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The polycrystal ductile-brittle transition

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Summary

The tensile stress-strain curves of a number of polycrystalline body-centred cubic (b.c.c.) and hexagonal-close packed (h.c.p.) materials show a transition from ductile to brittle behavior when they are compared at varying temperatures or strain rates. This transition may be directly specified in terms of the different stress-grain size parameters, σ_{0_ϵ} and k_ϵ, which apply for the stress, σ_ϵ, dependence on grain diameter, l, at constant values of strain, ϵ, in the relation

$$\sigma_\epsilon = \sigma_{0_\epsilon} + k_\epsilon l^{-1/2}$$

The experimental dependence of σ_{0_ϵ} and k_ϵ on strain, temperature, strain rate, impurities and solutes is reasonably well-documented. For the b.c.c. case, the principal temperature dependence is in the lower yield stress intercept, σ_{0_y}, whereas, for the h.c.p. case, k_ϵ at the onset of yielding is strongly temperature dependent. From a theoretical description which has been given for the stress-grain size relation, σ_{0_ϵ} and k_ϵ may be correlated with critical resolved shear stress measurements for single crystals. Lower limiting values of k_ϵ may be calculated for the brittle fracture stress from dislocation theory.

For b.c.c. materials, an expression for the tensile ductile-brittle transition temperature, T_c, obtains which may be usefully compared with previous theories. In this case, it appears that the tensile stress-strain behavior may be satisfactorily correlated with Charpy v-notch test results by consideration of the effective strain rate and the nature of the tri-axial stress-state. For h.c.p. materials, T_c may be written in an approximate form for small grain size as

$$T_c \approx A + Bl^{1/2},$$

where the constants A and B depend on the material.

Ductility versus brittleness

Iron and zinc are representative metals in the class of those crystalline materials which will break in a brittle glass-like manner when subjected to mechanical forces under certain conditions in a neutral environment. Cadmium and copper are examples of materials which are fully ductile over a similar range of conditions. On the other hand, iron and zinc behave in a ductile plastic manner in many other instances. Of course, iron and zinc crystals exhibit cleavage on certain planes whereas cadmium and copper, thus far, have not. Otherwise, all of these metals usually respond to mechanical loading by exhibiting plastic deformation by slip and twinning processes.

Only limited ideas have been proposed [1-3] to explain the fundamental reasons why under any circumstances some crystals may be made to cleave and others possibly may not. In fact, the variation in the overall strength properties of crystalline materials is sensitively dependent on the chemical composition, processing history and microstructure of each material and the temperature, strain rate and stress-state which apply for the mechanical system. The general understanding which does exist for this sort of range in material behavior involves the following considerations: (a), the atomic modes of the slip, twinning and fracture processes which a material may potentially exhibit; (b), the nature of internal (and external) stress concentrators; and, (c), the constitutive relations for crystal plasticity.

The fracture stress

The limiting tensile fracture strength, σ_p, which a volume of perfect crystalline material might just not sustain has been estimated some time ago and is given by

$$\sigma_p \approx \{E\gamma/a\}^{1/2} \tag{1}$$

where E is Young's modulus, γ is the specific surface energy of the crack faces and a is the crystal lattice spacing. Fracture stresses near to the theoretical limit have been measured for certain specially produced materials. Estimates have been made of γ for various cleavage systems in different crystals [4].

The calculated value of σ_p is larger than that measured for most materials because real materials contain cracks and/or dislocations. For the case of a crack-containing material, the limiting fracture stress, σ_c, becomes, in the absence of plastic flow, that stress which is required to catastrophically propagate the crack according to an energy criterion [5], and thence, for plane strain:

$$\sigma_c \approx \{E\gamma/a\}^{1/2}\ (c/a)^{-1/2} \tag{2}$$

In this case, the value of σ_p, given by equation 1, is reduced by the factor, $(c/a)^{-1/2}$, to give σ_c. The reciprocal of this factor is often taken as a measure of the stress concentrating multiplier for the applied stress at the tip of the crack. It is normally assumed that if plastic flow occurs during the fracture process, then σ_c should be increased simply by adding a positive term to γ for the plastic work associated with forming new crack surfaces.

Many polycrystalline materials contain dislocations which move in slip bands [6] or twin bands [7] to form cracks when the material is in a brittle condition. For this case, any one of a number of models of the stress

concentrating character of a slip or twin band gives the following relation for the tensile fracture stress, σ_F:

$$\sigma_F \approx \sigma_{0_F} + k_F\ (l/b)^{-1/2} \tag{3}$$

where σ_{0_F} is a measure of the average stress required to move the dislocations which produce the crack, k_F is a measure of the stress concentration at the tip of the slip or twin band, and b is the individual dislocation Burgers vector ($b \approx a$). For slip in a grain of the most favorable orientation, a lower limiting value of k_F has been estimated [8] as:

$$k_F \approx \{6\pi G\gamma/(1-\nu)b\}^{1/2} \tag{4}$$

where G is the shear modulus and ν is Poisson's ratio. The value of k_F corresponds to a stress concentration of the same order as that required to bring two dislocations to within a Burgers vector distance of each other. This value of k_F is theoretically demonstrated to be sufficiently large to allow the crack to propagate as a brittle cleavage crack if the influence of the change in grain orientation on crack propagation is neglected and no plastic flow is associated with the propagating crack [9].

The picture which emerges from the development of equations 1 to 4 is one in which the potential fracture process for a crystalline material becomes gradually more complex. The fracture stress is well determined for a totally perfect material or one imperfect in the sense that it contains a clearly specified crack of a given size, so long as neither material undergoes any plastic deformation in the fracture process. It is important that the two fracture stresses, σ_p and σ_c, are related to each other in a specific way. The manner in which plastic deformation is accounted for by changing γ in equation 2 appears somewhat arbitrary and this is particularly worrisome because large values of γ are often reported for experiments where insufficient evidence is given to show that equation 2 actually applies. The presence of a crack serves to give a low value of σ_c relative to σ_p but the influence of plastic work is suggested to increase σ_c again. The dislocation process of forming a crack that is unstable in the absence of additional plastic flow gives a fracture stress that is large for two reasons: σ_{0_F} is greater than zero because a finite component of stress is needed to drive the stress concentrators which produce the crack and, even so, k_F is large because it is relatively difficult to nucleate a crack by a dislocation mechanism. The value of σ_{0_F} has not been theoretically evaluated. Experiments have shown that it may represent an appreciable part of the brittle fracture strength for materials having a conventional grain size. The experimental values of k_F for several b.c.c. and h.c.p. metals compare favorably with the theoretical estimate in equation 4 [8].

Equations 1 and 2 give in terms of the known temperature dependence of E, γ and a rather temperature insensitive fracture stresses. The value of k_F, as determined from equation 4, behaves in a similar way. Experimental studies of the composition [6,10], temperature [11, 12], strain rate [12], and strain [13] dependence of σ_F, as expressed in equation 3, have indicated that σ_F is reasonably insensitive to these variables when the material fails in an essentially brittle mode.

The temperature insensitivity of σ_F and k_F means that σ_{0_F} is relatively constant also. This presents a still-unanswered problem for the dislocation theory because σ_{0_F} is purported to control the motion of the stress concentrating dislocations and their movement in the slip process is in many cases strongly temperature dependent, especially, for the b.c.c. metals undergoing a ductile-brittle transition. Against this is weighed the agreement between the experimental results for k_F and the calculated values which follow from the dislocation pile-up model. An alternative possibility which may apply for certain of the b.c.c. metals (or the h.c.p. ones) is that σ_{0_F} and k_F are actually determined by the twinning-fracture stress [7]. The stress-concentrating character of a twin is not easily assessed because of the variable shape of twins. The value of k_F can be estimated from a dislocation model for a penny-shaped configuration [14]. This configuration gives a maximum concentration of the tensile stress at the twin tip. The following value of k_{T-F} applies if the nucleation stress for this shape twin is presumed to cause fracture

$$k_{T-F} \approx m\{5{\cdot}4(2-\nu)G\alpha[\ln(l/2nh) - 3/2]/\pi(1-\nu)b\}^{1/2} \qquad (5)$$

where m is a product of direction cosines which relates the applied stress to the resolved shear stress, in this case, for twinning, α is the incoherent surface energy of the twin, n is the number of twinning dislocations separated by the planar height, h, and the radius of the penny-shaped twin is taken equal to half the grain diameter. For silicon-iron, the value of k_{T-F} given by equation 5, utilizing previous estimates for the various parameters involved in it [14], is in reasonable agreement with the value of k_F determined from equation 4 and that determined experimentally. For zinc, the estimate of k_{T-F} is less than the values which obtain for k_F. These preliminary calculations indicate that for a b.c.c. metal such as silicon-iron a twin may generate a sufficient internal concentration of stress to produce a crack and it should be preferred over that necessary to accomplish the same event by the slip process if the σ_0 value for it is lower than that for slip. The main advantage in this explanation, however, is that the twinning stress appears to be largely athermal and, therefore, the temperature and strain rate insensitivity of σ_F may thus be rationalized.

The yield stress

The plastic yield stress of a polycrystalline **aggregate** may be changed more easily than the brittle fracture stress, except for the influence on these stresses of altering the material grain size. Internal concentrations of stress do play a large part in determining the level of the yield stress and its dependence on a specific metallurgical structure. It has been proposed [15] that the polycrystal yield stress, σ_y, should be written in the following way,

$$\sigma_y \approx m\{\tau_{CRSS} + k_S\,(l/b)^{-1/2}\} \qquad (6)$$

where m is an average orientation factor to achieve the critical resolved shear stress, τ_{CRSS}, required to cause slip to occur in the absence of any resistance at the grain boundary and k_S is a measure of the average resistance to the propagation of plastic flow across the grain boundaries. The analysis underlying equation 6 has been shown to explain the grain size dependence of the yield stress of a number of the b.c.c., h.c.p. and face-centred-cubic (f.c.c.) metals and their alloys. The grain size independent term in equation 6, commonly denoted as σ_{0y}, has been correlated with single crystal measurements of τ_{CRSS} for iron [16], magnesium [17], α-brass and aluminum [19]. The value of k_S in equation 6 is dependent on the orientation relationship of the accommodating deformation systems operating in the grain boundary regions, the particular τ_{CRSS} values which apply for them, and on the segregation of impurities there. k_S has been evaluated according to one method for determining the stress concentration at the tip of a dislocation pile-up and the result obtained [17] is

$$k_S \approx c'\{m^*G\tau_c/(1-\nu)\}^{1/2} \qquad (7)$$

where m^* is an orientation factor related to the average misorientation of deformation systems in adjacent grains and τ_c is a critical shear stress for initiating plastic flow on them. The total coefficient of $(l/b)^{-1/2}$ in equation 6 is denoted as k_y. For magnesium, its value has been correlated through τ_c with the single crystal τ_{CRSS} measurement for the most difficult slip system, $\{10\bar{1}0\}\langle 11\bar{2}0\rangle$, proposed to control the accommodation of general plastic flow at the polycrystal grain boundaries, i.e. $\tau_c \propto \tau_{CRSS}$.

A review has recently been given [20] of a large number of the investigations which have been conducted to determine the values of the stress-grain size parameters, σ_{0y} and k_y, for various crystalline materials. Their dependence on temperature, strain rate, impurities and solutes is known in many cases. The same stress-grain size relation applies for the flow stress at a constant value of strain, σ_ϵ. In this case, the parameters are written as $\sigma_{0\epsilon}$ and k_ϵ to denote their values at various strain levels past the yield point or for even smaller strains when the material does not exhibit a well-defined yield point. In a way, the ductile yield stress and the final fracture stress give limiting forms of the stress-grain size relation. From numerous studies on several b.c.c. metals, the experimental evidence is that the principal temperature and strain rate dependence of plastic yielding is in the lower yield stress intercept, σ_{0y}. From a few studies of polycrystalline h.c.p. metals, the observation is just the opposite in that k_ϵ at the onset of general yielding is strongly temperature dependent. The general observation for b.c.c., h.c.p. and f.c.c. materials is that the influence of strain is mainly reflected in a change in $\sigma_{0\epsilon}$ through hardening of the grain volumes while k_ϵ changes very little. For mild steel showing a distinct yield point, k_y is greater than k_ϵ but k_ϵ remains essentially constant for strain values immediately following the Lüders strain and larger, until the onset of necking whence k_ϵ increases to approximately the same value for the true ductile fracture stress as applies for the brittle fracture stress.

The constitutive relations for the temperature and strain rate dependence of the yield stress of metals has been an active field of research for some time and the dynamical equations which have been proposed to explain these relations in terms of dislocation theory have also been the subject of a recent symposium [21]. The properties of single crystals are usually studied from the dislocation dynamics point of view. The indication is that the analysis of dislocation motion in terms of the thermal activation-strain rate analysis seems to have the widest range of applicability for the various materials studied thus far. Unfortunately, a large number of theoretical or experimental parameters are involved in this analysis and due to their interdependence, it is generally difficult to obtain even the temperature dependence of the τ_{CRSS} at a fixed strain rate. On the other hand, a simpler relation has been proposed [22] to describe rather directly the temperature and strain rate dependence of τ_{CRSS}, as follows,

$$\tau_{CRSS} \approx \tau_G + \tau_0 \exp\{-[\beta_0 + \beta_1 \ln(\dot{\epsilon}_0/\dot{\epsilon})]T\} \tag{8}$$

where τ_G is an athermal contribution which determines the value of τ_{CRSS} at high temperatures, T, τ_0 is the value of the thermal component of stress at $T = 0$, $\dot{\epsilon}_0$ is the strain rate and β_0, β_1, and $\dot{\epsilon}_0$ are experimental constants which may be related in an approximate manner to the parameters utilized in the thermal activation-strain rate analysis. For example, the constant, $\dot{\epsilon}_0$, is related to the number of dislocations participating in the slip process, the area they sweep out in a thermal fluctuation, and their vibrational frequency; $\dot{\epsilon}_0 > \dot{\epsilon}$.

The transition

As the temperature decreases or the strain rate increases, the yield stress of a material increases in a minor or major way according to the material parameters involved in equations 6 to 8. For b.c.c. and h.c.p. metals, these

dependences are usually greater than those observed for a typical f.c.c. metal. Yield stress levels comparable to and, then, even exceeding those which apply for the brittle fracture stress of the same material according to equations 2 to 4 should be obtained. At the lowest temperatures (or highest strain rates) a material will fail brittlely in tension before a condition of general plastic yielding occurs, whereas in compression testing, brittle fracture may be avoided and the yield stress continues to increase with decrease in temperature. This result has been clearly observed for tension and compression testing of polycrystalline molybdenum [11].

The preceding description of a ductile-brittle transition (d.-b.t.), as observed in tensile testing, should be regarded as an extension of the classical explanation of the d.-b.t. in terms of equal but separate ductile yield stresses and brittle fracture stresses [23]. In the classical theory, these two stresses were assumed to exist independently of each other. The fracture stress was fixed at an arbitrary but relatively constant stress level and fracture was presumed to occur without any prior plastic flow. The actual dependence of these stresses on any material or test condition were relatively unknown. It was demonstrated in one case, however, that the fracture stress could be altered in direct relationship to the yield stress. There was no understanding or explanation of the atomic mechanisms which were involved in determining the level of these stresses.

Recent analyses of the d.-b.t. have incorporated dislocation mechanisms for the crack initiation process and have dealt directly with material behavior in the transition region [24, 25]. Also, these analyses have been concerned with other ways of specifying a d.-b.t. in material behavior. An abrupt transition in the stress-strain behavior of a material may be characterized in terms of the temperature dependence of the uniform elongation or in the reduction in area of a specimen preceding its fracturing, or by observing other test quantities, such as the energy absorbed in an impact test, or the physical appearance of the fracture surface [24-26]. The approach to specifying a d.-b.t. by focusing primary attention directly on the transition itself is different from the classical one. The approach which naturally follows from the classical theory is to accurately specify the brittle fracture stress of a material under conditions whereby the material is completely brittle and to determine the yield properties of a material when it is fully ductile and, then, to extrapolate these results into the transition region where both behaviors should be equally predictable. This is the approach used in the following description of the d.-b.t. observed for b.c.c. and h.c.p. metals.

The b.c.c. case

In view of the major temperature dependence of the yield stress being in σ_{0y} for most b.c.c. metals, the other minor dependences may be neglected and the tensile d.-b.t. temperature, T_C, may be quantitatively determined utilizing equations 3, 6 and 8, at equal values for σ_F and σ_y, as

$$T_C \approx \{1/[\beta_0 + \beta_1 \ln (\dot{\epsilon}_0/\dot{\epsilon})]\}\{\ln \sigma_0 - \ln [(k_F - k_y) + (\sigma_{0_F} - \sigma_G)(l/b)^{1/2}] - \ln (l/b)^{-1/2}\} \tag{9}$$

where $\sigma_0 = m\tau_0$ and $\sigma_G = m\tau_G$. It follows from equation 9 that T_C is greater, the greater is the strain rate and, provided $k_F > k_y$, the smaller is the grain size. The inequality, $k_F > k_y$, should normally apply to most materials because of the expectation that a greater local concentration of stress is required to produce a crack than is required to cause a plastic nucleus to form. This inequality is in agreement with the observation that a decrease in grain size often leads to an increase in the strength of a material and improves its ductility, too. An increase in yield strength by increasing the strain rate contributes to increasing T_C. The transition temperature is raised by through increasing σ_{0_y} by increasing σ_0, m, τ_G and decreasing β_0, β_1 and $\dot{\epsilon}_0$ contributes to increasing T_C. The transition temperature is raised by decreasing σ_{0_F} and k_F.

The relation for T_C given above involves rather well-defined quantities. Though a complete experimental evaluation of them has not been undertaken in any one laboratory, all of them have been determined for reasonably similar grades of mild steel in one or another studies of their deformation and fracture properties. These measurements have been applied to numerically evaluating the major influences that grain size, strain rate, neutron irradiation and heat treatment by quenching should have on T_C [27]. The results have compared favorably with separate experimental measurements of T_C. In one investigation of the embrittlement of a structural steel caused by neutron irradiation, changes in T_C were directly correlated with changes estimated for σ_{0_y} and k_y [28]. The grain size dependence of T_C which follows from equation 9 agrees with that obtained from the preceding dislocation theories of the d.-b.t. only in the limit that σ_{0_F} is zero. Equation 9 shows that a large grain-size is to be avoided for low temperature strength applications for two reasons: T_C is already a high temperature for a material with a large grain size and T_C is increased more strongly by an increase in σ_G than applies for smaller grain sizes.

A notable application of the previous d.-b.t. theories has been towards analyzing the value of T_C determined by Charpy v-notch impact testing [24, 25]. The value of T_C measured in this test differs from that measured in a tensile test for, at least, two reasons: (a), the effective strain rate is large compared to that encountered in conventional tensile testing, on the order of 10^3 sec^{-1}; and, (b), the inhomogeneous stress system requires consideration of a plastic constraint factor to account for an increase in yield stress due to the localized deformation which is forced to occur at a specimen notch. The strain rate and the plastic constraint both contribute to an

increased yield stress and increased value of T_C. The former consideration is accounted for through a decreased value of β, whereas the influence of the second consideration on increasing T_C has been estimated by raising σ_y by a constant factor, approximately equal to 2·0. The value of T_C is sensitively dependent on this factor and this value gave consistent results for T_C and the influence on it of grain size and neutron irradiation [27]. The value of this factor is in reasonable agreement with theoretical estimates and independent experiments [29].

The analysis for T_C which has been given does rely on the brittle fracture stress being controlled by the nucleation of an unstable crack of size equal to the grain diameter. This certainly appears to be an accurate picture of the fracture process for a polycrystalline aggregate at temperatures below T_C. At temperatures just above T_C, however, it appears for large grain sizes that the temperature dependence of the ductile fracture stress, σ_F^*, is such that it may be extrapolated to equal the yield stress at a higher temperature, T_{DC} – the ductile cleavage temperature [26]. In the interval, T_C to T_{DC}, the fracture stress appears to follow the temperature dependence of the lower yield stress. A part of the reason for the observation that essentially brittle fracture occurs at temperature above T_C is undoubtedly due to the inherent experimental scatter and statistical nature of the brittle fracture process. It has been observed that $(T_{DC} - T_C)$ increases as l increases and this is in agreement with the prediction which follows from a fixed uncertainty in the fracture stress because T_C and $(d\sigma_{0_y}/dT)_{T_C}$ also increase as l increases.

An additional possibility exists that the experimental variation of $(T_{DC} - T_C)$ with l may be explained in terms of the $\sigma_{0_F}^*$ and k_F^* parameters obtained in the stress-grain size equation for the ductile fracture stress, [30]. Since the true strain at the ductile fracture stress increases as l decreases, the following inequalities should hold near to T_C: $\sigma_{0_F}^* \le \sigma_{0_y}$ and $k_F^* > k_y$. By comparing the ductile fracture stress versus $l^{-1/2}$ with σ_y and σ_F, it may be seen further that two other inequalities must obtain for $(T_{DC} - T_C)$ to increase with a decrease in $l^{-1/2}$: $\sigma_{0_F}^* < \sigma_{0_F}$ and $k_F^* > k_F$. Both of these inequalities appear to be significant. The first one, $\sigma_{0_F}^* < \sigma_{0_F}$, implies that near to T_C ductile fracture requires a propagation stress for existing cleavage cracks because, in the limiting case, $\sigma_{0_F}^*$ may be zero. The parallel interpretation for the second inequality is that for ductile cleavage, crack propagation under conditions involving appreciable plastic work at the crack tip is the important fracture mechanism to consider.

The h.c.p. case

Charpy impact tests on polycrystalline zinc show a transition curve similar to that which is observed for steel [31]. Zinc [32], magnesium [12] and beryllium [33] also show a transition from low to high ductility when deformed in tension at progressively lower temperatures; however, the ductility transition is not as abrupt as that which is characteristic of a b.c.c. metal such as steel. The transition which occurs in the stress-strain behavior at low temperatures has been depicted in terms of the uniform elongation versus the logarithm of the number of grains per unit area or in terms of the reduction in area versus temperature for different grain sizes [34, 35]. Either way, the transition in behavior is sensitively dependent on the grain size of these materials. Now, although the flow stress at any value of strain does depend on the grain size and the influence of strain is mainly reflected in an increase in σ_{0_ϵ}, a yield point is not often observed for these metals and, as mentioned earlier, equation 6 is usually shown to apply for the earliest proof stress which is measured. Values of k_ϵ nearly as large as those for steel are obtained because large values of m, m^*, and τ_c (c.f. equation 7) are required to achieve with the limited deformation systems which operate in these structures the compatibility of strains at the grain boundaries.

The relatively gradual nature of the ductility transition for these metals, their less-pronounced yield point behavior and the reasons that internal concentrations of stress are generated at their grain boundaries at all are inter-related. In contrast to the b.c.c. case where the segregation of impurities to all the grain boundaries makes each boundary about as effective an obstacle to the penetration of slip as any other boundary, the obstacle nature of each boundary in the h.c.p. case depends critically on the orientation relations particular to its adjacent grains and their deformation systems. Plastic flow is easily transmitted across some boundaries but is only accomplished with such difficulty for others that individual grains might elastically support the surrounding plastic deformation and work hardening of their neighboring grains. Because of the varying constraints due to the relative orientations of grains, an easily measurable plastic strain occurs even at the lowest temperatures for these metals in their most brittle condition.

As previously mentioned, the temperature dependence of σ_ϵ for reasonably small grain sizes is largely contained in k_ϵ and, for magnesium, k_ϵ has been correlated with the τ_{CRSS} for the most difficult slip system needed to operate at the grain boundaries in order to maintain continuity of material. If it is presumed that the major temperature dependence is only in k_ϵ and is due to τ_c, taken proportional to τ_{CRSS}, then, an approximate expression for T_C may be obtained by equating σ_F to σ_ϵ at a value of strain where the transition in ductility occurs. Thus, the following expression obtains for T_C specified from elongation measurements versus temperature for different grain sizes:

$$T_c \approx (1/\beta)\{\ln[cm^2m^*G\tau_0/(1-\nu)] - 2\ln[k_F - (\sigma_{0_\epsilon} - \sigma_{0_F})(l/b)^{1/2}]\}, \quad (10)$$

where c is a numerical constant, $\beta = \beta_0 + \beta_1 \ln(\dot{\epsilon}_0/\dot{\epsilon})$ and τ_G in equation 8 has been neglected. For very small grain sizes $(\sigma_{0_\epsilon} - \sigma_{0_F})/k_F(l/b)^{-1/2} \ll 1$ and T_C may be expanded in simpler form as

$$T_c \approx (1/\beta)\{\ln[cm^2m^*G\tau_0/(1-\nu)k_F^2]\} + \{2(\sigma_{0_\epsilon} - \sigma_{0_F})/\beta k_F\}(l/b)^{-1/2} \quad (11)$$

Equations 10 and 11 show that T_C decreases at a decreasing rate with decrease in $l^{1/2}$ until a linear dependence on it is obtained. The equations for T_C show that it may be altered in a predicted manner by changing various parameters: (a), the preferred orientation through m, m^* and τ_{CRSS}; (b), the fracture characteristics of the material through σ_{0_F} and k_F; (c), the slip processes within the grains through σ_{0_ϵ}; and, (d), the accommodating plastic flow mechanisms operative at the grain boundaries through τ_0 and β.

Until the present time, very limited data are available to test the analysis. The predicted grain size dependence has been shown to be in agreement with experiments on beryllium [17]. A similar result appears to apply for experiments on magnesium [34]. The limiting reduction of T_C that may be achieved by refining the grain size is determined by the first term in equation 11, of which only m and m^* should be able to be adjusted very much at all through following different metallurgical procedures.

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