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Fracture mechanics of rubber

Dr. G. J. LAKE, P. B. LINDLEY and A.G. THOMAS
Natural Rubber Producers' Research Association, 56 Tewin Road, Welwyn Garden City, Hertfordshire

Summary
Work on crack propagation in rubber has been examined by the energetics approach used in fracture mechanics. Emphasis is placed on the importance of establishing the validity of this approach by comparing the behaviour of test pieces of very different shapes. A novel test piece is described for which the release of strain energy with increase in crack length can be calculated in terms of the applied force.

Previous work on the crack growth, fatigue and corrosion of rubber is reviewed from this point of view. The unification of the various failure modes which has been possible by this approach and also the progress which has been made in giving a molecular explanation of some strength properties are described.

Introduction

The adaptation of Griffith's approach to the tear behaviour of rubber was made by Rivlin and Thomas in 1953 [1]. The terminology that has been used in the work on rubbers is somewhat different from that employed in the similar work on metals and plastics [2], largely because the initial work on rubbers was concerned with tearing. The energy to create new surfaces by tearing is termed the tearing energy, and denoted by T. It is defined in terms of the total strain energy in a test piece U by the relation

$$-\frac{1}{h}\left(\frac{\partial U}{\partial c}\right)_L = T \qquad (1)$$

where the sheet of material is of thickness h and the crack of length c. These dimensions are relative to the unstrained state. The derivative indicates that the differentiation is carried out at constant overall length L, or, more precisely, that the external forces do no work since their points of application do not move.

If the energy for tearing T is a characteristic of the material, which it must be if it is to be a useful concept in describing strength properties, then its value must be independent of the overall shape of the test piece. It was considered important to establish the range of validity of the tearing energy concept by carrying out experiments on test pieces of very different shapes but using the same material [1, 3]. This has apparently proved to be easier with rubber than with metal.

In hard materials much attention has been given to the case of an edge or centre crack in a strip deformed in simple extension, following Inglis' and Griffith's analyses. Although such a test piece has been widely used in experiments on rubber, it is not particularly suitable for a rigorous test of the theory as the relation between T and the applied stress is not easily calculable theoretically. This is because, in the case of rubber, the strains involved are not small and subsidiary elastic experiments have had to be made to overcome the difficulty. The high extensibility of rubber thus presents certain problems. However, the mitigating advantages compared with metals are (*i*) the absence of yielding, the rubber remaining essentially reversible elastically up to strains of several hundred percent, and (*ii*) the low modulus, which often enables bending or buckling stresses to be ignored.

In the early work [1] the quantity $-(1/h)\,(\partial U/\partial c)_L$ [$= T$] was derived experimentally from the measured force-extension relations for various crack lengths using numerical integration to obtain U in terms of c. However, by suitable choice of test piece it was found possible to calculate $-(1/h)\,(\partial U/\partial c)_L$ from easily measured forces or strains, which greatly simplified matters.

Three test pieces for which the calculation is possible [1, 3] are shown in Fig. 1. For example, the tearing energy for the 'trousers' test piece is given by

$$T = \frac{2F}{h} \tag{2}$$

provided the extension of the legs of the test piece is negligible. This test piece was used in some experiments which are described later. The behaviour of two of these test pieces, the pure shear and the trousers, was first examined using natural rubber (NR) as the test material [1]. With natural rubber it was found that an abrupt, catastrophic increase in the length of the crack occurred at a particular stress. When the results were expressed in terms of the tearing energy similar values were obtained from both test pieces. All three test pieces were later compared using a non-crystallizing rubber SBR (a styrenebutadiene copolymer) [3]. This material was found to differ from strain-crystallizing NR in that steady time-dependent tearing occurred at T values below that required to produce the very rapid 'catastrophic' tearing. Thus from each test piece a rate of crack growth versus tearing energy plot could be obtained. Comparison of the experimental results showed good agreement, all test pieces giving essentially similar relations.

Recently a new, 'angled' test piece has been developed. It provides a further check on the theory and is discussed in the following section.

Angled test piece

The test piece is shown in Fig. 2. It consists of a rectangular sheet of rubber rigidly clamped along its two longer edges. A cut is inserted to a

length somewhat greater than the distance between the clamps and these are then angled relative to each other. There is thus a portion of rubber which is effectively unstrained (in fact somewhat buckled) in region A. The crack must be sufficiently long to ensure that regions B are also unstrained. Between A and B there is a complex region of strain (D) in the neighbourhood of the crack tip.

If the angle (a) between the clamps is kept constant and a force F is applied to produce tearing then the tearing energy can be derived as follows.

The strain energy U in the test piece is a function of the crack length c and the distance [l] between the points of application of F. Thus

$$dU = \left(\frac{\partial U}{\partial c}\right)_l dc + \left(\frac{\partial U}{\partial l}\right)_c dl$$

and from this

$$\left(\frac{\partial U}{\partial c}\right)_l = \left(\frac{\partial U}{\partial c}\right)_F - \left(\frac{\partial U}{\partial l}\right)_c \left(\frac{\partial l}{\partial c}\right)_F$$

The term $(\partial U/\partial c)_F$ is zero for this test piece as, under constant force, the region of complex strain D merely moves on by a distance equal to the increment of the crack length and its strain energy remains constant, albeit unknown in magnitude. The quantity $(\partial U/\partial l)_c$ is equal to the applied force F and $(\partial l/\partial c)_F$ can be readily seen from the geometry of the test piece to be $2 \sin a/2$. The tearing energy is therefore

$$-\frac{1}{h}\left(\frac{\partial U}{\partial c}\right)_l = T = \frac{2F}{h} \sin a/2. \tag{3}$$

Thus, as is the case for the three test pieces discussed earlier, the relation for T for the angled test piece can be derived with no recourse to solutions of the complex strain distribution which exists around the crack tip.

Equation 3 shows that the relation between T and the tearing force does not depend on the elastic properties of the rubber, the test piece dimensions (other than thickness) or the crack length. For $a \doteq 180°$ this relationship reduces to that for the trousers test pice (cf. equation (2)). The particular characteristic of the angled test piece is that the force F for a given T value can be made large if a is sufficiently small.

The experimental arrangement is shown in Fig. 3. Because the line of action of the force moves as the crack grows, a long aluminium rod was attached rigidly to one clamp and restrained by pairs of strings from altering the angle a. The weight of the rod and clamp was taken by other

The fatigue failure of rubber is thought to be essentially a crack growth process from small flaws [9, 10]. If fatigue in simple extension is considered, then the case of a test piece with a small edge flaw must be solved. As the strains are finite, Inglis' and Griffith's theories cannot be applied precisely. For a semi-infinite strip in simple extension containing an edge crack of length c with a tip of diameter much less than c, it can be seen from geometric similarity that the reduction in strain energy due to the crack is given by [1]

$$U' = kc^2W \qquad (6)$$

where W is the strain energy density in the body of the test piece and k is a function of the strain. For classically small strains $k = \pi$, but the finite strain solution has not been found. Attacking the problem experimentally by, in effect, measuring U' as a function of c and bulk strain, Greensmith [11] found that k decreased steadily from π at zero strain to about 1·7 at 200% strain.
Differentiating equation (6) gives

$$T = 2kWc \qquad (7)$$

and using this relation with Greensmith's values for k the validity of the fracture mechanics approach has again been checked for crack growth under repeated stressing by comparison with results from other test pieces [9]. Agreement is satisfactory.

The crack growth behaviour can therefore be expressed as

$$\frac{dc}{dn} = f(T) \qquad (8)$$

where (dc/dn) is the crack growth per cycle, and T is the maximum tearing energy attained during each cycle. For natural rubber at fairly high tearing energies $f(T)$ is approximately proportional to T^2, whereas for non-crystallizing rubbers such as SBR it is proportional to T^4 or even higher powers [9, 10]. If

$$\frac{dc}{dn} = BT^2 \qquad (9)$$

is taken, then using equation (7) it can easily be shown that the number of cycles required to increase an edge crack in length from c_0 to c_1 is given by [9]

$$n = \frac{1}{B(2kW)^2}\left(\frac{1}{c_0} - \frac{1}{c_1}\right)$$

If the strip is repeatedly stressed until failure occurs, and the crack length at final rupture is much greater than c_0, then the number of cycles to failure, i.e. the fatigue life, is

$$N = \frac{1}{B(2kW)^2\, c_0} \qquad (10)$$

This predicted dependence on strain has been tested experimentally, and within the range of validity of equation (9) gives good agreement both for test pieces with artificially introduced flaws of known size and for test pieces containing no intentionally introduced flaws [9]. In the latter case the effective size of the flaws initiating failure can be estimated as about $2{\cdot}5 \times 10^{-3}$ cm. The fracture mechanics approach therefore enables the interrelation of crack growth and fatigue behaviour to be quantitatively expressed.

The particular form of the crack growth relation varies from one rubber to another [12], as mentioned above. It is believed that the form is largely governed by the mechanical hysteresis at the large strains in the region of the crack tip [13, 14]. The particular form of $f(T)$ given by equation (9) is appropriate to a material showing large high-strain hysteresis. It is interesting to note that a similar relation to equation (9) has been proposed for metals [15] and this suggests that, although the detailed mechanisms involved will be different, essentially similar factors are important in both cases.

The crack growth equation (9) does not hold for low tearing energies. Below a value T_0 the rate of crack growth under repeated stressing is effectively zero [16]. Corresponding to this is a fatigue limit below which the life of a test piece is virtually infinite. The T_0 value is found experimentally to be about 5×10^4 erg/cm^2. This is much greater than normal estimates of true surface energies. However, by considering the molecular constitution of vulcanized rubber (long flexible molecules crosslinked at intervals) it has proved possible to explain this magnitude purely theoretically in terms of molecular bond strengths and the flexibility of the polymer molecules [14]. The agreement is within a factor of two, which is very satisfactory considering the difficulties of such a theory.

Ozone attack on rubber

The fracture mechanics approach has also proved valuable in the study of crack propagation in stressed rubber in the presence of ozone. Ozone causes chemical scission of carbon-carbon double bonds in the rubber molecules and thus allows crack growth to occur. Braden and Gent [17] have studied this process using a tensile test piece with an edge crack. This test piece was necessary as they found that the energy required for crack propagation to occur was very low, being of the order of

100 erg/cm^2. This is about the magnitude expected for the surface energy of a liquid such as is likely to be produced at the tip by degradation of the rubber by ozone. At higher tearing energies the rate of ozone crack growth is independent of T but proportional to the ozone concentration. As this is another crack growth process which is superimposed on that due to mechanical action it can contribute to fatigue failure. Development of this approach along the lines described above shows that the combined effects of the corrosive atmosphere and repeated stressing can also be predicted satisfactorily [12].

Conclusions

We have attempted to show how the fracture mechanics approach involving the energetics of crack propagation can be successfully applied to rubber. By using test pieces of widely different shapes it has been possible to check fairly thoroughly the validity of the approach. Within the limits of the experimental variability it has been found to hold for the various types of crack growth and tearing that can occur for both crystallizing and non-crystallizing rubbers which have a marked difference in their strength properties. The range of energies involved is from about 10^2 to 10^8 erg/cm^2 corresponding to a factor of about 10^9 change in rate of growth. The crack growth characteristic has emerged as a key strength property, which has made it possible for not only fatigue failure but also tensile strength properties to be correlated and understood [8, 18, 19].

The relation between the strain concentration and tearing energy, which holds for any shape of test piece and for finite strains, has also enabled the practically important value T_0, which governs the fatigue limit, to be calculated approximately from purely molecular considerations.

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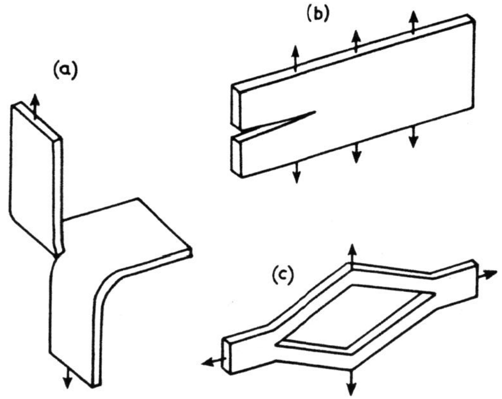

Fig. 1. Various tear test pieces: (a) trousers; (b) pure shear; (c) split.

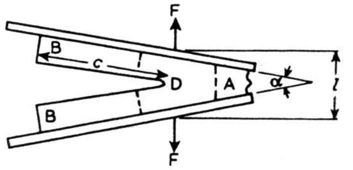

Fig. 2. Angled test piece.

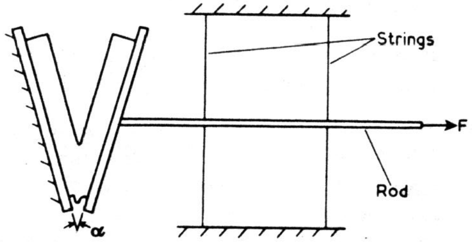

Fig. 3. Experimental arrangement for the angled test piece. The strings prevent changes in the angle (α) between the clamps as the crack grows.

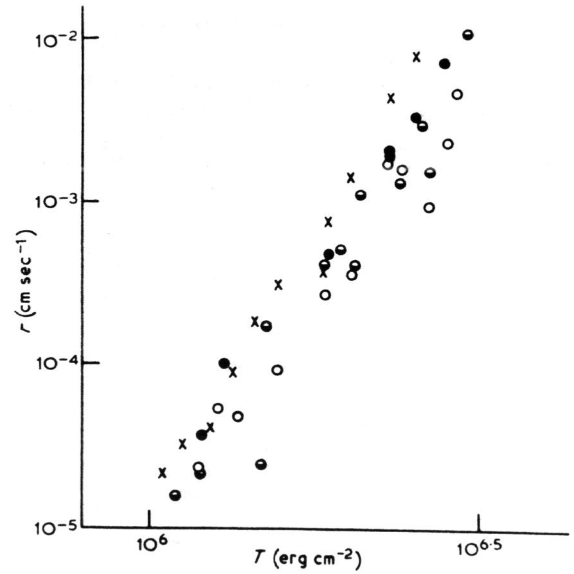

Fig. 4. Observed rate of crack growth r versus tearing energy T for the angled test piece (○ angle α between clamps = 24°; ⊖ α = 32°; ● various α) and for the trousers test piece (×).