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Low-cycle fatigue in relation to design

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Summary

Fatigue failure of engineering components almost inevitably occurs at details which cause stress concentrations. Under repeated loading, there may be plastic straining at a detail, the amplitude of which is determined by the general behaviour of the component. If this amplitude is known from experiment or calculation, the life can be forecast, due allowance being made for the state of strains if this is not the same as in a uniaxial test. Low-cycle fatigue of metals under biaxial straining follows a Coffin-Manson type relationship

$$\epsilon N^{\alpha} = C$$

but both α and C, while independent of the material, are functions of the state of strains. However, for lives above 300 cycles, the results for uniaxial tests correlated by the Von Mises equivalent strain for other states of strains may be used safely for predicting fatigue life.

The influence of a hole in a biaxial fatigue specimen has been investigated and found to decrease the life (defined as the onset of fatigue cracking) to about one tenth of the life of a plain specimen subject to the same strain amplitude. This result is correlated with the appropriate Coffin-Manson relationship.

Introduction

Conventional fatigue testing comprises the application of repeated or alternating stresses to specimens of fairly simple geometry, with the results usually expressed as the number of cycles to failure in terms of the stress range. Such data are sufficient for design against fatigue failure within a required lifetime of engineering components that also have a simple geometry and in which reliable estimates can be made of the service stresses.

In the region of low-cycle fatigue, i.e. lives of 10^5 cycles or less, operative stresses are usually above the yield stress of the material and plastic deformation is involved at every stress reversal. Under such conditions, the life can be related more simply to the strain range than to the stress range.

Most engineering components have far from simple geometry and fatigue failure almost always occurs where stress concentrations exist. If such a component has been designed with the intention that it should have a reasonable life, in terms of stress cycles to failure, then its overall behaviour should be elastic and, within any limitation due to complication of shape, the stresses at any point of detail could be calculated. If however the stress system in the region of that detail is such that the yield point would be exceeded, then there will be plastic straining in that region and the life may be limited by low-cycle fatigue behaviour. Extremely local plastic straining will have little effect on the overall elastic response of the component, but the magnitude of this local plastic strain will be governed by the general elastic behaviour of the component, and not greatly affected by cyclic strain hardening or strain softening. If this expected plastic strain range can be calculated or inferred from experiments, a reasonable estimate of the life could be made from the results of controlled low-cycle fatigue tests on the material.

Such tests are usually made by loading specimens uniaxially in push-pull or by bending wide cantilevers. If the state of strains (a term used in this paper to denote the ratio of the principal strains in the plane of the surface) is not the same as in a uniaxial test, then the question arises as to what consideration must be given to the application of uniaxial or wide cantilever test results in such circumstances.

Biaxial straining tests

Under low-cycle fatigue conditions, life can be related to the strain range according to the well-known Coffin-Manson law

$$\epsilon N^{\alpha} = C \qquad (1)$$

where ϵ is the strain range, N is the number of cycles to failure and α and C are constants.

For simple uniaxial specimens, Coffin [1] found that a value of $\frac{1}{2}$ for the exponent α fitted the results for many metals when the plastic range ϵ_p was considered. Other workers [2] [3] [4] have found that higher values of α were applicable, but that a value of 0·5 was more appropriate when the total strain range ϵ_t was considered.

Manson [5] [6] investigating the strain range—cycles to failure relationship for a large number of metals and for lives in the range 1-10^6 cycles found that both the elastic and the plastic components of the total strain range separately obeyed a law of the type given in equation (1), the plastic component having a value of 0·6 for α and the elastic component a much lower value in the range 0·06-0·16. For short lives, the plastic component is the dominant one and on a log ϵ – log N plot is indistinguishable from the total strain, whereas at long lives the same applies to the elastic component of the strain.

Various workers have investigated fatigue life for conditions of biaxial stressing or straining, usually giving results in terms of the strain ranges imposed. The effect of the type of loading can be described by the state of strains at the surface. This state of strains, described qualitatively in

the previous section, can be expressed as follows: if ϵ_1 and ϵ_2 are the ranges of the principal strains in the plane of the surface ($\epsilon_1 \geqslant \epsilon_2$) and ϵ_3 is the strain range in the direction perpendicular to the surface, then the state of strains is the ratios $\epsilon_1:\epsilon_2:\epsilon_3$. For uniaxial loading of a specimen with axial symmetry, $\epsilon_2 = \epsilon_3 = -\nu\epsilon_1$ where ν is Poisson's ratio in the elastic range and has the value 0·5 for purely plastic deformations, when volume remains constant. For low-cycle uniaxial fatigue tests, the state of strains is therefore (1:–0·5:–0·5). The states of strains for various loading conditions and some of the tests reported, other than those given by Coffin [1], are summarised in Table 1.

All this work has been carried out with different types of specimens, on different testing machines and with different materials. The criterion of failure was not the same in all cases, some tests having been taken to complete fracture, while others were only taken to an early stage of visible cracking. In bending and torsion tests there is a strain gradient into the material which is not present in the push-pull tests. In some cases, actual strains have not been measured, but calculated from the deflections of the specimens, these deflections having been varied between set limits. In such circumstances, strain hardening or strain softening may change the local strain pattern during the life. Also, as Miller [18] has shown, the variations in strain rate at the position of cracking, which will be influenced by both cycling rate and specimen geometry, have a significant effect on the values of the constants. Hence any comparison of the results is complicated and differences in values of α and C given in Table 1 could be due to causes other than purely material ones. It must be noted, as emphasised by Benham and Ford [2], that where the value of C is derived by extrapolation from points determined for longer lives this value is very sensitive to the choice for α.

Pascoe and de Villiers [19] have described a loading rig by which a wide range of different states of biaxial stress or strain can be imposed on the same shape of specimen. The specimen (Fig. 1) is of cruciform shape with spherical recesses on both sides of the central region. Both tensile and compressive loads can be applied through either or both pairs of arms. The strains at the centres of the specimens were measured with electrical resistance strain gauges and the loads were adjusted to give the desired ranges of strain. No attempt has been made to separate the elastic and plastic components of the total strain. The cycling rate was approximately 6 per minute. Failure was taken to be the stage at which relative movement of the sides of cracks could be seen with a medium-powered microscope. Most of the objections listed in the previous paragraph have therefore been avoided.

From a series of tests on a mild steel and a heat-treated steel of nearly three times the yield stress, very similar log ϵ – log N curves were obtained for each of three states of strains, viz: shear loading (1:–1:0), uniaxial loading (1:–0·6:–0·4) and equibiaxial loading (1:1:–2). The curves are shown in Fig. 2 from which it will be seen that the values of α differ considerably with different states of strain.

This work was with completely reversed strains. A limited number of loading tests have been performed with cruciform specimens and pulsating cycles, i.e. from zero to a given strain. The results, one of which for shear loading is shown on Fig. 2, fall within the scatter band of the completely reversed strain tests and suggest that strain range is the only significant variable, the value of the mean strain being unimportant. This bears out the results of Benham and Ford [2] on uniaxial specimens, and of Yokobori *et al.* [7] who found that varying the mean strain had no significant effect on the results both in tension and in torsion.

Criterion for design

It has been suggested by Ives *et al.* [14] on the basis of their results summarised in Table 1, that uniaxial and multiaxial test results can be correlated on a basis of the Von Mises equivalent strain

$$\epsilon_e = \frac{\sqrt{2}}{3}\sqrt{[(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2]}$$

Other workers [8, 20, 21] have quoted this in formulating a basis for short-life prediction in complex structural components. Jerram [8] does however say that it may not be the best correlation method. Dawson [20] and Pickett and Grigory [21] have based their opinion on the results quoted by Ives *et al.* [14].

To plot a log ϵ – log N curve on a basis of ϵ_e rather than ϵ_1 involves a mere parallel shift of the line so that this method of correlation is not sufficient where different states of strains give lines of different slopes. This may be seen from Fig. 3 in which the lines of Fig. 2 have been adjusted to refer to equivalent strain. However, it is obvious that to base the expected life under multiaxial stressing on the behaviour under uniaxial loading using equivalent strain as a correlating factor is satisfactory for lives in excess of 300 cycles. Only in rare circumstances are shorter lives likely to be considered in design. On the other hand, to base the expected life on equibiaxial behaviour could be unsafe for longer lives.

Uniaxial and wide cantilever tests gave closely similar results for many metals. Although similar information on non-ferrous metals does not yet appear to be available for multi-axial stressing, it is not improper to suggest that again a wide range of metals may show similar results and that strain range and state of strains are the primary factors affecting life.

The author proposes [19] that fatigue results for all states of strains could be displayed on a single plot which would embrace all metallic materials and would be the sole information about low-cycle fatigue needed for design purposes. The diagram would take the form shown in Fig. 4. Any particular state of strains would be represented by a straight line through the origin and all states of strains could be represented in the quadrant between OA and OD. The contours of equal life shown are drawn through points given by the author's results and the results of Gross [13] for steels. There is a large region in which no experimental results have yet been obtained with the cruciform specimen and where the contours shown are purely conjectural. This diagram will need revision as further data become available.

Criterion of failure

Most short-life fatigue data determined with uniaxial or wide cantilever tests are given in terms of life to complete fracture of the specimen. The life then depends upon two factors: crack initiation and crack propagation. In the cruciform tests, as stated earlier, the failure criterion was the stage at which relative movement of the sides of a crack could be observed with a medium power microscope. In these specimens and in many other forms of test specimen, such as pressure vessels, flat discs, etc., the spread of the crack is governed by a different geometry than in the simple specimens and final failure may be sudden when the crack has reached a certain size. Therefore, from the design point of view, the number of cycles necessary to cause a crack of the size that just shows visible movement is a reasonable criterion.

Applications to design

The author has carried out a few tests on cruciform specimens containing a hole of 1/32 in diameter drilled through the centre of the test section, but which were otherwise the same as the plain specimens illustrated in Fig. 1.

An electrical resistance strain gauge was affixed to each specimen with its centre-line parallel to one pair of arms and 3/32 in from the centre of the specimen. Equal loads were applied to all four arms and varied in such a manner that the strain gauge indicated the chosen strain amplitude, i.e. the specimen was subject to an equibiaxial alternating strain field. The criterion of failure was the same as that for the other cruciform specimen tests.

In each case, cracks originated at three or four points on the circumference of the hole. The results for two specimens so tested were lives of 944 and 225 cycles for measured total strain ranges of 0·4 and 0·6% respectively and are shown on Fig. 2 where they can be compared with the results for plain specimens. The lives have been reduced by a factor of about 10 due to the presence of the hole. No detailed calculation has as yet been performed to determine the strains at the periphery of the hole, but reasonable assumptions would be a strain magnification factor of approximately 3 and that the strain condition at the surface of the hole would be approximately that in a uniaxial specimen. Then considering the specimen strained at 0·4% total strain amplitude, the total strain range at the hole surface would be about 1·2%. At this strain range, the expected life for uniaxial loading would be almost exactly the expected value.

General consideration of design methods

The tests on cruciform specimens containing central holes show promise for a method of fatigue design based on the calculation of the expected strains at critical points in any component and then forecasting the life from charts such as Fig. 4. Such a design philosophy is a slight advance on that expressed elsewhere [8, 20, 21, 22] in which critical strains are to be calculated and life estimated from an experimentally-determined equivalent strain-cycles to failure curve, due allowances being made for the deleterious effects of surface conditions, inclusions, etc.

Kooistra has pointed out that in components such as pressure vessels one can determine the 'strain concentration factor' of the plate surface conditions, attachments, welds, etc. This factor is the ratio of the strain at the discontinuity to the mean strain away from the discontinuity. Once its value for a particular form of discontinuity is known from calculation or experiment, design can proceed. The factor should include any effects due to differing states of strain, though Kooistra does not mention this. He quotes the results of experimental determinations of such factors by a reverse process. From the observed life and the appropriate Coffin-Manson plot the actual strain amplitude at the discontinuity is estimated. This is compared with the mean strain away from the discontinuity, which can be measured by conventional methods. Values as high as 4·7 were determined for sharp-radius nozzles. Once determined for typical defects and attachments, the values could be used for design purposes in other cases.

Conclusions

Low-cycle biaxial straining fatigue tests on metals show that the constants in the Coffin-Manson law are dependent upon the state of strains and the strain amplitude, but are almost independent of the material or the mean strains.

Results for different states of strains cannot be correlated on a basis of equivalent strain, but the use of equivalent strain amplitude in estimating fatigue life is satisfactory for lives above 300 cycles if based on uniaxial test results.

A single chart is proposed including fatigue results for all states of strains. For design purposes, the strain amplitude at a discontinuity in an engineering component due to working loads should be calculated and the expected life read from the chart.

One test to verify this was the study of the effect of a small hole in an equibiaxial strain field. The life was reduced by a factor of 10 as would be expected.

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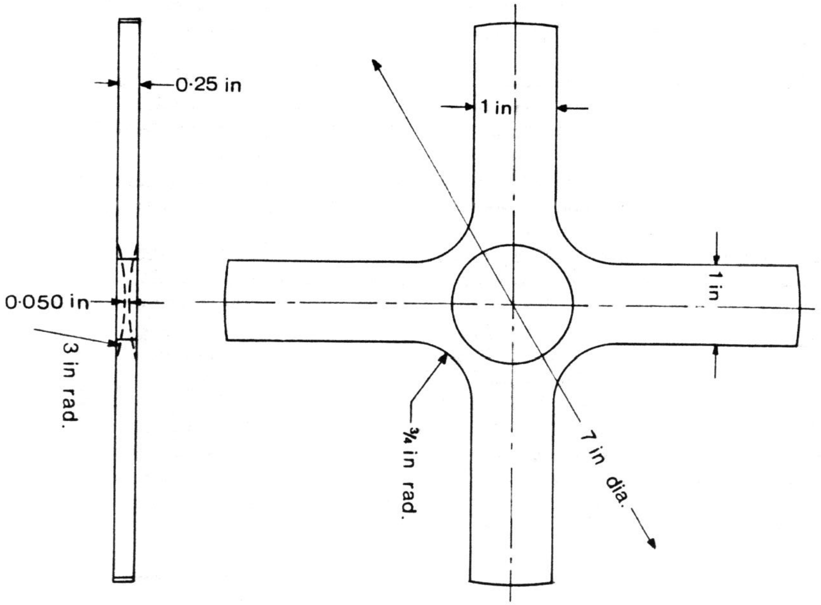

Fig. 1. Cruciform specimen.

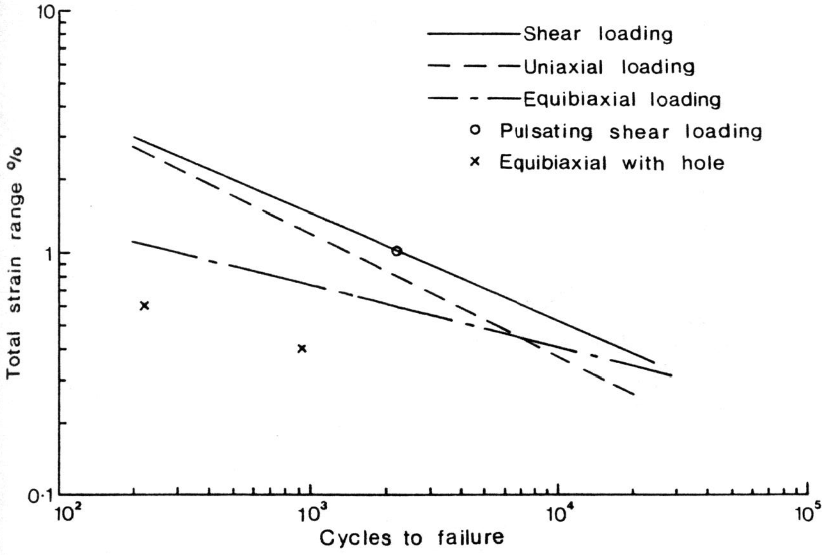

Fig. 2. Comparison log ϵ – log N curves for three states of strains and results of three other tests.

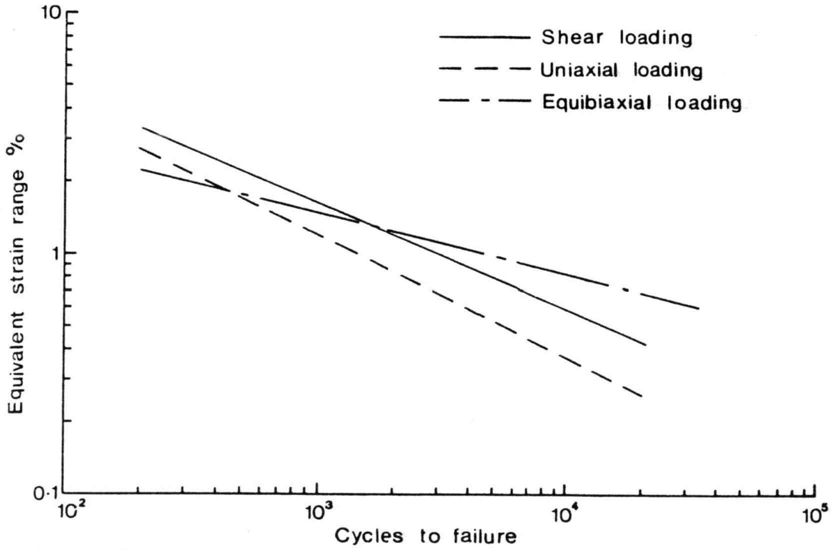

Fig. 3. Comparison of log ϵ – log N curves on a basis of equivalent strain.

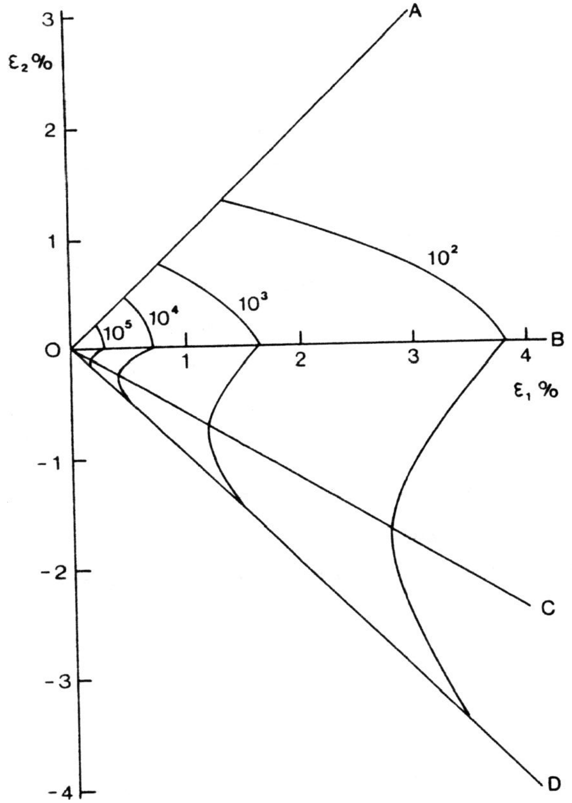

Fig. 4. Contours of equal fatigue life on $\epsilon_1 - \epsilon_2$ plane.
Line OA–equibiaxial tests.
Line OB–wide cantilever tests.
Line OC–uniaxial loading tests on cruciform specimen.
Line OD–shear or torsion tests.

Table 1

Room-temperature low-cycle fatigue tests

Type of loading	*State of strains*	*Material*	*Ref.*	*Total or plastic strain range*	α	*C*
Push-pull tension-compression	1:–½:–½	mild steel	[2]	p	0·61	0·66
				t	0·49	0·48
		A1 alloy	[2]	p	0·66	0·083
				t	0·22	0·065

Table 1 (cont.)

		mild steel	[3]	p	0·62	0·10
				t	0·58	0·12
		mild steel	[7]	p	0·50	0·39
		mild steel	[8]	t	0·50	0·52
		waspalloy	[9]	t	0·42	0·2
		inconel	[9]	t	0·31	0·15
Push-pull tensile strain only	1:–½:–½	mild steel	[2]	p	0·56	0·56
				t	0·43	0·32
		A1 alloy	[2]	p	0·58	0·062
				t	0·22	0·066
		stainless steel	[10]	p	0·66	0·21
		Mn steel	[11]	t	0·28	0·06
Bending-narrow cantilever	1:–½:–½	mild steel	[12]	p	0·35	0·3
Bending-wide cantilever	1:0:–1	Fe alloys	[13]	t	0·37	0·21
		A1 alloys	[13]	t	0·28	0·12
		Cu alloys	[13]	t	0·36	0·19
		Ni alloys	[13]	t	0·31	0·15
		mild steel	[14]	t	0·43	0·34
		Mn steel	[14]	t	0·24	0·062
		low alloy steel	[14]	t	0·25	0·092
		C and Mn steels	[15]		0·42	0·44
		low alloy steel	[15]		0·27	0·10
		complex alloy steel	[15]		0·21	0·06
		½% Mo steel	[16]	t	0·4	0·23
		¾% Cr-Mo-V steel	[16]	t	0·36	0·22
Equi-biaxial	1:1:–2	mild steel	[14]	t	0·30	0·057
		Mn steel	[14]	t	0·023	0·034
		low alloy steel	[14]	t	0·23	0·034
Torsion	1:–1:0	mild steel	[7]	p	0·50	0·7
		aluminium	[17]	p	0·58	0·044
		brass	[17]	p	0·52	0·07
		steel	[17]	p	0·60	0·022
		En 25 steel	[18]	p	0·73	1·15