PAPER 20 (SESSION II)

The relations between the applied load at fracture, the critical fracture stress, the proof stress and the plastic stress-concentration factor in high-strength steels

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Summary

This paper discusses the values and importance in the fracture at notches of high-strength steels, of the plastic stress concentration factor, $R = (\sigma_{1\,max}/\sigma_{Y'})$, where $\sigma_{1\,max}$ is the highest tensile stress in the volume beneath the notch and $\sigma_{Y'}$ is the plain tensile proof stress. It is assumed, following a number of authors, that fracture occurs when $\sigma_{1\,max}$ reaches a critical value σ_F

If R is put equal to a linear function of $(\sigma_{nb}/\sigma_{Y'})$, where σ_{nb} is the maximum fibre stress at the root of the notch, ignoring the elastic stress concentration, and σ_F is put equal to a linear function of $\sigma_{Y'}$, it is shown that σ_{nbs}, σ_{nb} at fracture, will, if fracture occurs before general yield, decrease with increase in $\sigma_{Y'}$ unless $(d\sigma_F/d\sigma_{Y'}) >$ about 0·3 to 0·8. Such an increase in fracture stress is unlikely and thus the model is consistent with the decrease in notch toughness often found with increase in proof stress. The critical value of $(d\sigma_F/d\sigma_{Y'})$ depends on R and therefore the determinations of R by other authors are discussed and it is decided that if the von Mises yield criterion holds a probable value of R up to general yield is given by

$$R = 0{\cdot}79\left\{\frac{\sigma_{nb}}{\sigma_{Y'}} - 0{\cdot}323\right\} + 1{\cdot}12$$

Introduction

Another paper discusses [1] the effect on the breaking load of the size and tensile proof stress of notched bend test pieces geometrically similar to a Charpy V-notch test piece (Fig. 1) and broken in three point bending before general yield i.e. before plastic deformation spreads across the testpiece. It was shown (Figs. 2 and 3) that the breaking loads in notch bend of the secondary-hardening steels reported upon in the paper, decreased with increase in tensile proof stress when this was greater than about 120 kgf/mm^2. This negative correlation existed whether the proof stress was varied by altering the tempering temperature or the temperature of test. (Some further details of the tests shown in Figs. 2 and 3 are given in an Appendix).

It will be assumed in this paper that fracture initiates when a critical tensile stress, σ_F, is reached at a point in the volume beneath the notch [2-10]. This concept gives rise to a number of difficulties, in particular, it does not explain the decrease in nominal breaking stress, σ_{nbs},(defined in Fig. 1) with increase in size of geometrically similar and metallurgically identical test pieces. One reason is that σ_F is taken as that calculated by continuum mechanics, and when properties of the real material are considered, a number of explanations of this size effect can be advanced, but the interrelation of these is complex and will be discussed in a separate paper [11].

However, leaving the difficulties on one side, useful deductions can be made, for fracture before general yield, using the concept of a critical fracture stress, σ_F, and these follow.

The effect of strain hardening prior to general yielding is ignored in this paper since the strains involved are small [1] and the maximum stress occurs at the boundary between the areas of elastic and plastic strain [6]. Thus measurements made on mild steel can be applicable to alloy steels provided that both materials are isotopic and the size of the grains is a small fraction of the notch root radius.

General relations between σ_{nbs}, σ_F, σ_Y and R

In a paper by Rendall and Earley [1], it is shown that the decrease in nominal breaking stress, σ_{nbs}, or in the net stress at fracture in notched tensile tests, often found with increase in proof stress can be explained using the relationship between $R = (\sigma_{1\max}/\sigma_Y)$ and (σ_{nb}/σ_Y) given by curve d of Fig. 4. However, since as shown below there are various calculations for R which differ widely, it is useful to consider some more general relations between σ_F, σ_Y, σ_{nbs} and R.

First, since the relation between σ_F and σ_Y is unknown it can be taken for simplicity as linear as in Fig. 5.

$$\sigma_F = k_1 + k_2\,\sigma_Y \tag{2}$$

k_1 being a positive parameter and k_2 a positive or negative parameter.

Similarly the relation between R and σ_{nb}/σ_Y can be taken as linear but, if the Tresca yield criterion holds, R must equal 1, when, owing only to the elastic stress concentration at the root of the notch, the stress at the root equals σ_Y. The elastic stress concentration for the standard V-notch Charpy test piece is according to Leven and Frocht [12] equal to 3·47 (the inverse of which is 0·288), therefore, a suitable equation is

$$R - 1 = k_3\left\{\frac{\sigma_{nb}}{\sigma_Y} - 0{\cdot}288\right\} \tag{3}$$

k_3 being a positive parameter. In Fig. 6, line OA of slope equal to the elastic stress concentration factor, 3·47, gives $\sigma_{1\max}/\sigma_Y$ when all the strains are elastic and line AB of slope k_3, gives R, the plastic stress concentration factor.

If conditions at fracture, where $\sigma_{nb} = \sigma_{nbs}$, the nominal breaking stress, and $\sigma_{1\max} = \sigma_F$, are considered equations [1] and [2] can be substituted into equation [3] giving:

$$\frac{k_1 + k_2\,\sigma_Y}{\sigma_Y} - 1 = k_3\left\{\frac{\sigma_{nbs}}{\sigma_Y} - 0{\cdot}288\right\} \tag{4}$$

re-arrangement gives:

$$\sigma_{nbs} = \frac{k_1}{k_3} + \sigma_Y\left\{\frac{k_2^{-1}}{k_3}\right\} + 0{\cdot}288$$

Differentiating with respect to k_1 taking σ_Y, k_2 and k_3 as constant, gives

$$\frac{d\sigma_{nbs}}{dk_1} = \frac{1}{k_3} \tag{5}$$

Since k_3 is positive, σ_{nbs} increases with increase in k_1. This is to be expected since any increase in the fracture stress should increase the load bearing ability of a notched component.

Differentiating equation (4) with respect to σ_Y (k_1, k_2, k_3 being taken as constant) gives

$$\frac{d\sigma_{nbs}}{d\sigma_Y} = \frac{k_2^{-1}}{k_3} + 0{\cdot}288 \tag{6}$$

therefore σ_{nbs} will only increase with increase of σ_Y.

if

$$k_2 + 0{\cdot}288\,k_3 > 1 \tag{7}$$

If the von Mises yield criterion and plane-strain conditions are assumed to hold, equation (3) and (7) are replaced by

$$R - 1{\cdot}12 = k_3\left\{\frac{\sigma_{nb}}{\sigma_Y} - 0{\cdot}323\right\} \tag{8}$$

$$k_2 + 0{\cdot}323\,k_3 > 1 \tag{9}$$

Now, it is shown later, Fig. 4, that a range of values of k_3 between about 0·6 and 2·5 will encompass the whole series of calculated values of R plotted against σ_{nb}/σ_Y – the larger values of k_3 being rather improbable. Thus, unless k_2 is greater than 0·3 for $k_3 = 2{\cdot}5$ and 0·8 for $k_3 = 0{\cdot}6$,

σ_{nbs}, if fracture occurs before general yield, will decrease with increase in σ_Y. Since such increases of σ_F with σ_Y are unlikely, the relationship in equation (7) is consistent with the decrease in load bearing ability with increase in yield strength, which is usually found.

These relations are based on the Charpy V-notch specimen, but any increase in notch sharpness will merely decrease the constants 0·288 or 0·323 in equation (7) and (9) and this change will only slightly affect the argument. Again a decrease in notch sharpness will not have a large effect as long as fracture occurs before general yield, but this last condition is not likely to be satisfied for a very blunt notch.

Estimations of the relation between R and σ_{nb}/σ_Y

It is useful to consider some estimates of the relation between R and σ_{nb}/σ_Y that are available for Charpy V-notch test pieces. The value of σ_{nb}/σ_Y at general yield is taken as equal to 1·95, the figure obtained by Alexander and Komoloy [13] using a slip-line field calculation. Curve *a* of Fig. 4 was derived by Wilshaw, Rau and Tetelman [7]; curve (*b*) is due to Wilshaw and Pratt [8]. Both groups of authors, Wilshaw, Rau and Tetelman [7] and Wilshaw and Pratt [8], used the Tresca yield criterion, but these curves have been recalculated by making $k = (\sigma_Y/\sqrt{3})$, where k is the yield stress in shear. The alteration was made in order that a direct comparison could be obtained with the calculated curves of Allen, Earley and Rendall [2]. curve c and Knott [5], curve *d*. The solid part of curve c was calculated by Allen *et al* [2] by relaxing the elastic stress fields of Hendrickson, Wood and Clark [6] to allow for plastic deformation. In the original calculation, which broke down at $R = 1{\cdot}93$ and $(\sigma_{nb}/\sigma_Y) = 0{\cdot}90$, the curve was continued to the point $R = 1{\cdot}93$, $(\sigma_{nb}/\sigma_Y) = 1{\cdot}95$ i.e. the value of R was calculated from the paper of Alexander and Komoloy [13] using the Tresca criterion. In fact, since the calculation was based on the von Mises criterion it should, if based on Alexander and Komoloy's calculation, finish at $R = 2{\cdot}23$ for $(\sigma_{nb}/\sigma_Y) = 1{\cdot}95$. This point has been joined to the calculated curve by a dotted straight line in Fig. 4.

The Wilshaw and Pratt [8-10] results used by Knott only allow R to be calculated between σ_{nb}/σ_Y from 0·90 to 1·95 and the line between this point and the limiting point of elastic failure is interpolated.

In Fig. 4, the point of elastic failure is plotted at $R = 1{\cdot}12$, $(\sigma_{nb}/\sigma_Y) = 0{\cdot}323$. The slope of the elastic line is that derived photo-elastically by Leven and Frocht [13]. The value of $R = 1{\cdot}12$ is derived, assuming plane-strain conditions, from the von Mises yield criterion using a value of Poisson's ratio of 0·28. If there is no strain hardening, at σ_x, the root of the notch will remain equal to $1{\cdot}12\ \sigma_Y$ as deformation progresses.

Application of the calculated R values to the results plotted in Figs. 2 and 3.

We can use the curves in Fig. 4 to calculate σ_F. The values of σ_F can be calculated either from the individual results given by Rendall and Earley [1] or from the least squares lines given by equation (1a) and (2a) in the appendix for the 10 and 25 mm square specimens. Figs. 7 and 8 give both these. The closed points in Figs. 7 and 8 represent the values used to calculate equations (1a) and (2a), the open points represent other steels tested by Rendall and Earley [1], which for various reasons were more brittle than those used to calculate equations (1a) and (2a). The values of σ_F have been calculated using curves *b* and *d* of Fig. 4; values from curves *a* and c were not used since curve *a* differs only slightly from curve *b* and curve c is incomplete in that it is only estimated for values of σ_{nb}/σ_Y above 0·9.

It will be seen from Fig. 7 that, using curve *b* of Fig. 4, i.e. the curve calculated by Wilshaw and Pratt [8], when σ_Y is less than about 170 kgf/mm², the value of σ_F is always $2{\cdot}36\,\sigma_Y$. The slope of the curve causes the σ_F values calculated from the least squares lines (i.e. the curves marked 10 mm and 25 mm) to rise at first to 387 and 419 kgf/mm² respectively and then fall. It is, of course, not impossible that σ_F would behave in this manner, but it is rather unlikely. The scatter of the σ_F values calculated from the individual tests is given in Table 1.

The calculated σ_F values in Fig. 8 derived from the least squares lines using curve *d* of Fig. 4 shows with increase in σ_Y, first a slight rise and then a fall; the scatter of σ_F values is given in Table 1. There is better correlation between σ_F calculated from the least squares lines and σ_F calculated from the individual points in Fig. 8 than in Fig. 7. Thus it can be argued that the relations between R and σ_{nb}/σ_Y is more likely to be represented by curves such as *d* in Fig. 4, than by *a* or *b*.

Calculations of R using least squares regression lines

There is a further method of investigating the relationship between R and σ_{nb}/σ_Y. The following assumptions are made:

(i) that the von Mises Yield Criterion holds and that plane strain conditions exist.

(ii) that at intersection of the extrapolated least squares line and elastic failure line in Fig. 2, $\sigma_F = \sigma_Y = 227{\cdot}8$ kgf/mm².

(iii) that σ_F is constant at the value of 227·8 kgf/mm² all along the least squares line in Fig. 2. If these assumptions are put into equations (2) and (8) we get the values for the parameters in these equations of

$$k_1 = 227{\cdot}8\ \text{kgf/mm}^2,\ k_2 = 0\ \text{and}\ k_3 = 0{\cdot}452.$$

The line for R based on $k_3 = 0{\cdot}452$ is plotted as line e in Fig. 4. This line lies below all the other calculated values of R in Fig. 4 which fact suggests that the hypothesis of a value of σ_F independent of σ_Y is incorrect.

Estimation of σ_F based on estimate of R by Knott [5]

If Knott's curve for the value of R against σ_{nb}/σ_Y is approximated by the straight line s in Fig. 4 then this line represents a value of $k_3 = 0{\cdot}76$. Using this value and the least squares lines in Figs. 2 and 3 the first two lines of Table 2 can be calculated. If the Tresca yield criterion is considered to hold then a line similar to s but passing through the co-ordinates in Fig. 4 of ($R = 1{\cdot}0$, $\sigma_{nb}/\sigma_Y = 0{\cdot}288$) and ($R = 2{\cdot}04$, $\sigma_{nb}/\sigma_Y = 1{\cdot}95$) is appropriate. The values of k_1, k_2, k_3 calculated using this line also appear in Table 2.

The curves of σ_F against σ_Y calculated from these values of k_1 and k_2 are given in Fig. 9. The differences in k_1 and σ_F are probably significant but not the differences in k_2 and the slopes of the curves in Fig. 9.

Conclusions

1. The reduction in nominal breaking stress in notched bend with increase in tensile proof stress found in these tests can be explained by the concept of a critical local maximum stress, σ_F, reached beneath the notch and resulting from plastic stress concentration due to triaxial stressing. It can be shown that unless $d\sigma_F/d\sigma_Y > 0{\cdot}3$ to $0{\cdot}8$ the nominal bending stress at fracture will decrease with increase in σ_Y.

2. It can further be shown that the most probable value of $d\sigma_F/d\sigma_Y$ is, in fact, about $-0{\cdot}76$ for 25 mm square test pieces; these values being obtained if the von Mises criterion of yield holds and lower values if the Tresca criterion is assumed to hold.

3. Of the various curves of plastic stress concentration factor, R against σ_{nb}/σ_Y for Charpy V-notch specimens discussed in this paper, that proposed by Knott [5], derived using data obtained by Wilshaw and Pratt [8,10], appear to be the most likely if the von Mises yield criterion is considered to apply to the data given in this paper.

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References

1. RENDALL, J. H. & EARLEY, C. C. *Notch bend tests on high strength steels*, to be published.
2. ALLEN, N. P., EARLEY, C. C. & RENDALL, J. H. 'Metallurgical and size effects in notched-bend tests', *Proc. Roy. Soc. A*, vol. 285, p. 120, 1965.

3. ALLEN, N. P. 'Effect of solute elements on the mechanical properties of iron' in *'Iron and its dilute solid solutions'*, ed. by Spencer, C. W. & Werner, F. E., Interscience Publisher, New York, 1963, p. 271.
4. KNOTT, J. F. & COTTRELL, A. H. 'Notch brittleness in mild steel', *J. Iron Steel Inst.*, vol. 201, p. 249, 1963.
5. KNOTT, J. F. 'On stress intensifications in specimens of Charpy geometry prior to general yield', *J. Mech. Phys. Solids*, vol. 15, p. 97, 1967.
6. HENDRICKSON, J. A., WOOD, D. S. & CLARK, D. S. 'Prediction of transition temperature in a notched bar impact test', *Trans. Amer. Soc. Metals*, vol. 51, p. 629, 1959.
7. WILSHAW, T. R., RAU, C. A. & TETELMAN, A. S. 'A general model to predict the elastic-plastic stress distribution and fracture strength of notched bars in plane strain bending', *Eng. Fracture Mech.*, vol. 1, p. 191, 1968.
8. WILSHAW, T. R. & PRATT, P. L. 'On the plastic deformation of Charpy specimens prior to general yield', *J. Mech. Phys. Solids*, vol. 14, p. 7, 1966.
9. WILSHAW, T. R. 'Deformation and fracture of mild steel Charpy specimens', *J. Iron Steel Inst.*, vol. 204, p. 936, 1966.
10. WILSHAW, T. R. & PRATT, P. L. 'The effect of temperature and strain rate on the deformation and fracture of mild-steel Charpy specimens', *Proc. 1st Inter. Conf. on Fracture*, Japanese Society for Strength and Fracture of Materials, 1966, p. 973.
11. RENDALL, J. H. *Size effects in the brittle fracture of steel'*, to be published.
12. LEVEN, M. M. & FROCHT, M. M. 'Stress-concentration factors for single-notch in flat bar in pure and central bending', *J. Appl. Mech.*, vol. 19, p. 560, 1952.
13. ALEXANDER, J. M. & KOMOLOY, T. J. 'On the yielding of a rigid/plastic bar with an Izod notch', *J. Mech. Phys. Solids*, vol. 10, p. 265, 1962.

Appendix

Details of tests plotted in Figs. 2 and 3

More complete details of the tests are given in an earlier paper [1], but the following is a brief outline. Composition wt%.

Steel	*C*	*Si*	*Mn*	*Ni*	*Mo*	*V*
1	0·30	0·75	1·54	–	2·07	0·51
2	0·46	0·76	1·51	–	2·12	0·48
3	0·31	0·03	0·54	2·60	2·02	0·51

The steels were laboratory made and after rolling were given the following heat treatment:

Steel	
1	Oil quenched from 1050°. Tempered at 400°, 500°, 600°C
2	Oil quenched from 1050°. Tempered at 400°, 500°, 600°C
3	Oil quenched from 1050°. Tempered at 500°, 600°, 690°C.

The steels were secondary hardening and were tempered at the peak hardness and at two other temperatures. They were machined into the 25 mm square test pieces shown in Fig. 1 and broken at a slow rate of strain in a three-point bending jig at three temperatures −196 °C, −78 °C and 20°C, twenty-seven tests in all. The 10 mm square test pieces (Fig. 1) and normal tensile test pieces were machined from the broken halves and tested at the same three temperatures. It was decided for the reasons given by Rendall and Earley [1] to analyse the results on the basis of the 0·5% tensile proof stress, though, in fact, the analysis would not have differed significantly if the 0·1% proof stress or the tensile strength had been used.

When the results were plotted in Figs. 2 and 3 it was seen that the nominal breaking stresses, calculated from the formula in Fig. 1, seemed to depend on the 0·5% proof stress, independently of whether the proof stress was varied by altering the tempering temperature or the temperature of test and in the paper the results are analysed on this basis.

The least squares regression lines calculated from the results in Figs. 2 and 3 are:

10 mm square test pieces $\sigma_{nbs} = 634{\cdot}5 - 2{\cdot}340\ \sigma_Y$ kgf/mm², equation (1a)

25 mm square test pieces $\sigma_{nbs} = 556{\cdot}1 - 2{\cdot}153\ \sigma_Y$ kgf/mm², equation (2a)

where σ_Y is taken as the 0·5% proof stress.

The average difference between the σ_{nbs} for the two sizes of test piece is 43·8 kgf/mm².

Table 1

Highest and lowest values of σ_F calculated using the points in Fig. 2 curves b and d of Fig. 4

Size of test piece	*Curve in Fig. 14*	*Based on calculations by*	*F kgf/mm²* *highest value*	*lowest value*	*range*
25 mm square	*b*	Wilshaw & Pratt 8	422	295	127
,,	*d*	Knott 5	339	238	101

Table 2

Values of k_1, k_2 and k_3 from values of R estimated from curve s of Fig. 4

Test-piece size (square)	*Yield criterion*	k_1 *kgf/mm²*	k_2	k_3
10 mm	von Mises	481·5	−0·90	0·76
25 mm	von Mises	421·8	−0·76	0·76
10 mm	Tresca	395·4	−0·64	0·62
25 mm	Tresca	346·9	−0·52	0·62

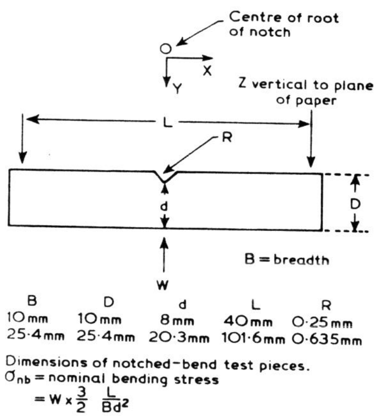


Fig. 1.

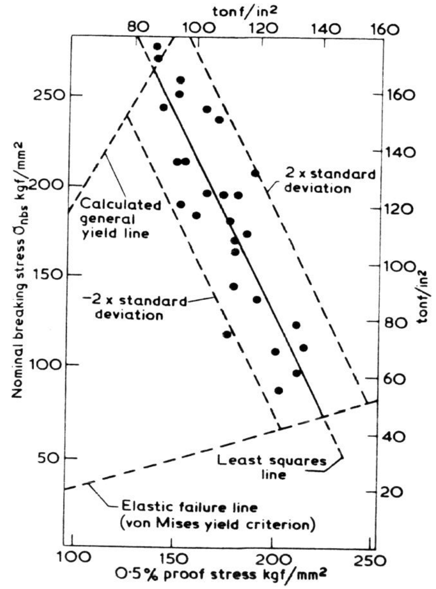


Fig. 2. Nominal breaking stress plotted against 0·5% proof stress, 25 mm test pieces.

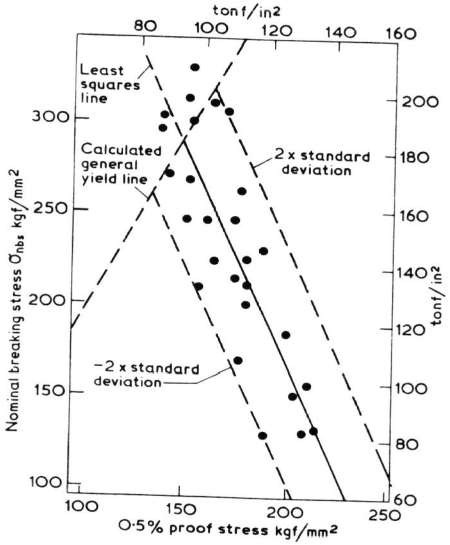


Fig. 3. Nominal breaking stress plotted against 0·5% proof stress, 10 mm test pieces.

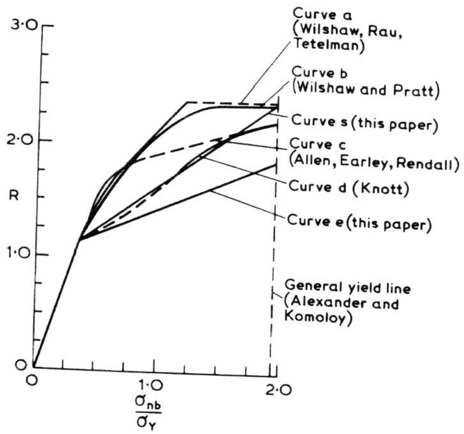


Fig. 4. Plastic stress concentration factor R (von Mises Criterion) plotted against σ_{nb}/σ_Y.

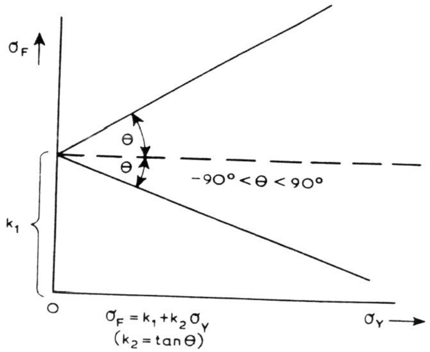
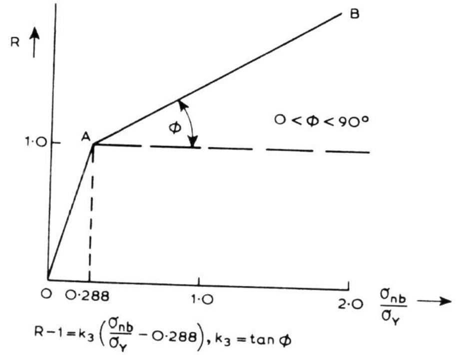


Fig. 5.



RELATIONSHIPS BETWEEN σ_F AND σ_Y AND R AND $\frac{\sigma_{nb}}{\sigma_Y}$

Fig. 6. Relationships between σ_F and σ_Y and R and σ_{nb}/σ_Y.

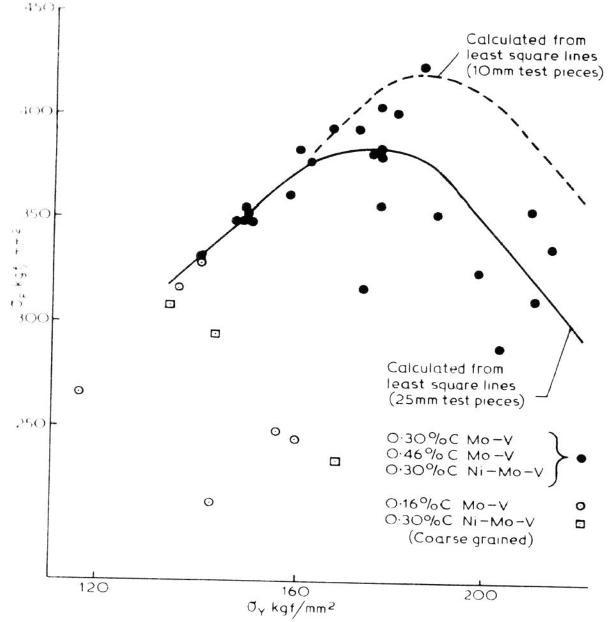


Fig. 7. Values of σ_F (individual points for 25 mm square test pieces) calculated from least squares regression lines (using calculation of Wilshaw and Pratt) plotted against 0·5% proof stress σ_Y.

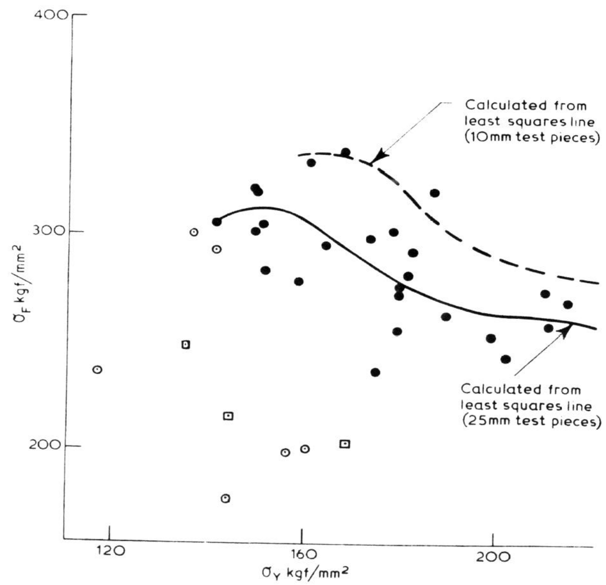


Fig. 8. Values of σ_F (individual points for 25 mm square test pieces) calculated from least squares regression lines using lines (calculated by Knott from Wilshaw) plotted against 0·5% proof stress σ_Y.

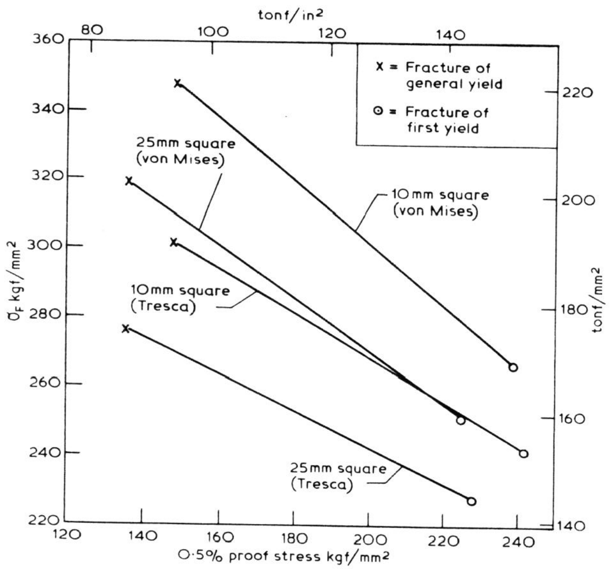


Fig. 9. σ_F Calculated from k_1, k_2 and k_3 of Table 3 plotted against σ_Y.