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Predictive parameters and their application to high temperature, low cycle fatigue

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Summary

Existing predictive methods for determining high temperature, low cycle fatigue life are re-examined, based on experiments on 'A' nickel. The relationship $N_f^{\alpha} \Delta\epsilon_p = C$ is found to exist, where α is a function of strain rate and temperature. Assuming $\Delta\epsilon_p = \epsilon_f$ at $N_f = \frac{1}{4}$, as for low temperature, where ϵ_f is the tensile ductility, this point falls well below the fatigue curve. The present results and those of other investigators are shown to be expressible in the form $C_1^{\beta} \Delta\epsilon_p = C_2$ where C_1 is a quantity defined as the frequency-modified fatigue life and β and C_2 depend only on temperature for a given material. The frequency-modified fatigue life, defined as $\nu^{K} t$, where ν is the frequency and t the time to failure, has applicability to the prediction of long hold-time fatigue life from short time tests. Fatigue crack propagation was both transgranular and intergranular and this bi-modal failure process is self-consistent with the concept of a frequency-modified fatigue life, and with the dependence of β on temperature. An upper bound fatigue curve is predicted for very low lives where crack propagation is by ductile, transgranular fracture. It employs the tensile ductility and assumes $\alpha = \beta = \frac{1}{2}$. The curve intersects with the bi-modal curve at very low cycle life, and a method for predicting this intersection is suggested, based on experimental observation.

Introduction

It has been widely shown that the low cycle fatigue phenomenon can be represented by the relationship

$$N_f^{\alpha} \Delta\epsilon_p = C \tag{1}$$

where N_f is the number of cycles to failure, $\Delta\epsilon_p$ the plastic strain range and α and C are parameters. The simplicity of the relationship and its repeated demonstration over a broad range of testing conditions have resulted in attempts to attach special significance to the parameters α and C, as, for example, their relationship to other physical properties of the metal involved. This approach has the attraction that complex fatigue failure prediction might be possible from some simple experiments or from physical data rather than detailed experimentation. It is this approach which the author has followed in earlier work in predicting low cycle fatigue behavior at low temperatures (below the creep range) [1]. A similar approach was taken by Manson and co-workers [2]. Here, in the absence of time-dependent effects, a was treated as a material independent constant, while C has been related to the tensile ductility of the specific material.

Methods have also been developed for applying equation (1) to high temperature, where time dependent effects are present. Manson, for example, considers a lower bound, average and upper bound approach, which is based directly on his earlier method, or a modification taking into account creep-rupture effects [3]. The author has developed a predictive method for applying equation (1) whereby a is maintained as 0·5 and C is again related to the tensile ductility, but where the ductility employed is that for the same time or strain rate, as well as temperature of the fatigue test [4, 5]. This so-called parametric method was compared to experiment, for two different materials [4, 6]. For this purpose the concept of a fatigue ductility [4, 7] was introduced. This is defined as the extrapolation of $\Delta\epsilon_p$ to $N_f = \frac{1}{4}$ from a specific fatigue test where $\Delta\epsilon_p$ and N_f are known, using equation (1) where $a = 0{\cdot}5$. Agreement of the fatigue and tensile ductilities under equivalent parametric conditions is confirmation of the method.

An aspect of this approach is the introduction of the ductility parameter to account for temperature and strain rate effects. Here it is assumed that a relationship exists between the ductility (tensile or fatigue) and some function of time and temperature or strain rate and temperature. By this means, short time test results could be used to predict long time behavior. Experimental support for this approach was found in a 2% molybdenum steel [4].

In a recent study the fatigue ductility was compared to the tensile ductility for a 1·5% manganese steel over a broad range of temperatures and strain rates [6]. It was found that agreement in the two ductilities existed at room temperature and 300 °C, whereas at intermediate temperatures the tensile ductility fell below, while at temperatures higher than 300 °C, it fell above the fatigue ductility. Since insufficient tests were carried out at each temperature and strain rate to obtain a, the source of the difference could not be determined. Of particular concern was the lack of agreement at temperatures of the order of 500 °C, where, from tensile ductility considerations, a life four times in excess of that actually experienced was predicted.

One problem in the interpretation of the results of the 1·5% Mn steel experiments was the inability to obtain a tensile ductility in a test under conditions of constant strain rate to fracture. The employment of constant crosshead velocity leads to ever-increasing strain rates as necking proceeds. This, it can be argued, leads to tensile ductilities which are not directly comparable in strain rate to cyclic strain experiments, particularly if a strong rate sensitivity exists. Thus, an aspect of the present

work is the attaining of tensile ductilities at constant strain rate to fracture.

The present paper reconsiders the question of the predictive possibilities of equation (1) and re-evaluates, from experiments on a simple, single-phase metal at high temperatures the earlier parametric approach developed by the author for certain low alloy steels. Many investigators have reported values of α well in excess of 0·5 under conditions where temperature and strain rate are well controlled. Further, under these circumstances, tensile ductility does not appear to predict fatigue behavior in any reliable way. Forrest has demonstrated this [8], for example, on a molybdenum-vanadium steel at 500°C (his Fig. 1). Thus, the use of $\alpha = 0{\cdot}5$ would appear to be unduly restrictive. The question of whether α is, in fact, related to other physical properties is considered in what follows, as well as the role of ductility, in those cases where $\alpha > 0{\cdot}5$. Additionally the roles of strain rate and frequency in fatigue life prediction are considered.

Material and test procedure

Test specimens were prepared from $\frac{5}{8}$ in diameter bar stock of 'A' nickel, the nominal composition of which is 0·2% Mn, 0·15% Fe, 0·1% Cu, 0·1% C, 0·05% Si, 0·005% S, balance Ni. Specimens were of the same type employed earlier [4, 6], having a minimum diameter of $\frac{1}{4}$ in. Following careful surface preparation involving final longitudinal polishing to remove all circumferential scratches, the specimens were annealed in dry hydrogen for one hour at 750 °C.

Uniaxial cyclic and monotonic tests were performed in a 20,000 lb capacity, reverse loading Instron machine following procedures reported earlier [6]. In the cyclic tests strain rate experiments were conducted by controlling the *at* constant crosshead speed, $\dot{x}$, between 0·0005 and 0·2 inches/minute. The tension tests were conducted by means of closed loop control of specimen minimum diameter such that the true longitudinal strain rate was held constant until fracture. Strain rates of 0·00025 to 0·25 min^{-1} were employed.

Diametral strains were measured and recorded, and served as the basis for control of all cyclic and monotonic tests. The gauge for these experiments has been described previously [6]. An *X-Y* instrument was provided to record hysteresis loops of load and diameter change for each experiment. Additionally both load and diameter were recorded as a function of time.

All tests were conducted in air. Induction heating was used in all experiments and temperature was controlled by means of two fine-wire thermocouples spot-welded to the specimen about $\frac{1}{4}$ in from its minimum diameter; one was for temperature control, the other for temperature measurement. Temperatures up to 750°C were investigated.

Constant strain rate to fracture experiments

Until the advent of closed loop control systems in mechanical testing, experiments involving constant true strain rate to fracture were extremely difficult to conduct. Very little is to be found on the subject in the literature [9]. With closed loop control, one method for carrying out such tests is to program the relationship of diameter to time on a curve follower. This relationship is derived from the condition of constant true-strain rate, the true strain being defined as 2 $\log_e (d_0/d)$ when d_0 is the initial diameter.

Results for the tests are included as Table 1. In all cases the fracture nucleated and grew internally after necking had developed. This was evidenced in the load-diameter curve by an abrupt change in slope. This point was considered to be fracture. Longitudinal sections of such failures indicate extensive voids in the constrained section of the specimen.

Early in the program considerable emphasis was placed on diametral strain control. As the program developed, the emphasis shifted and subsequent tensile tests were conducted with constant crosshead rate control. These results are shown in Table 2. Fracture is defined in the same manner as above.

Cyclic strain experiments

Cyclic strain tests were conducted in the same manner as in earlier work [6]. Fatigue results for the various temperatures and crosshead rates are shown in Fig. 1. This figure indicates several facts of interest. First it is observed that the data essentially plots as straight lines, confirming equation (1). Secondly it is observed that the exponent a of equation (1) is greater than 0·5, despite the fact that the strain rate is approximately constant for each curve. Thirdly, there is a substantial rate effect such that the curves are displaced to the left as the rate is decreased. Fourth, the temperature increase plays a role similar to that of a rate decrease in that an increase in temperature causes a displacement of the curves to the left. Finally it appears that for this material under these conditions the tensile ductility is not a satisfactory indicator of the fatigue behavior as can be seen by fitting the values of Table 1 or 2 on Fig. 1 where $\epsilon_f = \Delta\epsilon_p$ at $N_f = \frac{1}{4}$.

The effect of frequency on life

Although strain rate is a meaningful parameter when considering the physical behavior of a material, from an engineering viewpoint the frequency appears to be more meaningful since the most important concern is generally how the structure will behave under cyclic operation where the strain rate is complex. The present test results may be presented in terms of frequency. To this end it is of interest to pursue further the

approach of Eckel [12], Forrest [8], and Coles *et al.* [13]. Here for a given total strain range the relationship

$$\nu^K t = C_1 \qquad (2)$$

is found to apply, where ν is the frequency of the test, t the total time to failure and C_1 a constant. It would appear that the preferable approach would be to consider a specific plastic strain range and examine the frequency effect. Fig. 2 shows this behavior for the present data at 550 °C, where t is in minutes and $\nu = N_f/t$ in cycles per minute. As a first order approximation, the constant K is seen to be independent of the magnitude of the plastic strain range, and further that the constant C_1 becomes a simple function of the plastic strain range. Fig. 3 gives a plot of the computed value of C_1 for each test for nickel A, vs. the plastic strain range at three temperatures, 550, 650 and 750 °C.

From Fig. 3 it appears that the relationship

$$C_1{}^\beta \Delta\epsilon_p = C_2 \qquad (3)$$

applies. Values of K, β and C_2 are given in Table 3 for nickel A at the various testing temperatures. It is of interest to note here that Coles *et al.* [13] obtained the value 0·85 for K on two low alloy steels at 550 °C.

In examining equations (2) and (3), it is readily seen that if $K = 1$, that is, if there is no frequency effect, then $C_1 = N_f$. Hence equation (3) becomes equivalent to equation (1). More generally, C_1 can be considered as the frequency-modified fatigue life.

Comparison with other investigations

It is of interest to examine some other published results of high temperature cyclic strain for support of the frequency-modified fatigue life approach. The work of Forrest [8] on a 0·5% molybdenum, 0·25% vanadium steel at 500 °C is pertinent. He employed reversed bending and presented data of plastic strain range versus cycles to failure at three frequencies – 10 cycles per minute, 1 cycle per 10 minutes and 1 cycle per 10 minutes with 1 hour dwell periods at each zero strain. Following Forrest, in this last instance the frequency is considered as 1 cycle per 130 minutes. Using the two strain ranges given in his Fig. 3, and converting to plastic strain range, the frequency modified life versus plastic strain range is given here as Fig. 4. Since his more detailed date of plastic strain range versus life falls on parallel sets of curves for different frequencies, the balance of his test data can be expected to fit in the same straight line of Fig. 4. The constants of equations (2) and (3) for these data are given in Table 3.

Berling and Slot have recently presented data on elevated temperature, low cycle fatigue of AISI 304, 316 and 347 stainless steel employing controlled diametral strain and strain rate [14]. Temperatures were 430°, 650° and 816°C. Longitudinal strain rates were approximately 4×10^{-3}, 4×10^{-4} and 4×10^{-5} sec $^{-1}$. At least three strain ranges were examined for each of the above conditions. Testing procedures were similar to those employed here, with the exception that closed-loop control of strain was used, permitting constant strain rates with each cycle. When plotting their test results in the form of equation (1), they found a strong effect both of temperature and strain rate, particularly at 650° and 816°C. In some instances a four-fold decrease in life occurred with a 100-fold decrease in strain rate. These data have been converted to plastic strain versus frequency-modified fatigue life coordinates and are included here as Figs. 5, 6 and 7. The pertinent constants of equations (2) and (3) are listed in Table 3.

Role of tensile ductility

In fitting the tensile ductility to curves of $\Delta\epsilon_p$ vs. N_f, the procedure developed earlier [1] by the author was to let $\Delta\epsilon_p = \epsilon_f$ when $N_f = \frac{1}{4}$. In the present representation, C_1 can be related to N_f by equation (2). Here, if t is the time for failure in simple tension, the period for an equivalent cyclic test (that is, for a full cycle) is assumed to be $4t$ such that

$$(C_1)_{\epsilon_f} = (4t)^{-K}t \tag{4}$$

Thus for a material with $K = 0{\cdot}85$ and $t = 60$ minutes, $(C_1)\,\epsilon_f = 0{\cdot}569$. Letting $\Delta\epsilon_p = \epsilon_f$, the tensile ductility can then be located on $\Delta\epsilon_p$ vs. C_1 coordinates. In Table 3 are given values of tensile ductility, fracture time and the corresponding $(C_1)\,\epsilon_f$. The fit of these ductility quantities in Figs. 3 to 7 was found to be poor. A detailed examination reveals the fact that the more β increases, the further the fatigue curve is *above* the tensile ductility. Since the general trend is for β to increase with increasing temperature (Table 3) one can only conclude that, by this procedure, the higher the temperature, the poorer the fit.

One interesting fact that comes from these considerations is seen schematically in Fig. 8. Because the short-time tensile ductility falls below the fatigue curve F for comparable values of C_1, a line T drawn through this point with $\beta = 0{\cdot}5$ will also be below the fatigue curve for low values of C_1 (<25 in Fig. 8). It will be argued later that the curve T represents an upper bound of plastic strain for a given fatigue life. Assuming for the moment that this is the case, then curve F has meaning only to the right of the intersection point. For lives less than 25 cycles, then curve T will be followed. Thus an approximate method exists for

identifying the location of curve T, since the intersection point A can be found from the tensile ductility.

The procedure involves first locating the tensile ductility at $(C_1)_{\epsilon_f}$ and $\Delta\epsilon_p = \epsilon_f$. Since $\beta = 0{\cdot}5$, point A can be found along curve T by the co-ordinates $C_1 = 10(C_1)_{\epsilon_f}$, $\Delta\epsilon_p = \epsilon_f/\sqrt{10}$. The value of C_1 at A is arbitrary, but is chosen both for convenience in locating $\Delta\epsilon_p$ and from experimental evidence.

By this procedure the assumption is made that, provided the tensile ductility remains constant, as β increases above 0·5, the line F rotates about A. If $\beta = 0{\cdot}5$, line F is merely a continuation of line T. Hence point A can be located on Figs. 3 to 7 by the procedure outlined above. It will be noted that in most instances the fit is quite reasonable.

Discussion

Some evidence for the existence of a transition in slope when $N_f < 60$ cycles is suggested by the work of Raraty and Suhr [15] on a magnesium-0·8% aluminum alloy in which a change in the fatigue process, from high strain fatigue to diffusional cavitation is reported. In the present case it can be argued that for very high cyclic strains and short lives and times the fracture mode is entirely by ductile, transgranular fracture, while at lower strains and longer times the fracture mode is mixed. Evidence for the mixed fracture mode is given below; further work is needed to support the existence of the fully ductile, or upper bound curve.

With respect to the fatigue results on A nickel, microstructural evidence indicates that the initial crack growth, starting at the surface, is intergranular, while in the later stages of life the mode is more transgranular. This bi-modal form of crack growth may explain why the high temperature fatigue results can be represented in terms of a frequency-modified fatigue life. If a single mode of crack growth exists, then crack growth per cycle should be largely independent of frequency. With bi-modal cracking, lowering the frequency lowers the strain rate, increases the relative amount of intergranular fracture and hence increases the crack growth rate. In this instance a frequency effect on fatigue life exists, that is, $K < 1{\cdot}0$.

It is seen in Figs. 3 to 7 that, for a variety of materials strain cycled at elevated temperatures, good support is given to the concept of a frequency-modified fatigue life. This result has several implications. First, it enables a direct determination of the frequency effect. By combining equations (2) and (3) it can be shown that

$$N_f = (C_2/\Delta\epsilon_p)^{1/\beta\nu^{1-K}} \tag{5}$$

Using the constants of Table 3, N_f can be directly calculated. Secondly, the constants β, C_2, and K can be determined from relatively short time experiments to permit predictions of life when the frequency is, say, one per day and the life is much longer than can be tolerated for laboratory tests. Thirdly, equation (5) presumes that the frequency is defined in terms of a full cycle without regard to the wave shape. If this could be verified, the resulting simplification in experimentation and design could be of significant value. Thus, provided $\Delta\epsilon_p$ and ν remain constant, the life would be the same whether, for example, the straining is sinusoidal, or contains hold periods or other complications. Experimental attention needs to be given to this question.

Conclusions

From the experimental evidence presented here, the following conclusions can be drawn:

(1) At temperatures well into the creep range low cycle fatigue data can be expressed in the form of equation (1). The exponent a was found to depend on the strain rate and temperature, and was well in excess of 0·5. The tensile ductility, when plotted at $N_f = 1/4$, falls substantially below the fatigue curve.

(2) The fatigue data presented here, and those of other investigators, can be represented on logarithmic coordinates by the plastic strain range and the frequency-modified fatigue life, C_1, defined by equation (2). Expressing this relationship as equation (3), the exponent β is dependent only on the temperature and is independent of the frequency.

(3) The concept of a frequency-modified fatigue life has application in predicting long hold time fatigue behavior from short time data.

(4) Crack propagation was found to be both transgranular and intergranular. This bi-modal form of crack propagation was viewed as consistent with the existence of the frequency-modified fatigue life and with the high value of α.

(5) The existence of an upper bound or purely transgranular fatigue curve is postulated, defined in terms of the short term tensile ductility and $a = 0{\cdot}5$. This curve intersects with the bi-modal fracture curve at very low cyclic life. An approximate method for determining this intersection point, and thus locating the position of the bi-modal curve is suggested based on experimental observations.

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Table 1

Controlled strain rate to fracture tension tests – nickel A

True strain rate min^{-1}	True strain at fracture 500°C	550°C	600°C
0·25	1·64	1·74	–
0·025	1·63	1·57	–
0·0025	1·43	1·23	1·31
0·00025	0·87	0·92	

Table 2

Constant crosshead rate tension tests nickel A

Temp. °C	*True strain at fracture*	*Time to fracture min*
550	1·85	6·4
650	2·09	6·7
750	3·90	7·5

Table 3

Constants for high temperature low cycle fatigue

Material	*Temp.* °C	K	β	C_2	ϵ_f	t min	$(C_1)_{\epsilon_f}$	n'	*Reference*
A Nickel	550	0·83	0·79	2·65	1·85	6·42	0·44	0·26	This paper
A Nickel	650	0·82	0·78	1·80	2·09	6·70	0·47	0·19	This paper
A Nickel	750	0·88	0·77	1·55	3·90	7·45	0·38	0·35	This paper
Mo-V Steel	500	0·85	0·92	4·50	1·20	210	0·69	—	[8]
304 SS	430	0·81*	0·50	0·48	1·02	4·3	0·43	0·42	[14]
304 SS	650	0·81	0·70	1·10	0·55	2·3	0·38	0·23	[14]
304 SS	816	0·81	0·84	1·40	0·71	3·0	0·40	0·12	[14]
316 SS	430	0·90*	0·52	0·59	0·97	4.0	0·38	0·39	[14]
316 SS	650	0·90	0·58	0·63	0·89	3·7	0·37	0·25	[14]
316 SS	816	0·81	0·73	1·00	0·96	4·0	0·38	0·15	[14]
348 SS	430	0·85*	0·50	0·48	1·09	7·3	0·47	0·37	[14]
348 SS	650	0·85	0·57	0·57	1·16	4·8	0·34	0·20	[14]
348 SS	816	0·85	0·72	1·00	1·75	4·6	0·33	0·18	[14]
Mn Steel	500	0·85*	0·82	4·50	2·46	1·1	0·35	—	This paper

* Assumed

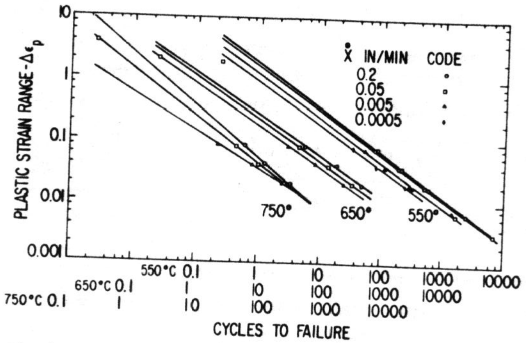

Fig. 1. Plastic strain range versus cycles to failure – A nickel at various temperatures and crosshead rates – all tests.

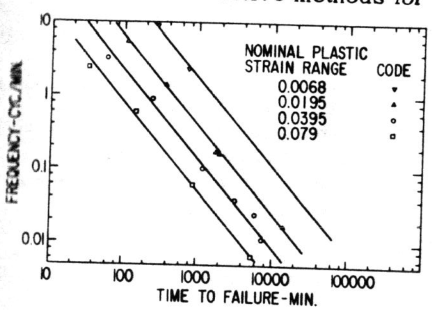

Fig. 2. Frequency versus time to failure – A nickel, 550°C, various plastic strain ranges.

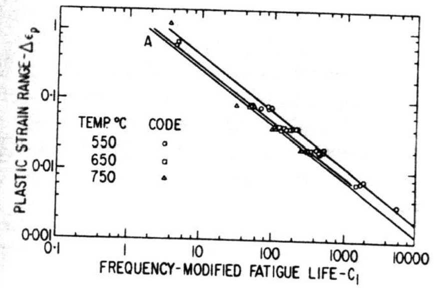

Fig. 3. Plastic strain range versus frequency-modified fatigue life – A nickel, four crosshead rates – point A indicates tensile ductility.

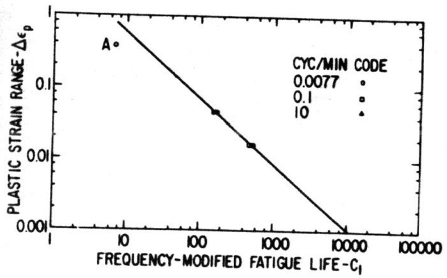

Fig. 4. Plastic strain range versus frequency-modified fatigue life – Mo-V steel at 500°C after Forrest [8] – three frequencies – point A indicates tensile ductility.

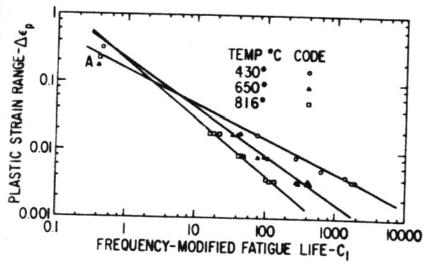

Fig. 5. Plastic strain range versus frequency-modified fatigue life – 304 stainless steel, various temperatures and strain rates, after Berling and Slot [14] – points at A indicate tensile ductility.

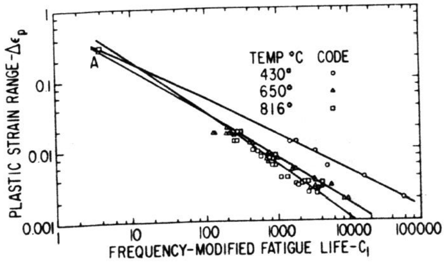

Fig. 6. Plastic strain range versus frequency-modified fatigue life — 316 stainless steel, various temperatures and strain rate, after Berling and Slot [14] — points at A indicate tensile ductility.

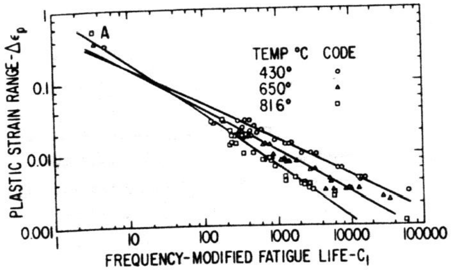

Fig. 7. Plastic strain range versus frequency-modified fatigue life — 348 stainless steel, various temperatures and strain rate, after Berling and Slot [14] — points at A indicate tensile ductility.

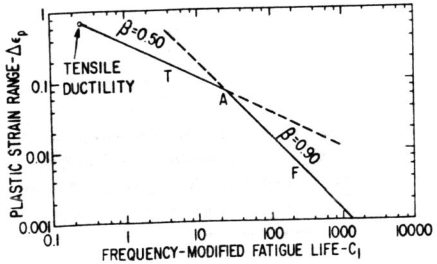

Fig. 8. Schematic representation of high temperature, low cycle fatigue behavior — *F* represents fatigue curve at intermediate lines. *T* is upper-bound curve.