The analysis of fatigue test results for butt welds with lack of penetration defects using a fracture mechanics approach

J. D. HARRISON
The Welding Institute, Abington, Cambridge, England.

Summary
Published fatigue test results for butt welds, incorporating lack of penetration defects, have been analysed on a crack propagation basis. An approach based on linear fracture mechanics has been used and the published results agree with the theory. Proposals for using the theory in the form of curves linking allowable defect size, material thickness, service stress and required endurance are made.

Introduction

For some years now, The Welding Institute has been carrying out an extensive research programme into the effect of weld defects on the fatigue strength of butt welds. The object of this programme has been to derive a logical basis for an acceptance standard using a fitness for purpose approach as opposed to a good workmanship approach.

One type of defect studied as part of this programme at The Welding Institute is lack of penetration at mid thickness in aluminium alloy butt welds. The same type of defect in steel has been investigated elsewhere.

In the past, it has been suggested that a satisfactory correlation exists between the percentage reduction in fatigue strength and the percentage loss of area caused by a lack of penetration defect. While this may be true for laboratory specimens, it is dangerous to extrapolate the relationship to large butt welds. To take an extreme example, a defect extending through 90% of the thickness over a 12 in length of a circumferential weld in a 30 ft diameter pressure vessel represents less than 1% of the area of that weld. There is, however, no doubt that such a defect would seriously reduce the fatigue strength of the vessel. It seems likely that both the absolute size of the defect and its size relative to the dimensions of the weld are significant factors.

The science of fracture mechanics has involved a study of the stresses in the vicinity of cracks. Since lack of penetration is similar in nature to a crack, the number of cycles to initiate a fatigue crack from such a defect will be small in relation to the overall life. The rate of propagation of a fatigue crack from the initial defect will depend on the stresses near its tip, so that an approach based on fracture mechanics principles offered some hope of producing a more universal answer. In the present report, such an approach has been used to analyse the results published in the literature and the degree of success achieved leads one to expect that a similar approach to the problem of cracks themselves would yield useful results.

The nomencleture is shown in Fig. 1.

Published information on the effect of lack of penetration defects on fatigue performance

Aluminium alloy

Dinsdale and Young [1] reported an extensive investigation of the effect of lack of penetration in NP5/6 material. The initial defect size ($2a_i$) through the thickness ranged from 0·030 in to 0·350 in and the material thickness at the plane of the defect $2t$, from 0·17 in to 0·73 in. In all cases the defect was continuous through the width of the specimen. Fatigue strengths varied. They were not correlated with defect size so that the results could be used only if a particular geometry examined approximated with one found in service. Dinsdale and Young [2] have also investigated the effect of double operator defects in NP5/6 and pure aluminium. This defect is similar in nature to lack of penetration and the results have been used in the present investigation.

The Dutch delegation to Commission XIII of the IIW [3] have reported tests on welds in an aluminium magnesium alloy with lack of penetration defects having initial sizes of $2a_i = 0{\cdot}02$ and 0·12 inch in material approximately 0·5 in thick.

Steel

Guyot *et al.* [4, 5] reported an investigation of lack of penetration defects in 0·8 in and 0·4 in thick mild steel. Some 150 specimens were tested with defects whose dimension $2a_i$, measured through the thickness, ranged from 0·25 in to 0·03 in and whose initial length $2b_i$, varied from a defect continuous across the width of the specimen, i.e. $2b_i = W$ to one 0·12 in long. A correlation between defect length and fatigue strength was attempted by these authors, but it should be noted that in this work the longer defects also tended to be larger in the through thickness dimension. The correlation found is perhaps fortuitous. In the second series [5] specimens with two defects and with defects near the surface were also tested.

Wilson *et al.* [6] tested thicker specimens (0·875 in to 1·05 in) with defects having much larger values of $2a_i$ (0·37 in to 0·55 in). In this case the defects were continuous. These authors showed that, as might be expected, a lack of penetration defect in a weld running parallel to the direction of stressing did not lower the fatigue strength of the joint. Newman and Dawes [7] reported two series of tests in which the defects had the same values of $2a_i$ but different lengths. In this work the material thickness was 0·5 in. Correlation between defect area and

fatigue strength was attempted, but this approach suffers from the drawback indicated in the introduction. Warren [8] used only one defect size in 0·5 in thick mild steel. Reggiori and Erra [9] tested two series of specimens with defects of approximately the same through thickness dimension but with lengths $2b_i = W$ and $W/2$.

Finally the Italian delegation to Commission XIII of the IIW [10] have reported an investigation in which the defect was formed by causing a brittle fracture in the root run of the weld and then completing the weld with the fractured surfaces in contact. Because the defect surfaces were so closely matching, the defect would have been more cracklike than those used by other investigators. The material thickness was 0·4 in and the defect was continuous with $2a_i$ approximately equal to 0·1 in.

Determination of fatigue life using a fracture mechanics analysis of crack propagation

Defects continuous through the length of the weld ($2b = W$)

A lack of penetration defect only differs from a crack in that the radius at the tip is greater. The important consideration is the number of cycles necessary to propagate a crack from the original defect size to failure.

Assuming that the propagation rate of a crack at any given stage will depend solely on the cyclic strain range at the crack tip at that stage, if the material were perfectly elastic this strain range would be infinite, but inevitably yielding occurs. This leads to a local redistribution of stress and the strain range which occurs at the crack tip is limited by the surrounding elastic material. If the size of the plastic zone is small in relation to the specimen or structure, the concepts of linear fracture mechanics may be used to describe the elastic stress field surrounding the plastic zone. The stress at any given point in this field is proportional to K, the stress intensity factor, a function of nominal applied stress and defect size. Therefore, under fatigue loading conditions, the strain range at the crack tip is proportional to ΔK, the range of stress intensity factor.

As stated above, the linear fracture mechanics concept can only be used if the plastic zone size is relatively small, so it is not normally applicable to low strength ductile materials such as mild steel and aluminium alloys. But its use in the analysis of high cycle fatigue behaviour in such materials is justified by the fact that the stresses are very much lower than those which have to be applied in normal fracture toughness testing. In the later stages of fatigue crack growth, the plastic zone enlarges and the use of linear fracture mechanics then becomes suspect; but by this time the crack is propagating relatively rapidly so that major errors in the final answer are eliminated.

The dependence of crack propagation rate on the range of K can be written as:

$$\frac{da}{dN} = f(\Delta K)$$

Paris and Erdogan [11], in an analysis of a volume of published crack propagation results, suggested that

$$\frac{da}{dN} = B(\Delta K)^4 \qquad (1)$$

gives the best agreement with all the available results, where B = constant.

This relationship has been further confirmed experimentally by Paris [12].

For a crack in which b and t are both large with respect to a, it can be shown that

$$K = \sigma(\pi a)^{1/2} \qquad (2)$$

Substituting (2) in (1) we get

$$\frac{da}{dN} = B(\Delta\sigma)^4 (\pi a)^2 \qquad (3)$$

However, in most cases of lack of penetration defects, t is not large with respect to a and in these circumstances it is necessary to apply a correlation factor. The so called tangent formula, although not exact, is suitable in the present case. This gives, in place of (2),

$$K = \sigma(\pi a)^{1/2}\left(\frac{2t}{\pi a}\tan\frac{\pi a}{2t}\right)^{1/2} \qquad (4)$$

Substituting (4) in (1) we find that

$$\frac{da}{dN} = B(\Delta\sigma)^4\left(2t \tan\frac{\pi a}{2t}\right)^2$$

For reasons which will become apparent later, this equation will be rewritten as

$$\frac{da}{dN} = C\left(\frac{\Delta\sigma}{E}\right)^4\left(2t \tan\frac{\pi a}{2t}\right)^2 \qquad (5)$$

where C, a new constant = $B.E^4$, E being Young's Modulus.

Integrating (5), the number of cycles required to propagate the crack from a size $2a_1$ to $2a_2$ is given by

$$2\pi Ct\left(\frac{\Delta\sigma}{E}\right)^4 N = \cot\left(\frac{\pi a_1}{2t}\right) - \cot\left(\frac{\pi a_2}{2t}\right) - \frac{\pi}{2t}(a_2 - a_1) \qquad (6)$$

If $2a_1 = 2a_i$, the initial defect size, and $2a_2 = 2a_{cr}$, the critical crack size at which failure occurs, then (6) gives N, the endurance to failure.

Because, when a is large the crack will be propagating rapidly, the value of N determined from (5) is not sensitive to assumptions made in defining a_{cr}.

For example, in a real structure, $2a_{cr}$ might be taken to be the critical crack width for brittle fracture, if this mode of failure was likely, or it might be taken to be the material thickness in a pressure vessel since leakage would occur.

For the purpose of analysing experimental results in small specimens, it will be sufficient to assume that a_{cr} is reached when the stress on the remaining net section is equal to σ_u the UTS of the material,

$$\text{i.e.} \quad a_{cr} = \frac{t(\sigma_u - \sigma)}{\sigma_u}$$

Equation (6) can be rewritten

$$\left(\frac{\Delta\sigma}{EX^{1/4}}\right)^4 . N = \frac{1}{\pi C} \qquad (7)$$

where X is a parameter dependent solely on the geometries of the defect and specimen or structure, and

$$X = \frac{1}{2t}\left[\cot\left(\frac{\pi a_i}{2t}\right) - \cot\left(\frac{\pi a_{cr}}{2t}\right) - \frac{\pi}{2t}(a_{cr} - a_i)\right] \qquad (8)$$

If (7) is correct, we should be able to predict the life of the defective joint, knowing the constant amplitude service loading and the defect size and material thickness. To check the validity of (7), the results from references 1, 2 and 3 have been plotted in Fig. 2 in the form

$$\log\left(\frac{\Delta\sigma}{EX^{1/4}}\right) \text{ against } \log N.$$

If (7) is correct, a straight line relationship of slope = $-\frac{1}{4}$ should be obtained. The lines showing the extremities of the scatter band in this figure are drawn with this slope.

Although the scatter is considerable, it is not excessive in view of the number of results used, the variety of material thickness and defect size combinations and the inevitable variations in initial root radius for this type of defect. It will also be seen that the upper edge of the scatter band is fixed by four high results and that if these were ignored the scatter band would be considerably narrower. It may be noted that the assumed slope of $-\frac{1}{4}$ agrees well with the results.

Effect of defect length ($2b<W$)

So far it has been assumed that the value of $2a$ (defect width) is small with respect to $2b$ (defect length), but this assumption is not always valid. It is justifiable to assume that $2b$ is never less than $2a$ and lack of penetration defects are therefore never elongated through the thickness.

Irwin [13] has shown that the effect of having a defect length to width ratio which is not large is to divide the stress intensity factor, K by ϕ, the complete elliptic integral,

$$\text{where } \phi = \int_0^{\pi/2} \left[1 - \left(\frac{b^2 - a^2}{b^2}\right)\sin^2\theta\right]^{1/2} d\theta \qquad (9)$$

Since the crack will propagate at different rates from the ends of both the major and minor axes of the defect, ϕ will generally change throughout the life. There are two instances where it will remain constant – where $2b$ is large with respect to $2a$ and $2t$. In this case $\phi = 1$. The other is the case of a circular defect where $2a = 2b$, This type of defect will grow uniformly in all directions and here $\phi = \pi/2$ throughout the life. (There will be a tendency to grow faster through the thickness as the crack approaches the surface; but this will only affect the later stages of propagation). For initial defect shapes between these extremes, ϕ will change, but always $1 \leqslant \phi \leqslant \pi/2$. Clearly, one way of dealing with the problem is to assume that all defects are long and this will always give a conservative answer. The maximum error introduced into equation (7) by this assumption means that for a given value of $2a_i$ and a given stress range, the life of a circular crack is underestimated by a factor of 6·1, or alternatively, for a given value of $2a_i$ and a given endurance, the allowable stress is underestimated by a factor of about 1·6. This could be satisfactory but it would mean that some defects would be rejected unnecessarily. Many of the results published in the literature are for defects where $2b$ is not large with respect to $2a$ and for the purpose of comparing these results with the theory, it is necessary to investigate the effect of defect length further.

Irwin [13] has shown that the stress intensity factor at any point on the circumference of an elliptical defect is given by:

$$K = \frac{\sigma(\pi a)^{1/2}}{\phi}\left(\sin^2\beta + \frac{a^2}{b^2}\cos^2\beta\right)^{1/4} \qquad (10)$$

where ϕ is given by (9) .

Two points of particular interest on the circumference of the defect are the ends of the minor and major axes where:

$$K = \frac{\sigma}{\phi}(\pi a)^{1/2} \qquad (11)$$

and

$$K = \frac{\sigma}{\phi}(\pi a)^{1/2}\left(\frac{a}{b}\right)^{1/2} \qquad (12)$$

Equation (3) can now be rewritten for crack propagation at these two positions

$$\frac{da}{dN} = C\left(\frac{\Delta\sigma}{E\phi}\right)^4 (\pi a)^2 \qquad (13)$$

and

$$\frac{db}{dN} = C\left(\frac{\Delta\sigma}{E\phi}\right)^4 (\pi a)^2 \left(\frac{a}{b}\right)^2 \qquad (14)$$

From (13) and (14) it is clear that, for an elongated defect, the crack will propagate faster in the minor axis direction than in the major axis direction. This is clearly illustrated in Figs. 3 and 4.

Equations (9), (13) and (14) are simultaneous equations in ϕ, a and b but are not capable of an exact solution. An iterative solution is possible, in which the value of ϕ is calculated at various stages during crack growth. Use is made of the fact determined from equations (13) and (14) that:

$$a^2 da = b^2 db$$

$$\text{i.e.} \qquad a^3 - a_i^3 = b^3 - b_i^3 \qquad (15)$$

The value of ϕ is calculated at the end of each of a number of small steps in crack propagation. The mean of these values, ϕ_m, is then calculated for the complete endurance. ϕ_m is dependent on the ratio of initial crack length to crack width, $2a_i/2b_i$, and on the ratio of initial crack width to material thickness, $2a_i/2t$. The dependence of ϕ_m on these two ratios is shown in Fig. 5.

Equation (7) can now be modified to take account of defect length and becomes:

$$\left(\frac{\Delta\sigma}{\phi_m E X^{3/4}}\right)^4 . N = \frac{1}{\pi C} \qquad (16)$$

The results from references [4, 5, 6, 7, 8, 9 and 10] are plotted on this basis in Fig. 6. Because there were too many results from [9] to include in the figure the mean results and the extents of the scatter bands have been shown. Although the scatter for all the results plotted is considerable, it is encouraging that it is not greater than the scatter from this one investigation [9] using one defect geometry and one material thickness. Excluding the results from [9] there are some 180 results in Fig. 6 and including the results from [9] which all lie within the overall scatter band, the total is approximately 350. The thicknesses cover a range of from 0·4 in to 1·05 in and the initial defect size range is from 0·03 in to 0·55 in. The results shown agree with the theory.

It has been shown by Gross [14] that, for a given total strain range per cycle, the life in low cycle fatigue is the same for a considerable variety of materials. Although we are dealing with high cycle fatigue, the strain range which occurs at the crack tip will be considerable and there seems no reason why the independence of material properties should not be found. The parameter used has been K/E and not K and it is gratifying to find that when the scatter bands for aluminium alloys from Fig. 2 are superimposed on Fig. 6 which shows results for steels, there is close agreement. While writing this report, the author's attention was drawn to an aritcle by Pearson [15] who found that the same equation $(da/dN) = f(K/E)$ can be applied to a variety of materials.

The two extremities of the scatter band in Fig. 6 are such that the constant C in Equation (16) is $8{\cdot}0 \times 10^7$ for the lower boundary and $6{\cdot}12 \times 10^5$ for the upper boundary. Pearson obtained $3{\cdot}43 \times 10^7$ for this constant. This lies within the scatter of the present investigation but is close to the lower boundary. This is reasonable since Pearson was dealing with fatigue cracks throughout, whereas in the results used here the crack started with a variety of root radii only the sharpest of which would approach the sharpness of a fatigue crack.

Multiple defects

Multiple lack of penetration defects separated by regions of sound material are not common. It would be conservative to assume that such defects are continuous between the outer extremities of the complete group; but in fact such defects have to be close together for any interaction to take place between them. The tangent formula equation (4) can be applied to an array of defects of length $2b$ separated by sound material of length d, (Fig. 7). Using this formula it can be shown that if $d > 2;5\ b$, interaction will raise the value of K by less than 10%. To ensure that interaction is below this level throughout the life, the value of b used here should be that at failure. A good approximation for the value of b at failure can be found by putting $a = t$ in (15). It is

therefore possible to say that if d_i, the initial length of sound material separating the defects exceeds $4{\cdot}5\ (b_i^3 - a_i^3 + t^3)^{1/3} - 2b_i$, interaction effects will be negligible. The values of b_i and a_i used in this should be those for the largest defect. If d_i is greater than this value, the assessment can be based on the dimensions of the largest defect. Where this is not so, rather than attempting to calculate what the interaction is, it is satisfactory to assume that the defect length is the overall distance between the outside extremities of defects not separated by such a length of sound material.

The second series of tests reported by Guyot *et al.* [5] include six tests of specimens with two separate defects in each specimen. The above interaction criterion was applied to these and it was found that in only two of them would interaction occur. The results for these two specimens are plotted in Fig. 6 on the assumption that the defect length was the overall length from the outside extremities of the two defects. The results for the other four specimens are also plotted in Fig. 6, but in this case are based on the more severe of the two defects in each specimen.

Photographs of fracture surfaces in [5] show that where the defects are close together, the fatigue cracks have run into each other, whereas with the defects further apart, each has propagated separately.

Application in practice

If the validity of the approach presented here is accepted [16] becomes a most useful equation. It enables one to define an acceptance limit for lack of penetration defects for given service conditions. Equally, for those structures where it is decided that no such defects can be accepted, it permits one to specify the sensitivity required of the non-destructive test selected.

One of the points arising from this investigation is that the length of the defect is considerably less important than its through thickness dimension. Unfortunately, radiography is insensitive to this latter measurement though ultrasonic testing offers some possibilities in this direction. In the absence of any direct means of measuring the through thickness dimension, it would not be unreasonable to assume that this was no greater than the original root face.

There are several ways in which the information from equation (7)· can be presented. One given here is that in which the maximum defect size is plotted against thickness for a variety of stresses and lives, (Figs. 8 and 9). The curves plotted here are based on the lower limit of the scatter band in Fig. 2. When making the calculations for these curves, a_{cr} was taken to be equal to t. If the stress and/or life is unknown, it is still possible to use these curves by saying that the fatigue strength of the defective butt weld must not be less than that of other critical details in the structure. For example, if there are fillet welds (for which a typical fatigue strength at 2×10^6 cycles is 5½ tons/in²), subjected to the same stress as the stress on the butt weld under consideration the use of the curve appropriate to 7·5 tons/in² for steel and 2×10^6 cycles will be conservative and ensure that failure does not take place from that butt weld. This will hold for endurances other than 2×10^6 cycles because the *S-N* curves for fillet welds and defective butt welds are approximately parallel.

The curves presented assume that the defects are continuous. Account may be taken of the length of the defect by dividing the stress by the appropriate value of ϕ_m determined from Fig. 5.

A simple way of taking account of interaction for multiple defects has been derived as follows. In the paragraph on Multiple Defects above it was seen that interaction would not take place if

$$d_i > 4{\cdot}5\,(b_i^3 - a_i^3 + t^3)^{1/3} - 2b_i$$

This can be simplified and the result will be conservative if a_i^3 is assumed to be negligible compared with $b_i^3 + t^3$. If the inequality is divided by t we get for no interaction that

$$\frac{d_i}{t} > 4{\cdot}5\left(\frac{b_i^3}{t^3} + 1\right)^{1/3} - 2\,\frac{b_i}{t}$$

This is presented in graphical form in Fig. 10. For points lying above the curve, interaction can be ignored. For points below the line, the defect should be taken to extend from the outer extremities of any defects not separated by a sufficient length of sound material.

Conclusions

A fracture mechanics approach has been used to calculate the expected rate of crack propagation from a central lack of penetration defect. All the experimental results published in the literature agree with the theory within broad scatter bands, but the scatter bands are no wider than those obtained in one investigation for a single geometry. The width of the scatter band probably results from the inevitable variations in initial radii at the defect tips. Agreement is obtained between the results for steels and aluminium alloys if the parameter used is the strain intensity factor K/E. Curves are presented showing how the results of the investigation could be put to practical use.

Although it is not strictly correct to use linear fracture mechanics for the later stages of fatigue crack growth, the approach is justified by the good agreement found between theory and experimental results.

As yet, the work only covers central lack of penetration defects. Guyot *et al.* [5] published results for defects which break the surface,

i.e. defects which might arise with single *V* or single *U* preparations. There is no reason why a similar argument should not be developed for such defects.

The experimental work supporting the theory was based on lack of penetration defects, but the theory was developed for cracks. The use of the lower limit of the scatter band should mean that the results can be applied with caution to pre-existing cracks, but their use for analysing the effect of other less deleterious defects such as slag inclusions will be conservative.

No direct measurement of crack propagation has been possible so that the experimental results supporting the theory have had to be based on integration. It would be more satisfactory if crack propagation measurements could be made on specimens of the type considered.

In applying the results of this work, consideration must be given to the possibility of premature failure resulting from brittle fracture.

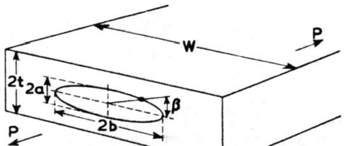
Acknowledgment

The author is glad to acknowledge the useful discussions which he has had with his colleague, Dr. F. M. Burdekin.

References

1. DINSDALE, W. O. & YOUNG, J. G. 'Significance of defects in aluminium alloy fusion welds. Part 1. Lack of fusion in ¼ in and ½ in NP5/6', *Brit. Weld. J.*, vol. 9, no. 8, pp. 482-493, 1962.
2. DINSDALE, W. O. & YOUNG, J. G. 'Significance of defects in aluminium alloy fusion welds. Part 2. Double operator defects in ½ in NP5/6 and pure aluminium', *Brit. Weld. J.*, vol. 11, no. 5, pp. 229-232, 1964.
3. ANON. 'The effect of discontinuities on the behaviour of aluminium welds', IIW Doc. XIII-262-61.
4. GUYOT, F., MARTELEE, J. & SOETE, W. 'Essais de fatigue sur soudures bout à bout transversales contenant des manques de penetration', CRIF report MT23 April 1966.
5. GUYOT, F., MARTELEE, J. & SOETE, W. Second series. CRIF report MT30, June 1967.
6. WILSON, W. M., MUNSE, W. H. & SNYDER, I. S. 'Fatigue strength of various types of butt welds connecting steel plates', *Illinois Univ. Exp. Sta. Bull.* 384, March 1950.
7. NEWMAN, R. P. & DAWES, M. G. 'Exploratory fatigue tests on transverse butt welds containing lack of penetration', *Brit. Weld. J.*, vol. 12, no. 3, pp. 117-120, 1964.
8. WARREN, W. G, 'Fatigue tests on defective butt welds', *Welding Research* (British), vol. 6, no. 6, pp. 112_r-117_r, 1952.
9. REGGIORI, A. & ERRA, A. 'Research on the mechanical characteristics of welded joints', Private report 2439 Institute de Richerche Breda, Milan, July 1957.
10. ANON. 'Compte rendu des essais de fatigue sur soudure bout à bout avec defaut continue à la racine du chanfrien en X', IIW Doc. XIII-452-66.
11. PARIS, P. C. & ERDOGAN, F. 'A critical analysis of crack propagation laws', *J. Bas. Engng.*, vol. 85D, no. 4, pp. 528-534, 1963.
12. PARIS, P. C. 'The fracture mechanics approach to fatigue', Proc. 10th Sagamore Army Materials Research Conference. Fatigue, an interdisciplinary approach.

13. IRWIN, G. R. 'The crack extension force for a part through crack in a plate', *Trans. ASME. J. Appl. Mech.*, vol. 29, no. 4, Series E, pp. 651-654, 1962.
14. GROSS, M. R. 'Low cycle fatigue of materials for submarine construction', *Naval Engineers Journal*, Oct. 1963, pp. 783-795.
15. PEARSON, S. 'Fatigue crack propagation in metals', *Nature 1966*, vol. 211, 5053, 1077.



2a = Defect width through thickness
2b = Defect length
2t = Material thickness at defect
W = Material width at defect
σ = Gross area stress = $\frac{P}{2Wt}$
Δσ = The cyclic range of stress σ
K = Stress intensity factor
ΔK = Range of stress intensity factor
σ_u = Ultimate tensile stress of the material
Φ = Complete elliptic integral
$= \int_0^{\pi/2} \sqrt{1 - \frac{(b^2 - a^2)}{b^2} \sin^2\theta} \, d\theta$
β = Angle defining a point on the circumference of the defect
Subscript i : e.g. a_i = initial value of the dimension concerned

Fig. 1. Nomenclature.

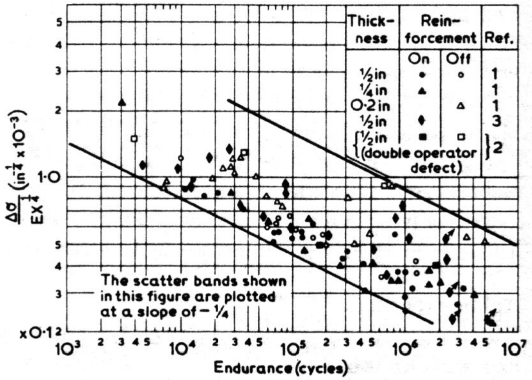


Fig. 2. Test results for aluminium alloys.

Fig. 3. Failure from elongated defect. Note propagation in minor axis direction.

Fig. 4. Failure from short defect. Note approximately equal propagation in the direction of both axes.

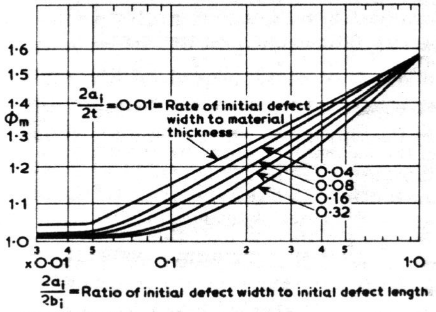


Fig. 5. Values of ϕ_m against $2a_i/b_i$ for various values of $2a_i/2t$.

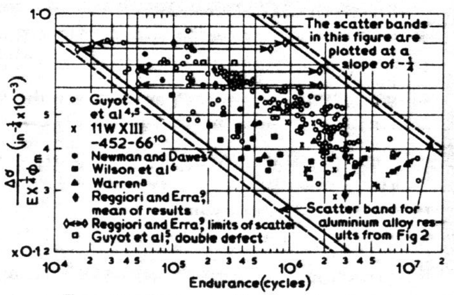


Fig. 6. Test results for steels.

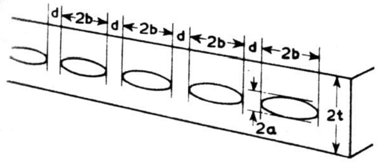
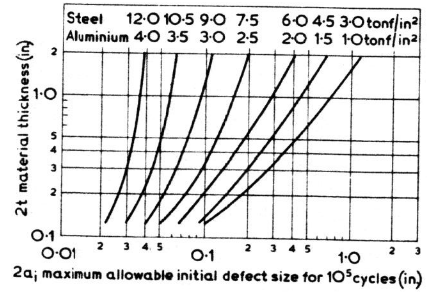


Fig. 7. Multiple defects.

Fig. 8.



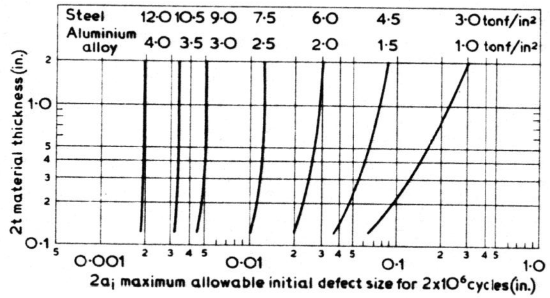


Fig. 9.

Fig. 10. Curve for determining whether account must be taken of interaction between two defects.