PAPER 65 (SESSION V)

Combined stress crack propagation in thin panels

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Summary

Measurements have been made of the rates of fatigue crack propagation and associated crack tip stresses in tensioned cracked panels subjected to acoustic loading. The alternating stresses in such a situation derive wholly from the lateral vibration of the panel. The basic fatigue crack propagation data is readily obtainable but, contrary to the usual situation in fatigue testing, it is not at present possible to deduce from the applied loading conditions alone the magnitudes of alternating stresses in the panel.

Both direct and bending components of alternating stress are present and, as crack growth occurs, absolute and relative magnitudes of these components change. The signals from strain gauge pairs placed ahead of the crack tip have been arranged to give outputs proportional to total stress according to the Ang-Williams theory, and these have then been extrapolated to the crack tip. Crack tip stress intensities have been compared to the corresponding crack propagation rates, and the comparison shows consistent agreement between the data points and a trend which agrees well with that found by other investigators using simpler test configurations.

Introduction

It has been established that fatigue crack growth may take place from the tips of existing cracks in thin metal panels subjected to severe turbulent boundary layer loading [1]. Such loading may occur in certain fuselage and wing panels of modern aircraft, particularly those in the neighbourhood of the engines. The acoustic fatigue facility [2] was originally devised to investigate the phenomenon on simple specimens representative of the full scale case.

In the presence of severe boundary layer excitation, lateral vibration of the panel occurs giving rise to dynamic stresses which are characteristically of low magnitude and high frequency. The paper discusses work carried out to determine crack propagation rates in simple sheet specimens, and the associated crack tip dynamic stresses. Since there is a well established correlation between crack propagation rate and dynamic stress intensity in simple loading configurations, a similar comparison for the present complex loading conditions has been the main aim of this investigation.

Acoustic test facility

The test facility consists basically of an acoustic tunnel and a specimen loading frame. The latter is designed for panels 10 in wide with 14 in between grips, and for the normal specimen thickness of 0·064 in a gross uniaxial stress up to 15,000 lb/in^2 may be applied. The frame is located in the side of the tunnel which comprises a long duct with a siren at one end. Sound levels up to 150 dB may be generated in the frequency range 90-1200 Hz. Broad band noise in this range is applied to the siren so that an essentially flat response is obtained in the tunnel. The specimen acts as a filter, responding only to those frequencies corresponding to natural modes of vibration. These response frequencies are all well within the frequency range of the exciting noise.

Panel behaviour

It is not possible to initiate a fatigue crack in a solid panel in the present test facility. Thus, an artificial slit is always inserted to provide a starting point for the crack propagation process. In a typical test situation beginning with a crack 3 in long, the panel vibrates in an essentially first mode manner and crack propagation occurs at a very low rate. As the crack grows the natural frequency of vibration decreases and the amplitude of the central region surrounding the crack increases. Crack propagation rate also steadily increases. When the crack reaches a certain critical length buckling of the panel occurs, but unlike the static case the direction of the buckle continually changes, so that a large amplitude vibration occurs. This continues for some time, during which crack growth occurs at an increasing rate. A stable buckled position is finally established and, with the panel permanently buckled, the crack continues to grow until final static failure occurs. The duration of a single test is usually of the order of 10-50 hours depending upon load levels.

Thus, crack propagation rates may be readily determined, but because the test configuration is but an approximation to any practical situation, and vibration characteristics (and therefore stresses) are critically dependent upon edge conditions as well, any direct application of the results would be extremely uncertain. The vital information which is missing is that of dynamic stress, without which no real progress is possible.

Panel stresses

In conventional fatigue testing the configuration is such that the gross alternating stress level is known, and that at the crack tip is readily calculable. In the present case such a desirable situation does not exist – it is not possible to calculate dynamic stresses at any point in the panel from the input parameters of applied tensile stress, and noise level. Considerable previous work has been carried out to predict the mode shape and frequency response characteristics of the panel [3], and present

work is aimed at using this information to predict the stress pattern. The problem is, however, extremely difficult, and it will be some time before predicted stress data is available. In order to provide the essential verification of the theory and to give currently needed estimates, the dynamic stresses in the crack tip region have been measured experimentally.

Measurement of crack tip stress

The basic scheme used to determine the alternating crack tip stress relies on extrapolation of the stress measured at three points ahead of the crack tip. The uncertainties associated with this scheme will be clear enough, but it is, perhaps, the only practical method available for this particular experimental situation.

A series of stress plates were prepared, each with a different crack length covering the range over which fatigue crack propagation occurs. Each plate was provided with sets of three 3 mm strain gauges on each side of the sheet and placed close to and ahead of the crack tip at an angle of 30° to the line of loading (60° to the line through the crack extended), as in Fig. 1. Since one of the principal axes is in the crack-extended direction, the strain gauges give an output which is proportional to principal stress [4], in this case the stress in the direction normal to the crack.

In this test situation, both direct (σ_d) and bending (σ_b) components of alternating stress are present. It has been shown [5, 6] that for combined loading the stress intensity close to the crack tip is

$$\{\sigma_d + [(1+\nu)/(3+\nu)]\,\sigma_b\}\sqrt{(a)}$$

where ν is Poisson's ratio and a is the semi-crack length. For the aluminium alloy tested, $\nu = 1/3$, so that the above becomes $(\sigma_d + 0{\cdot}4\sigma_b)\sqrt{(a)}$ very closely. Thus, as a first step, bending and direct stresses may be measured at each gauge position, extrapolated to the crack tip, and then combined as above to give the crack tip alternating stress intensity.

However, since random stresses are involved, there is an additional problem. The random direct and bending stresses are not acting independently. In fact there is a very high degree of correlation between them. To determine the sum of non-independent combined random processes the correlation between them must also be included.* Since the separate

* In terms of probability theory, the variance (mean square) of the sum of two quantities is the sum of each separately plus twice the co-variance between them, or, in terms of spectral densities, that of the combined quantities must also include the two cross spectral densities.

individual components are not required, but only the resultant, it is preferable to combine the signals before analysis, rather than after it. This may be done as follows.

At a given instant, suppose the output from one gauge is $\sigma_d + \sigma_b$. Then, by definition, the output from that on the other side of the sheet is $\sigma_d - \sigma_b$. These two may be combined so that

$$(\sigma_d + \sigma_b) + c_1(\sigma_d - \sigma_b) \propto \sigma_d + 0{\cdot}4\sigma_b$$

i.e.

$$(1 + c_1)\sigma_d + (1 - c_1)\sigma_b \propto \sigma_d + 0{\cdot}4\sigma_b$$

This gives $c_1 = 0{\cdot}428$. Thus by applying $0{\cdot}428$ of the voltage applied to the first strain gauge circuit to the second, and adding the two resulting signals, a single output proportional to $\sigma_d + 0{\cdot}4\sigma_b$ is obtained.

The combined signal at each gauge position is proportional to $(\sigma_d + 0{\cdot}4\sigma_b)\sqrt{(a/2r)}$, where r is the distance of the strain gauge from the crack tip. By dividing each measured r.m.s. value by its corresponding $\sqrt{(a/2r)}$, there remains only the r.m.s. of the reduced combined stress level, $\sigma_d + 0{\cdot}4\sigma_b$. In any usual situation this reduced stress will be the same at each strain gauge position, so that extrapolation to the crack tip represents no problem – in fact, a single strain gauge pair would suffice. In the present case, however, the reduced stress is not identical at each position, but increases towards the crack tip. This is because the central region surrounding the crack vibrates predominantly in a first mode manner and both σ_d and σ_b increase towards the crack tip. This fact makes extrapolation a little more difficult, although, it is believed, does not introduce any substantial error. Multiplication of the extrapolated value of $(\sigma_d + 0{\cdot}4\sigma_b)$ by $\sqrt{(a)}$ gives the required alternating stress intensity at the crack tip.

There is one final modification to be made. Although the vibration characteristics are mainly first mode, there are also many higher modes present. This means that there is an excess of extrema over zero crossings. A typical spectral density plot of stress is shown in Fig. 2. Rice, Beer and Paris [7] have shown that the mean rise, $\bar{h}$, in such a signal is given by

$$\bar{h} = \sqrt{(2\pi)}\,\sigma\,\frac{N_0}{N_e}$$

where σ is the r.m.s. level and N_0 and N_e are the numbers of zero crossings and extrema respectively per second. Further, it was shown that these latter may be readily obtained from moments of the power spectrum.

In addition to the r.m.s. level of each strain gauge pair, the signal was recorded for subsequent analysis. It has been found that, for any given loading condition, the shape of the power spectrum at each strain gauge position is identical, and thus it may reasonably be presumed to have this shape also at the crack tip. Values of the ratio N_0/N_e range from $0{\cdot}5$ to $0{\cdot}8$, but are commonly in the region of $0{\cdot}6$. Inclusion of $\sqrt{(a)}$ in the above expression gives the final estimate of alternating stress intensity at the crack tip as

$$\Delta K = \sqrt{(2\pi a)}\,N_0/N_e$$

where σ is the extrapolated r.m.s. value. Values of ΔK fall within the range 500 to 1500 $\text{lb/in}^2\sqrt{(\text{in})}$, and are thus in the lower regions of published fatigue stress intensity data.

Fatigue crack propagation

Several plain panels of 5070 aluminium alloy have been tested in the facility to provide the basic crack propagation data. Each specimen was provided with an initial slit and a pair of crack length gauges, one at each end of the slit. The crack length gauges consist of a grid of fine parallel foil strands mounted on a thin plastic backing. These are bonded to the specimen and each strand is connected electrically to a clock. As the crack passes each strand, that strand breaks, and the corresponding clock stops. By this semi-automatic means an incremental crack length versus time history is obtained, from which time rates of crack propagation are measured. Using the number of peaks per second, N_e, from the appropriate stress panel power spectrum, these time rates are readily converted into more meaningful crack propagation rate units of inches per stress rise.

Comparison of crack propagation rate with crack tip stress intensity

The comparison for the fatigue data to hand so far is shown in Fig. 3. It is seen that there is consistent agreement between the data in that the slope of the curve is positive. This fact is strong evidence of the importance of the Poisson factor $(1+\nu)/(3+\nu)$ in the combined stress case as becomes clear from the following. At the lowest crack propagation rates (corresponding to short crack lengths), the stress intensity is almost wholly composed of the bending component only. At higher rates of crack propagation (corresponding to longer crack lengths) the direct component assumes progressively more importance, until at the highest crack propagation rates (longest crack lengths) it becomes the dominant component. Thus, were the data derived without the Poisson factor on bending stress, those points at the left side of Fig. 3 would plot at an ordinate level approximately $2{\cdot}5$ times higher, and the intermediate points would also plot at higher but reducing values. It is possible, in fact, that a curve of negative slope would result, thereby making interpretation somewhat difficult. It may be concluded that the Ang-Williams theory provides the essential correlation required for crack propagation data in the combined stress case. The pure bending case has already been examined [8] and when modified by the Poisson factor gives good agreement with direct stress results.

Data from other sources on the same material [9] are also shown in Fig. 3. The relative positioning of the present data suggests that the extrapolated estimates of crack tip stress intensity are certainly of the correct order. The data evidently fall a little below those of other investigators, indicating a higher rate of crack growth for a given stress intensity. This is to be expected, since the relative mean stress here (15 to 25) is rather higher than usual. The fact that the present data are mostly at rates of propagation substantially lower than usual makes direct evaluation of the effect of mean stress uncertain at this stage.

If it is generally true that crack propagation data obtained under given overall conditions where the only variable is the type of dynamic loading (i.e. direct, bending, or any combination) fall on a single stress intensity versus propagation rate plot, then this has the greatest ramifications so far as data collection is concerned – it would be necessary to obtain this basic data only under those loading conditions which are most convenient.

Conclusions

The present work suggests that the direct application to fatigue of the Ang-Williams theory for combined stresses results in a most satisfactory correlation between crack propagation rate and alternating stress intensity when both direct and bending stresses are present. Satisfactory correlation has already been indicated between comparative direct and bending results, and it therefore appears likely that a single stress intensity – crack propagation rate relationship will hold, irrespective of the relative proportions of the stress components, for given overall parameters. Should this conclusion prove to be a general result, there seems to be no reason why reliable estimates of crack propagation rate should not be made in any practical situation, provided only that the basic data is available and the (crack tip) stress intensities calculable. It is clear that as such information is required for increasingly more complex situations, accurate estimation of stress intensity is likely to become the limiting factor.

Acknowledgements

The author is indebted to the Public Service Board of the Commonwealth of Australia for the granting of an overseas scholarship. He also expresses

his gratitude to his supervisor, Professor B. L. Clarkson, for his guidance and advice. In addition, the research has been sponsored in part by the Ministry of Technology under Ref. No. PD/40/040/ADM, and the Air Force Materials Laboratory, Research and Technology Division, through the European Office of Aerospace Research (OAR), United States Air Force, under contract AF 61(052)-862.

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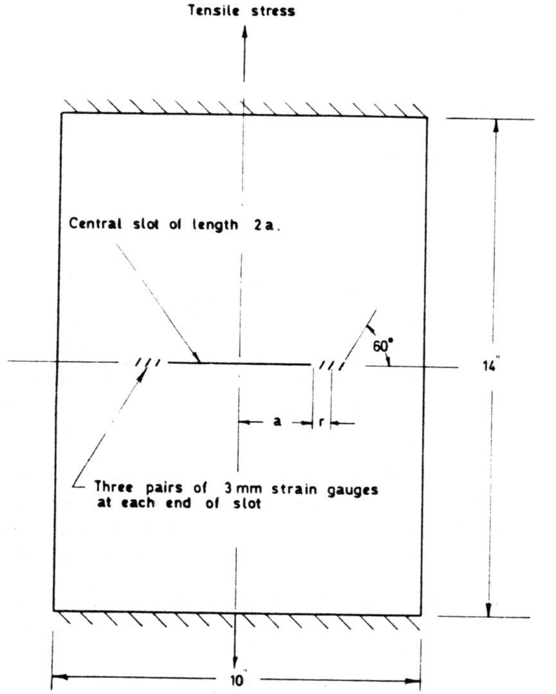

Fig. 1. Stress piate configuration.

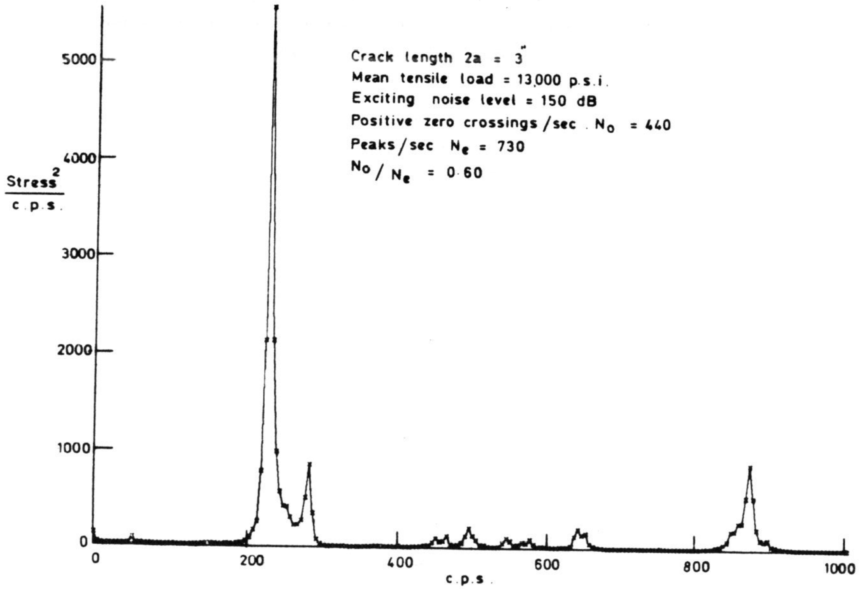

Fig. 2. Spectral density plot of stress.

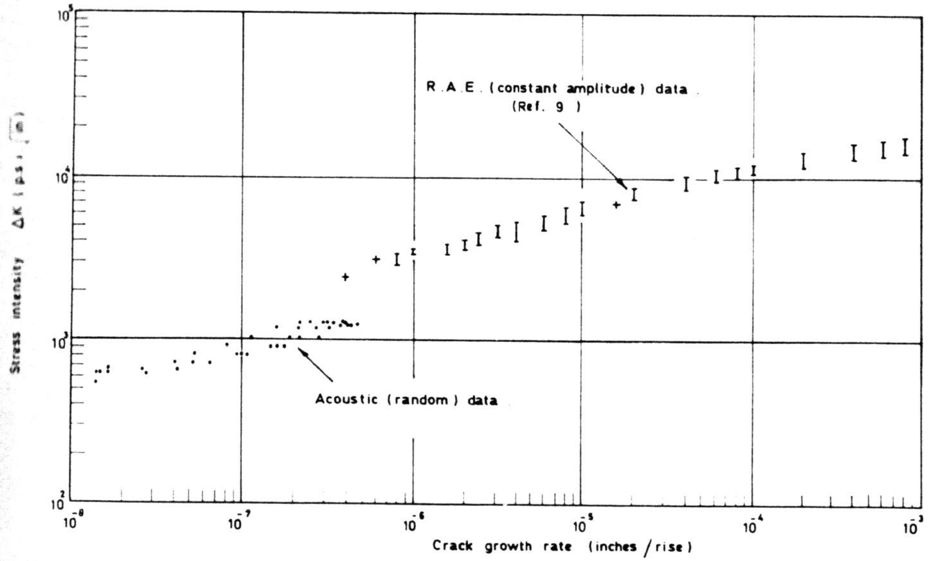

Fig. 3. Stress intensity vs. crack propagation rate (5070 Al. Alloy).