

Fracture propagation rates and times to fail following proof stress in bulk glass

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Summary

Glass buoyancy spheres and capsules for manned occupancy in deep diving work boats are under research and development in several U.S. Navy laboratories. There is great concern for the safety and reliability of structures made of glasses and ceramics in the face of their known susceptibility to delayed catastrophic fracture. In spite of designers' intentions some areas may be under tensile stresses and some surface cracks or scratches may be present or acquired during service. Proof stressing may put an upper limit to the size of such defects and time to fail under subsequent stresses is determined by the rates at which cracks can grow as affected by the environmental conditions and by the applied stresses. The predictability of a safe envelope of stress-time-temperature-humidity and prior proof stress is best approached through a characterization of the material using fracture mechanics measurements. Materials parameters take the form of constants in formulae for crack length or depth vs time and applied \mathcal{G} or K (Irwin driving force and stress intensity parameters).

Data on propagation rates were obtained using a new and simple technique in which \mathcal{G} and K are independent of crack length if the applied load is constant. Integration or summation of the formula for varying \mathcal{G} vs crack rate in order to obtain total length vs time is found to be valid.

The safe life or minimum time to fail may be expressed in closed form.

The crack opening dimensions in glass are discussed in relation to the mean free path of the environmental vapor as this may or may not affect the rate of transport of water to the root of a surface crack where 'stress corrosion' may be occurring.

Introduction

Subcritical crack growth rates govern the long term strength and reliability of bulk glass. Recent attempts to fabricate and test large glass pressure capsules for deep submergence applications have resulted in disappointing performances in spite of the expected high compressive strength of the material. Although an effort is made to design so as to eliminate tensile stresses by using prestressed glass and special joint designs, it is difficult to avoid tensile stresses entirely in service, particularly if the design includes hull penetrations and butted joints. In the case of windshields for aircraft the failure rate vs time of service is not well documented.

The present investigation is concerned with measuring subcritical crack rates as affected by the environment, the glass composition and the fracture mechanics driving force \mathcal{G} .

Experimental method

A specimen and loading system were selected which maintains a constant \mathcal{G} or energy input per unit area of crack for a constant load, independent of crack length. Fig. 1 illustrates the glass plate specimen with a surface scratch to guide the crack. The two halves of the specimen are subjected to equal torque by a four point application of dead weight loading and support. The loads are applied through steel spheres.

The fracture mechanics driving force ' \mathcal{G} ' is developed from the elementary formula

$$\mathcal{G} = \frac{P^2}{2} \frac{dC}{dA} \quad (1)$$

where P is the load, C is the elastic compliance and A is the area of the crack. For constant plate thickness t we have

$$t \, dx = dA$$

where dx is the increment of crack length. ' P ' is the load.

For the geometry of Fig. 1, equation 1 converts to

$$\mathcal{G} = \frac{P^2}{2t} \frac{dC}{dx} = \frac{3}{8} \frac{P^2 W_m^2}{G t^4 W} \quad (2)$$

G is the shear modulus of the material and W_m is the moment arm for one of the halves of the plate. ' W ' is the entire width of the plate including both halves. Equation 2 may be expressed in terms of the deflection ' y ' or displacement of an inner load point with respect to the outer load point on one of the specimen arms.

$$\mathcal{G} = \frac{2Gt^2 y^2 W}{3W_m^2 x^2} \quad (3)$$

Here x is the crack length measured from the load point position to the end of the crack. Equation 2 is usually preferable to equation 3 since the load can usually be measured more accurately than ' y '. For soda lime glass ' G ' is taken as 4.25×10^6 psi and $G = 3.86 \times 10^6$ psi for Pyrex.

The humidity is controlled by various convenient means such as saturated salts and for a dewpoint of -50°F and below a material known as Molecular Sieve was used. The apparatus is shown in Fig. 2.

Crack length vs time under constant \mathcal{G}

In order to make predictions of crack length or depth from rate data it is desirable to have a quick response of crack velocity when the driving force \mathcal{G} is changed. The quick response characteristic in this experiment is demonstrated by the sharp change in slope of the line showing crack length vs times in Fig. 3. The data are for soda lime plate glass. The two driving forces ' \mathcal{G} ' for the two line segments are shown on the figure.

The data are highly reproducible and the crack growth rates are determined with an accuracy rarely achieved in brittle fracture tests.

Another desirable characteristic of the experiment is that within the range of crack speeds of current interest, the speed dx/dt is not only constant at constant \mathcal{G} but can be plotted over much of the range of interest as a straight line on semi-log paper, thus

$$\log \frac{dx}{dt} = C_1 \mathcal{G} + \log C_2 \quad (4)$$

represents the data well with the constant C_1 being about the same for a considerable range of humidities on a given glass. Fig. 4 shows crack speeds in soda lime glass at four different humidity levels plotted as functions of applied \mathcal{G} . The functional form of equation 4 does not allow for sudden changes in the transfer rate of moisture to the crack tip because of a possible critical ratio of crack opening to mean vapor free path or to changes from emphasis between surface diffusion and volumetric (vapor) diffusion. Fig. 5 shows crack opening displacements for semielliptical cracks as compared with the mean free vapor path. Selected \mathcal{G} values are assumed imposed at various nominal stress levels σ on semielliptical cracks, whose depths are assumed determined by prior proof test stresses as given in the legend. The numerical values of η , the displacement from mid plane, were calculated from elasticity formulas given by Irwin [1].

According to [1] a semielliptical surface crack opens up to an semiellipsoidal cavity when tension is applied normal to the crack surface. The equation for the ellipsoid is

$$\frac{\eta^2}{\eta_0^2} + \frac{x^2}{a^2} + \frac{z^2}{c^2} = 1 \quad (5)$$

where η is the distance from the mid plane to the ellipsoidal surface and is in the direction of the applied tensile stress σ . η_0 is the maximum value of η . The total crack opening at the maximum is $2\eta_0$ at $x = z = 0$. Also

$$\eta_0 = \frac{4(1-\nu)(-A)}{ac} \quad (6)$$

$$\text{where } (-A) = \frac{\sigma ca^2}{2G 2\phi} \quad (7)$$

where G is the shear modulus and ' a ' and ' c ' are semimajor axes of the ellipsoid, and ν is Poisson's ratio. The depth of the crack is measured in the x direction. According to [2]

$$\phi = \int_0^{\pi/2} \sqrt{1 - \left[\frac{c^2 - a^2}{c^2} \right] \sin^2 \theta} \, d\theta, \quad (8)$$

and the stress intensity factor K_I is

$$K_I = \frac{1.1 \sigma \sqrt{\pi a}}{\left[\phi^2 - .212 \left(\frac{\sigma}{\sigma_{ys}} \right)^2 \right]^{1/2}} \quad (9)$$

Our choice of the 10 to 1 crack shape, ϕ is such that for σ well below yield strength,

$$K_I = \sigma \sqrt{(\pi a)}.$$

As shown in Fig. 5 the maximum crack opening at the surface $2\eta_0$ can be either more or less than the mean free vapor path for a given $\mathcal{G} = 0.02$ in.lb/in² depending on the crack depth. For equal crack depth, curves B and D show that the maximum opening $2\eta_0$ is proportional to the applied stress or to the stress intensity factor K . This is for the semi-elliptical crack assumed.

The depth of the crack in the plate specimen ranges from zero to something slightly greater than the plate thickness at any \mathcal{G} , thus this experiment is not directly suited to testing the hypothesis that a law expressing rate as a function of \mathcal{G} might change when the crack opening at the surface is more or less than the mean free path. No rate change suggesting this is evident in the data up to the present.

Prediction of times to fail following proof test and assuming semi-elliptical surface cracks

By integration of equation 4 it follows that for a 10 to 1 surface crack of semielliptical shape the largest such defect surviving proof stress σ_p will fail in time τ_{fail} where

$$\tau_{fail} = \frac{E \mathcal{G}_c}{C_2 \pi \sigma^2} \left\{ 1 - \left(\frac{\sigma}{\sigma_p} \right)^2 \right\}. \quad (10)$$

This is an approximation formula which predicts values within a few percent as given in previous report [3]. This agreement depends on using $\mathcal{G}_c = 0.08$ in.lb/in² and a value of $C_2 = 1.1071 \times 10^{-4}$ in/sec for wet soda lime glass as deduced from Fig. 4.

Crack arrest possibilities

For detecting crack arrest we plot applied \mathcal{G} vs velocity on a linear scale as shown in Fig. 6 for soda lime glass at several humidity levels. Up to the present we find the \mathcal{G} values for arrest are somewhat uncertain and probably below 0.004 in.lb/in² for wet conditions and below 0.006 in.lb/in² for R.H. = 40% at room temperature. It is suggested by Fig. 7 that if any appreciable moisture is available the arrest values of \mathcal{G} may be the same

independent of humidity. This implies the possibility that slow condensation at the tip of the crack may always result in wetted fracture surfaces. The more dry the ambient atmosphere the more sudden is the downturn in \mathcal{G} vs velocity as the arrest is approached. On the basis of a 72 hour period without movement it appears that the \mathcal{G} for arrest is between 0.004 in.lb/in² and 0.006 in.lb/in² for both wet and 40% R.H. The value of $\mathcal{G} = 0.004$ in.lb/in² indicated tentatively for crack arrest in soda lime glass is equivalent to 700 ergs/cm² for the pair for fracture surfaces. This corresponds closely to the usually accepted value for the surface energy of a pair of clean glass surfaces and to the expected value of the energy barrier for a fracture process without moisture. If the surfaces are completely wetted, the energy of the pair of surfaces would be about 120 ergs/cm².

In [4] the authors proposed an equation combining fracture mechanics and chemical rate theory which predicts the existence of a crack arrest given by the condition

$$(p + \beta) \mathcal{G} \leq N_0 f_s$$

where β is the fraction of the applied fracture energy \mathcal{G} actually used in breaking bonds on the main fracture plane. Most of the \mathcal{G} applied apparently goes into plastic flow which gives stress relief at the crack tip. 'p' is the fraction of \mathcal{G} that must be restored in healing broken bonds at the crack tip and the lower the value of β , the lower the expected value of p. $N_0 f_s$ is the effective value of the energy per unit area of a single surface of the fracture where it is being created and as influenced by moisture. In the present experiments the data indicate

$$(p + \beta) \cong 0.2$$

at crack arrest for soda lime glass. The equation in which $(p + \beta)$ is used in [3] is a mathematical model as follows:

$$\frac{dx}{dt} = \frac{d_0^3}{h} \exp \left[\frac{-f^*}{kT} + \frac{\beta \mathcal{G}}{N_0 kT} \right] \{ (p + \beta) \mathcal{G} - N_0 f_s \}. \quad (11)$$

where d_0 is the bond spacing

f^* is the energy barrier for one bond

$N_0 = \frac{1}{d_0^2}$ = number of bonds per unit area

$N_0 f_s$ is the effective surface energy at the tip where moisture is present.

Effect of composition

Although good reproducibility of data giving crack speed vs \mathcal{G} at fixed humidity is obtained, it is clear that compositional differences are important as shown in Fig. 7. Here wet Pyrex is shown to propagate a crack at about

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10 times the speed as wet soda lime glass. The differences in speed resulting from small compositional differences in a given type of glass have not yet been measured.

Conclusions

- (1) A new technique and apparatus for measuring crack speeds as a function of arbitrarily fixed driving force \mathcal{G} is described. This has advantages of providing a constant \mathcal{G} while under constant dead load. The test method was suggested by Prof. J. Outwater, University of Vermont.
- (2) Subcritical crack speeds were measured and found to be linear in \mathcal{G} vs log speed over a range of crack speeds varying by factors up to 10^5 .
- (3) The subcritical crack speeds, although all small, were extremely sensitive to ambient humidity.
- (4) It was shown that to a useful degree of accuracy a prediction of time to fail under constant stress following proof stress could be made based on the measured subcritical crack speeds.
- (5) Crack speeds in wet Pyrex were found to be about ten times those in soda lime glass at equal values of driving force \mathcal{G} . Effects of composition although evidently important remain to be explored.

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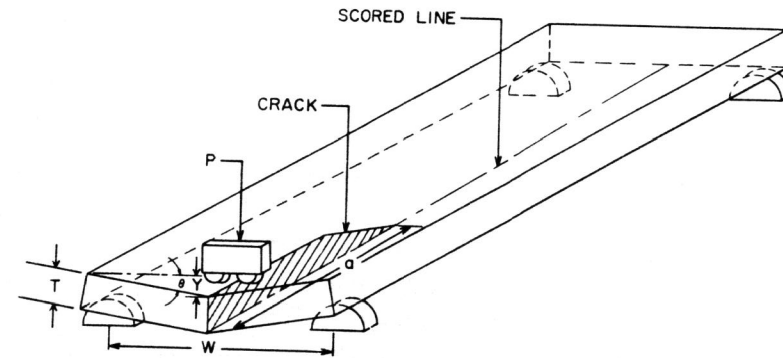
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References

1. IRWIN, G. R. *Analytical aspects of crack stress field problems*. University of Illinois, Theoretical and Applied Mechanics Report 213, March, 1962.
2. TIFFANY, C. F. and LOREZ, P. M. *An investigation of low cycle fatigue failures using applied fracture mechanics*. Tech. Documentary Report ML-TDR-64-53, May 1964. Contract AF 33(657)-10251 by the Boeing Co., May 1964.
3. SINCLAIR, G. M. and WITHROW, S. P. *Environmentally assisted crack growth in glass*. NRL Report 6635, 22 Nov., 1967.
4. KIES, J. A. and CLARK, A. B. J. *Mechanism of Strength Degradation and prediction of times to fail for bulk glass*. Presented at ASTM, Atlanta, Ga., 3 Oct., 1968.

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SCHEMATIC OF TEST APPARATUS

Fig. 1. Double torsion 'Outwater' specimen in which \mathcal{G} is constant if the load P is constant.

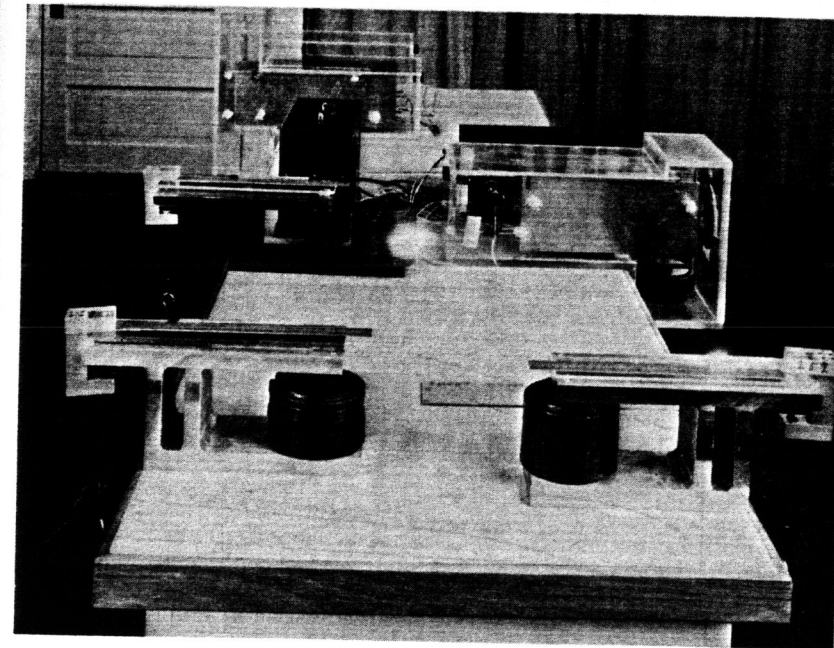


Fig. 2. Apparatus for maintaining constant humidity during cracking rate experiments on glass.