimens that have been loaded beyond general yield.

Slip line theory analysis has been extensively used to predict the local stresses.(2) However, the assumption of perfectly plastic material limits the application of this method to cases where the strains ahead of the notch are small. On the other hand, strains along the notch center line are usually obtained by measuring the microhardness.(3) The disadvantage of this technique is that the individual components of strains cannot be evaluated away from the notch center line. The finite element technique recently used by Griffiths and Owen(4) provides a complete solution of a notched bar in plane-strain bending. However, the solution was given only up to the general yield load and the analysis was based on a linear work-hardening rate, which rarely occurs.

It is the main objective of this investigation to apply the technique of "visioplasticity" to determine accurate stress and strain distributions near the root of notched bars loaded in fully plastic plane-strain bending. The visioplasticity technique allows for the actual work-hardening of the material and triaxialty. The results are then used to develop macroscopic fracture criteria in terms of microscopic ductility.

II. Determination of strain and stress distribution in fully plastic notched bars

The technique of visioplasticity has been extensively used in the analysis of plane-strain plasticity problems associated with metal forming operations,(5) and is easily adopted to the analysis of notched bars in fully plastic bending,

Half thickness Charpy bars (0.20" thick) of low carbon steel were prepared. The notch root radius ρ of these bars is 0.010, 0.005 and 0.002 in.; all other dimensions are identical to the standard Charpy V specimen. Each specimen consists of two similar notched bars bonded together along their faces. In order to help avoid separation of the two bars during loading, the specimens were also bolted together at points removed from the notch.

The matched faces of the bars were carefully polished before a square mesh of lines was inscribed using a microhardness indentor equipped with a stage. In this way, fine grid lines were drawn at intervals of 0.006, 0.004 and 0.002 in. for the different root radii, respectively. Eastman 910 adhesive was used to glue the two bars together. The specimens were deformed in a slow three-point bending test to an angle of bend $\theta = 1.5\text{-}3.0^{\circ}$ before the load was released. Magnified photographs of the grid lines before and after deformation were taken (Figure 1). These incremental deformations were repeated to obtain the stress and strain distributions at higher angles of bend. All the experiments were carried out at room temperature.

The coordinates of the grid points were registered automatically on computer cards, by the use of a scanning machine equipped with a punching machine. The displacement components in X and Y directions were determined from the positions of the nodal points before and after each incremental deformation. The incremental strains were calculated from the incremental strain/displacement relationships.

The total strain components at different angles of bend were evaluated from the integration of the respective incremental strains along the individual flow line of each point. By considering the equilibrium equations, plasticity conditions, and the flow properties of the material, the instantaneous stress components were determined.

An extensive study on stress and strain distributions has been presented elsewhere,(6) and only a summary of pertinent results is presented here. The results of such experiments on low carbon steel, A533B steel, and 4340 steel showed that variations in the work-hardening rate and the yield strength of the material have a negligible effect on the strain distribution around the notch root at a given angle of bend. The root radius, however, does affect the strain distribution. Figure 2 shows the variation of the maximum longitudinal strain ϵ_{yy} at the notch root with the plastic angle of bend θ for different root radii.

III. Development of a criterion for fracture initiation

In order to derive a criterion for fracture initiation, four steel specimens with different yield strengths and work-hardening rates were used. Three of the specimens were made from 4340 steel which was quenched and tempered to give yield strengths of 160, 132 and 104 Ksi. The fourth specimen was A533B steel of 60 Ksi yield strength. Several standard Charpy specimens having 0.010, 0.005 and 0.002 in. notch root radii were prepared for each of the four yield strength levels.

The angle of bend at which fracture initiated θ_i was determined in the following way.(7) Four specimens of the same root radius and yield strength were deformed beyond general yield to different angles of bend in slow bending tests. These angles of bend were chosen such that a small crack is produced at the notch tip at room temperature. After the slow bend test had been performed, the specimen was broken in impact at -196°C. In this way, the fracture surface of the specimen was composed of two separate areas. The area close to the notch root is composed of ductile rupture while the rest of the specimen contains cleavage. By the use of the scanning electron microscope the size of the ductile region Δ could be measured. From plots of Δ versus θ, Figure 3, the initiation angle of fracture θ_i was determined. Initiation was considered to occur when the size of the ductile region was 0.002 in. The experiments were conducted on specimens of all four strength levels and all three root radii.

Figure 2 was used to evaluate the maximum longitudinal strain at fracture initiation ϵ_i, from measurements of θ_i. Figure 4 shows a plot of ϵ_i versus ρ for steels having 160, 132, 104 and 60 Ksi yield strength.

The normal longitudinal stress σ_{yy} distribution along the notch center line for different root radii at the initiation of fracture is shown in Figure 5, for σ_y = 104 Ksi.

IV. Discussion

It can be clearly observed from Figure 4 that the maximum longitudinal strain at fracture initiation ε_i is independent of the notch geometry. However, the maximum local stress σ_{yy}^{max} is dependent on the notch root radius. For steel having 104 Ksi yield strength, the normal longitudinal stress reached its maximum at distances of 0.010, 0.012, and 0.018 in. from the notch root for root radii of 0.002, 0.005 and 0.010 in. respectively. The maximum values of the stress are 249, 225 and 222 Ksi respectively. It was also noted that σ_{yy}^{max} increased with increasing yield strength level.

Figure 4 indicates that ε_i is independent of root radius ρ. Also, ε_i decreases with increasing yield strength, as does fracture toughness K_{Ic}. Alternatively σ_{yy}^{max} is a function of ρ (Figure 5). Futhermore, σ_{yy}^{max} increases as K_{Ic} decreases. These facts suggest that in these materials, where fracture initiates by rupture rather than cleavage, fracture initiation is controlled by a critical local strain ε_i rather than a critical local tensile stress σ_{yy}^{max}. Figure 6 shows the variation of the Charpy V shelf energy C_v with ε_i. The increase in ε_i is probably responsible for the increase in shelf energy toughness as yield strength is decreased.

In summary, the visioplasticity method allows a determination of the local stress and local strain distributions in sharply notched bars loaded in fully plastic bending. These distributions may then be used to determine the local plastic strains at fracture initiation ε_i which correlate well with measurements of Charpy energy (or fracture toughness). Further studies are being conducted to derive expressions for K_{Ic} in terms of ε_i for a variety of materials.

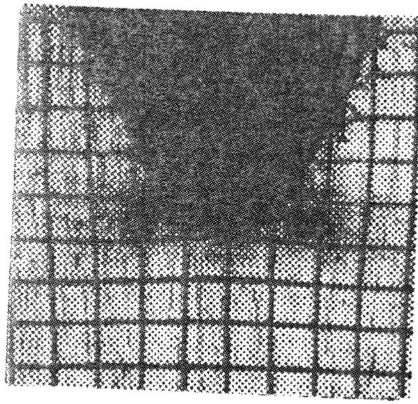

Figure 1. Distorted Grid of 0.006 in Original Spacing at Angle of Bend $\theta = 1.2°$

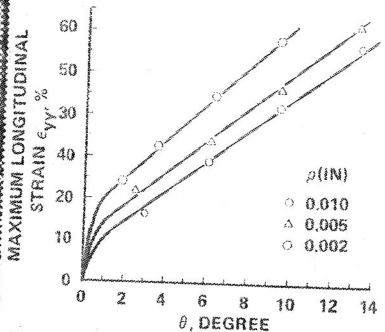

Figure 2. Variation of the Longitudinal Strain at the Notch Root with Angle of Bend for Different Root Radii

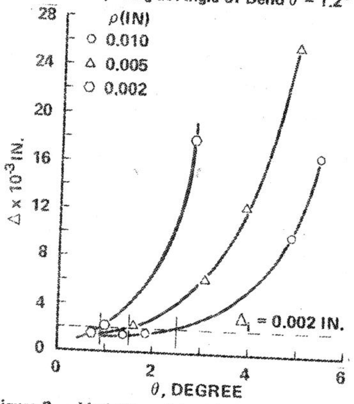

Figure 3. Variation of the Size of the Ductile Region with Angle of Bend for Different Root Radii, for a Steel Having a Yield Strength of 104 Ksi

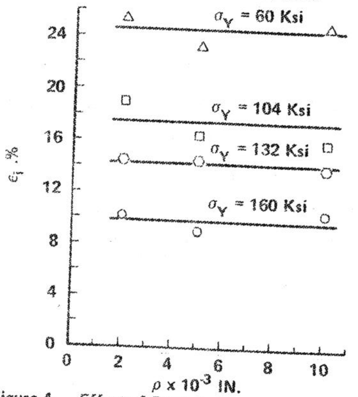

Figure 4. Effect of Root Radius ρ on the Local Strain ϵ_i Required for Fracture Initiation, for different steels

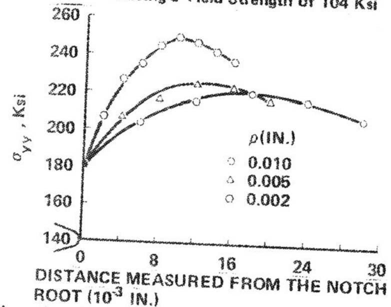

Figure 5. Distribution of the Normal Longitudinal Stress Component σ_{yy} Along the Notch Center Line for Different Root Radii at Fracture Initiation, for a Steel Having a Yield Strength of 104 Ksi

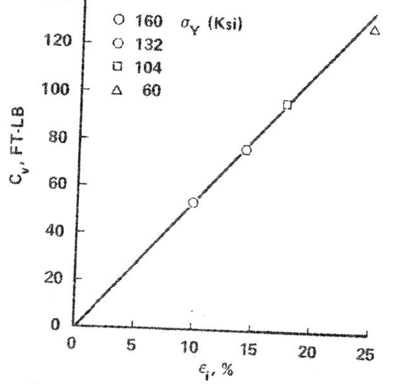

Figure 6. Effect of ϵ_i on the Charpy V Notch Shelf Energy C_v

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