PAPER 17 (SESSION II)

Effects of notch depth on the toughness of mild steel

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Summary

The paper examines the deformation and fracture behaviour of thick and thin bend specimens containing notches of various depths. The deformation behaviour of thick specimens is shown to be very sensitive to notch depth, and the experimental results are shown to agree well with the conclusions of some recently developed slip-line field theory. Emphasis is placed on the way in which notch depth can affect the fracture behaviour, and hence toughness, of thick specimens and attention is drawn to the entirely different behaviour in thin pieces. It is concluded that very careful consideration will have to be given to the geometrical design of testpieces used for measuring toughness in low-strength materials intended for heavy-section applications.

Introduction

One of the most exciting features of modern engineering design in high strength materials is the use of the methods of Fracture Mechanics to provide a quantitative relationship between size of defect and applied working stress [1]. The success of these methods when applied to design in thin sheet has resulted in endeavours to obtain equally useful design information for materials of lower strength and for thick sections. In particular, interest has been shown in the 'crack opening displacement' or C.O.D. [2, 3] as a possible means of measuring fracture toughness. The principle is that the C.O.D. measured an a small thick testpiece which fails after general yield is taken as identical to that in a large thick structure which fails before general yield.

This paper is concerned with the fracture of mild steel notched bars, which show a transition from low-energy cleavage to high energy fibrous initiation as the temperature is increased. It can be argued that the useful toughness of mild steel at temperatures above the transition depends on the ease with which fibrous fracture is initiated, since the cleavage fracture which produces the final instability is a consequence of the accelerating growth of the fibrous fracture [4, 5]. Above the transition, a C.O.D. therefore measures the crack-tip displacement associated with sufficient fibrous growth to produce instability. Below the transition, at high strain-rates, or if embrittlement occurs in service, instability may be caused by direct cleavage initiation at the crack-tip and the C.O.D. is then a measure of the amount of crack-tip strain required to satisfy the cleavage initiation criterion [see 6].

The problem with which this paper is concerned is whether such C.O.D. values can always be used to predict, from measurements on small (albeit thick) specimens, the relationship between defect size and applied stress in a large piece. In particular, a situation is examined where the defect length (notch depth) determines the hydrostatic component of stress in the fracture process zone. Here, a C.O.D. measured from a testpiece with, say, a deep notch, would be used to predict the critical defect size for a large piece loaded to some specified fraction of its yield stress. However, since the perdicted defect length might in service produce a different hydrostatic component of stress from that in the notched testpiece, the fracture mode (or C.O.D. associated with a single mode) could also be different. The C.O.D. measured from the testpiece would then be irrelevant to service design. Since an effect of this sort had been encountered in a 3% CrMoV steel [7], the present work was started to examine the general problem, in both thick and thin section.

Experimental details

The material used was a high nitrogen Acid Bessemer rimming steel in which plastic deformation could be revealed by the Fry's etch technique [10, 12]. The composition was as follows:

element	C	Si	Mn	S	P	N
wt.%	0·07	0·005	0·33	0·053	0·049	0·021

and it was supplied as 1 in^2 hot-rolled bars. 'Thick' notched bend specimens were machined as bars 0·5 in thick; 'thin' specimens were prepared by waisting a thick specimen to a gauge-length thickness of 0·05 in (cf. 8). Both types of specimen were provided with sharp 45° V-notches*, mainly of depths 0·15, 0·08 or 0·04 ins. Some intermediate depths were used. The depth of the cross-section below the notch was 0·35 in. in all cases. Further thick specimens were provided with notches of included angle, 20°, 90° or 120°. Tensile specimens of the Hounsfield no. 13 type were also tested.

Specimens were deformed in either four-point bending or uniaxial tension over a range of low temperatures, using 'cages' attached to the crosshead of an Instron Universal Testing Machine of 5 tons capacity: these cages could be surrounded by constant temperature baths capable of control to ± 1 deg. C. The crosshead speed was 0·08 in/min for all tests.

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* Slots or fatigue cracks are not necessary to show the general effects of notch depth, and the 45° V-notched bar geometry is more amenable to treatment by slip-line field analysis.

Effects of notch depth on deformation

Three aspects have been examined: primarily, they way in which the loads and slip-line fields at general yield in thick 45° V-notched bars are affected by the depth of the notch; then, in less detail, how the results are altered by changes in specimen thickness or notch angle. The basic effect of notch depth on general yield load at room temperature is shown in Fig. 1: the load increases with notch depth until a 'depth-ratio', r_d (gross/net section depth) of about 1·4 is attained.

The behaviour of the general yield loads may be related to the forms of the slip-line fields as revealed by etching in Fry's reagent (Figs. 2 (a), (b) and (c)). For deep notches yield is confined to the notched cross-section and so must spread in a region of triaxial tensile stress: this necessitates a correspondingly high applied load at general yield. As the notch depth is decreased, a point is reached where the gross specimen depth becomes too small to support the stresses of the deep-notch field and yielding spreads from the top surface of the bar. Qualitatively, the yielding is then a mixture of the deep-notch ('hinge') and plain bar (45° slip-lines) fields [see 10]. The general yield load therefore decreases with notch depth as the difference between the size of the gross and the notched cross-section becomes smaller, until the two are equal, and the specimen is smooth.

To support this argument, comparison may be made between the experimentally observed general yields loads and those calculated using slip-line field theory. The main features of the slip-line fields have been described by Green [10], but detailed calculations have recently been made by Ewing* [9], who has obtained figures for the constraint factors ($L \equiv$ general yield load/yield load of unnotched specimen) of specimens with 0·04 and 0·08 in deep notches. He has also found the 'critical depth ratio' above which general yield is confined to the hinge field (r_d crit = 1·408 for a 45° notch) and has confirmed the value of constraint factor obtained for a deep notch by Green [10]: the 0·15 in notch may be regarded as deep ($r_d > 1{\cdot}408$). The experimental values are given in Fig. 1.

There is difficulty in defining a general yield load which is meaningful in the slip-line field sense for an unnotched bar of material which work-hardens It cannot be defined as a 'proof' offset from the load vs. angle-of-bend curve (i.e. and engineering 'yield') because this in fact corresponds to only small amounts of yielding along the tension and compression faces of the bar. On the other hand, the load at which plasticity first reaches the neutral axis involves such large surface strains and work-hardening that it cannot be equated to the yield load of a

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bar of rigid/plastic material. This problem does not arise for even moderately deep notches, because the proof offset then corresponds to the general yield slip-line field pattern. The proof offset load is given in Fig. 2 for unnotched bars, but values of constraint factor based on this would be much higher than those predicted by Ewing [see also 11].

Consequently, values for the general yield load were predicted for the deep-notched specimens from the uniaxial lower yield stresses [as in 12] using Green's formula [10], which gives $L = 1{\cdot}26$. From Fig. 1 it can be seen that the experimental values agree with this formula quite well, and that Ewing's theoretical predictions for shallow notches (based on the Mises' yield criterion) agree equally well with the experimental points for the 0·08 and 0·04 in deep notches. It can therefore be concluded that the effect of notch depth on yielding in thick specimens is a direct consequence of the change from yielding on the notched section to yielding on the gross section.

The situation is different for thin specimens as can be seen in Table 1 and Figs. 2 (d), (e) and (f), which show the constraint factors and deformation patterns close to general yield for the 0·05 in thick bars having the same three notch depths: 0·15, 0·08 and 0·04 in. There is now little change of constraint factor with notch depth, because even the deeply notched bars do not have high constraint factors. Equally, there is virtually no sign of yielding starting on the top surface of any bar. These results are to be expected, because the high constraint factors in thick specimens are due to the presence of triaxial tensile stresses below the notch, the hydrostatic component of which does not contribute to yielding. In thin specimens, the stress in the thickness direction is reduced, so that the applied load required to produce general yield need not be so high. Since the constraint factors are not high, there is also less tendency for the production of yielding over the gross cross-section.

The way in which the critical depth ratio varied with the included angle of the notch was also investigated briefly. For 20° and 90° angles $r_{d\ \mathrm{crit}}$ lay between 1·36 and 1·5 (cf. $r_{d\ \mathrm{crit}} \simeq 1{\cdot}4$ for a 45° angle), whereas for a 120° notch it lay between 1·24 and 1·36. The similarity between the behaviour of the 20° and 90° notch is further illustrated by Figs. 2 (g) and (h), which show yielding just starting from the top surfaces of specimens with 0·08 in deep notches which have undergone similar amounts of deformation on the notched cross-section: cf. also Fig. 2 (b) which has deformed slightly more.

Effect of notch depth on fracture

The effect of notch depth on the fracture of thick and thin specimens was investigated over a range of low temperatures for the three depths 0·15, 0·08 and 0·04 in and the two thicknesses 0·5 and 0·05 in. The general yield and fracture loads are shown for thick specimens in Fig. 3 and for thin specimens in Fig. 4. The curves are superimposed (using different load scales) in Fig. 5, which shows the general yield loads as a scatter-band, although at any one temperature they vary systematically with notch depth and thickness.

There is a marked effect of notch depth on the fracture of thick specimens (Fig. 3): there are large differences in the temperatures at which the 'cliffs' in the fracture load curves occur (these are indicative of the cleavage/fibrous fracture initiation transition) and in the temperatures, T_{GY}, at which the specimens fracture at general yield. For thin specimens, the effects are much less pronounced (Fig. 4): the temperatures, T_{GY}, are much closer, and the fracture load curves after general yield for the deep and shallow notches almost superimpose.

The variation of general yield load with temperature may be related to that of the uniaxial yield stress (Fig. 6), using slip-line field theory from Green [10] for the deep notches in plane strain, from Ewing [9] for shallow notches in plane strain, and from Ford and Lianis [13] for (deep) notches in plane stress. The agreement between theory and experiment was good for thick specimens, indicating that these were deforming virtually in plane strain, but the constraint factors for the thin specimens were slightly higher than Ford and Lianis would predict: deformation in the thin specimens is therefore not quite plane stress.

It is also possible to calculate from the fracture results values of stress intensification* at general yield as a function of notch depth in both thick and thin specimens. The method employed is described briefly below [see also 6].

The criterion for the (slip-initiated) cleavage fractures at general yield is that failure occurs when the maximum local tensile stress beneath the notch, $\sigma_{1\,(\max)}$, attains a critical value σ_F which for this steel is independent of temperature [for experimental confirmation see 14]. For deeply notched specimens in plane strain, $\sigma_{1\,(\max)}$ at general yield is given by [10]:

$$\sigma_{1\,(\mathbf{max})} = \frac{\sigma y}{\sqrt{3}}\left(1 + \frac{\pi}{2} - \frac{\theta}{2}\right) = R\sigma_y \qquad (1)$$

on the Mises yielding criterion, where θ is the total included angle of the notch. ($\theta > 6{\cdot}4°$). For the deep-notch thick specimens, θ is 45°, so $R = 2{\cdot}51$ at general yield. These specimens break at general yield at a

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* Stress intensification, R, is defined as the ratio of the maximum local tensile stress beneath a notch to the uniaxial yield stress: it is a function of the geometry of the specimen and of the amount of deformation undergone, and is a quantitative measure of the hydrostatic component of stress beneath the notch [see 6].

temperature T_{GY} ,so the critical fracture stress for the (slip-initiated) cleavage, θ_F, may be calculated, since:

$$\sigma_F = \sigma_{1\,(\mathrm{max})} = 2{\cdot}51\ \sigma_y \tag{2}$$

where σ_y is measured at T_{GY} . Thus, taking the appropriate value of σ_y from Fig. 6, σ_F may be calculated as 53·2 tons/in^2 [see also 14].

Values of stress intensification at general yield, R_{GY}, may now be calculated for the other five specimen geometries, using the relationship: $R_{GY} = \sigma_F / \sigma_y$, (where σ_y is measured at the appropriate temperature, T_{GY}), since σ_F is independent of temperature [14]. The results are shown in Table 1, together with figures for the constraint factors. It can be seen that R_{GY} decreases significantly with decrease in notch depth for thick specimens, but that the effect is much less pronounced in thin specimens, where the values of R_{GY} are uniformly much lower.

Discussion

There are two main aspects to the results: firstly, the information given on the general deformation behaviour and stress analysis of notched bars; secondly, the specific effects on the fracture behaviour of these specimens, and the implications of these with regard to C.O.D. and toughness testing.

Deformation and stress analysis

The first point which will be enlarged upon is the insensitivity of the critical depth ratio ($r_{d\ \mathrm{crit}} \simeq 1{\cdot}4$) to notch angle for acute notches (see e.g. Figs 2 (g) and (h)). Previously, only two situations had been considered [15]: a 60° V-notch in pure bending, where $r_{d\ \mathrm{crit}}$ was calculated as 1·39, and a 45° V-notch deformed in three-point or cantilever bending, as in a Charpy or Izod test: here $r_{d\ \mathrm{crit}}$ was calculated as 1·22. In view of the apparent anomaly raised by these two figures when tested by the present results, Ewing [9] calculated the critical depth ratio throughout the whole range 10° – 180° for bars deformed in plain strain four-point bending. His results, which agree well with the present experiments, are shown in Fig. 7, together with the deep-notch constraint factors [10]: it can be seen that the two curves have similar form, emphasizing the relationship between the two terms. The two conclusions from the results are:

(i) specimens of Charpy geometry have insufficiently deep notches for 'deep-notch' deformation in four-point bending (although they are satisfactory in three-point bending: see Fig. 8).

(ii) the critical depth ratio for a sharply cracked specimen will be virtually identical to that for the 45° V-notch ($r_{d\ \mathrm{crit}} \simeq 1{\cdot}41$).

Table 1 indicates that the stress intensification at general yield, R_{GY}, decreases with notch depth. This seems reasonable in that, for specimens with subcritical notch depths, the premature yielding from the top surface can 'freeze' the stress intensification below the notch at a value less than that which would have obtained had constrained deformation been able to continue. Estimates made by Ewing of values of R_{GY} for 0·08 and 0·04 in deep notches are 2·4 and 1·96 respectively, in good agreement with the experimental results in Table I and a previous experimental value for R_{GY} in specimens of Charpy geometry [14]. They are understandably at variance with theoretical figures derived for Charpy specimens in four-point bending by Alexander and Komoly [16], because these authors did not allow for the possibility of gross-section yielding.

The stress intensification calculated for the deeply notched, 0·05 in thick specimen may be compared with previous results on the effect of thickness on stress intensification [6, 17]: the present figure is slightly higher than the mean of values for plane stress (theoretical) and for a thickness of 0·1 in (experimental). For this particular depth geometry, therefore, the curve of stress intensification vs. thickness increases rapidly between 0 and 0·1 in, but then increases much more gradually. (The 0·1 value is about 85% of the plane strain figure.)

Fracture

The fracture results on the thick specimens (Fig. 3) show clearly two points made in the introduction. Firstly, the fracture mode, and hence the notch-root strain at fracture, can be altered at a constant temperature by changing the notch depth. Secondly, even if the fracture mode is the same for all notch depths, the notch-root strain can still vary. For example, at –50 °C, the 0·04 in deep notch produces fibrous initiation with local strains of over 100%: the 0·08 and 0·15 in deep notches produce cleavage with strains of about 20% and 12% respectively. These effects can be explained satisfactorily in terms of the different maximum local tensile stresses attained in these specimens at general yield. (For a similar situation involving variation of notch angle, see [6].)

The constraint factor results in Fig. 1 indicate that increasing the notch depth beyond a depth ratio of 1·4 will produce no further increase in the hydrostatic tensile stress and hence will produce no larger value of R_{GY}. This has been confirmed [18] and it has also been shown that even after general yield, very deep notches (r_d = 2 and 2·5) are no more severe, because a new form of stress relaxation occurs.

The above conclusions of course hold mainly for the thick specimens. For the 'thin' (0·05 in thick) specimens, the effect of notch depth is greatly reduced (Figs. 4 and 5, Table 1), because the values of R_{GY} are all fairly similar. In true plane stress, the hydrostatic component of stress would be almost identical to that in an unnotched specimen, and independent of notch depth. There would then, in plane stress, be no effect of notch depth on C.O.D. even for a steel showing a fibrous/cleavage transition.*

Implications

Using the above results, it is possible to comment on the effect of notch depth on C.O.D. and hence fracture toughness. The notch depth problems have seldom arisen for high strength materials, because these have been used primarily in plane stress applications, where differences in equivalent values of R are small. In plane strain, however, it does seem that for subcritical depth ratios (see Fig. 7) the toughness measurement made on a small test specimen may not be a parameter relevant to service application, unless the *depth ratio* is the same in the two cases. For some applications, this could mean, for example, that conventional fracture toughness test specimens provide an unnecessarily severe stress state.

The common toughness measurement made on material for use in thick section will be the 'plane-strain' fracture toughness, G_{IC}, or equivalent C.O.D. What may in fact be measured, however, is the lowest toughness value obtainable by specimen design alone: i.e., in the present context, the deep (parallel-sided) notch plane-strain toughness. (Incidentally, in tension this necessitates a depth ratio of 9·6 [19] in a very thick specimen). Alternatively, specimen design may give the toughness corresponding to the highest elastic stress concentration factor: however, this testpiece geometry does not seem to be particularly meaningful for C.O.D. tests in small specimens which break after general yield.

Mild steels and tempered low-alloy steels possess high energy barriers to fracture initiation under most service temperatures and loading rates: provided that embrittlement does not occur in service, there seems to be no reason for under-estimating material toughness by designing too severe a test-piece. The whole basis of Fracture Mechanics has been a rational design code derived from the limits of material toughness: it would appear that, for heavy-section low-strength steel, relevant toughness values can only be measured in test-pieces which are carefully designed to reproduce exactly the stress systems existing in each particular service application.

Conclusions

The general effect of notch depth on the toughness of mild steel in thick section resides in the relaxation of triaxial tensile stresses by plastic deformation on the gross cross-section of the notched piece. Once the

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* The situation is unclear if the cleavage fracture has to be initiated by twins, because fracture may then become mechanism-controlled.

critical depth ratio has been exceeded, differences between notch depths are removed [18]. Since conditions in service often correspond to the subcritical region, it is necessary to design test specimens with the same depth ratio as that in service, if meaningful toughness values are to be obtained. This procedure is probably unnecessary for thin components where the situation is close to plane stress.

Acknowledgments

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References

1. IRWIN, G. R., *Trans. A.S.M.*, 1948, vol. 40, pp. 147-163.
2. COTTRELL, A. H., *Iron & Steel Inst. Spec. Rep. 69*, 1961, pp. 281-296.
3. WELLS, A. A., *Brit. Welding Jnl.*, 1963, vol. 10, pp. 563-570.
4. KNOTT, J. F., *Metallurgia*, 1968, vol. 77, pp. 93-99.
5. FELBECK, D. K., & OROWAN, E., *Welding Research Supplement*, Nov. 1955, pp. 570s-575s.
6. KNOTT, J. F., *Jnl. Iron & Steel Inst.*, 1966, vol. 204, pp. 104-111.
7. KNOTT, J. F., *ibid* 1967, vol. 205, pp. 970-971.
8. KNOTT, J. F., *ibid* 1967, vol. 205, pp. 285-287.
9. EWING, D. J. F., *Jnl. Mech. Phys. Solids*, 1968, vol. 16, pp. 205-213. And private communication.
10. GREEN, A. P. & HUNDY, B. B., *ibid*, 1956, vol. 4, pp. 128-144.
11. HOBBS, R., & OSBORNE, C. J., *Jnl. Australian Inst. Metals*, 1968, vol. 13, pp. 88-93.
12. KNOTT, J. F., & COTTRELL, A. H., *Jnl. Iron & Steel Inst.*, 1963, vol. 201, pp. 249-260.
13. FORD, H., & LIANIS, G., *Zangew Math. Phys.*, 1957, vol. 8, pp. 360-382..
14. KNOTT, J. F., *Jnl. Mech. Phys. Solids*, 1967, vol. 15, pp. 96-103.
15. GREEN, A. P., *ibid* 1956, vol. 4. pp. 259-268.
16. ALEXANDER, J. M., & KOMOLY, T. J., *ibid* 1962, vol. 10, pp. 265-278.
17. KNOTT, J. F., *Proc. Roy. Soc.* 1965, vol. A285, pp. 150-159.
18. KNOTT, J. F., *Jnl. Iron & Steel Inst.* 1967, vol. 205, pp. 288-291.
19. EWING, D. J. F., & HILL, R., *Jnl. Mech. Phys. Solids* 1967, vol. 15, pp. 115-124.

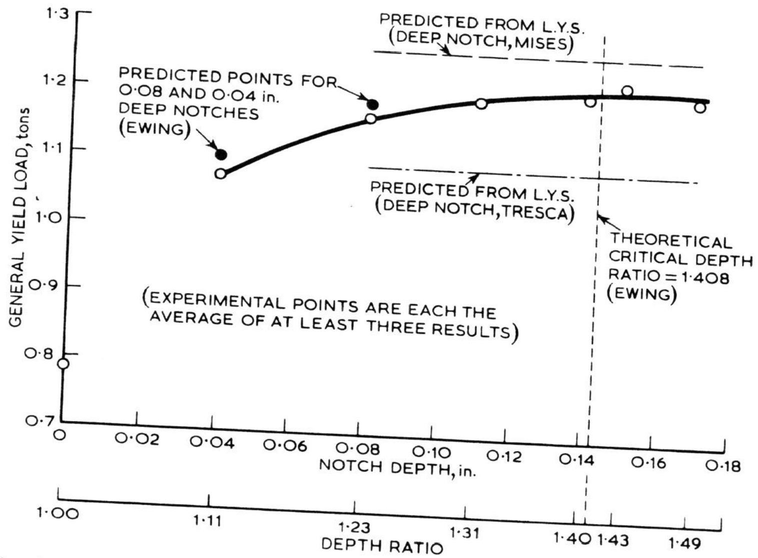
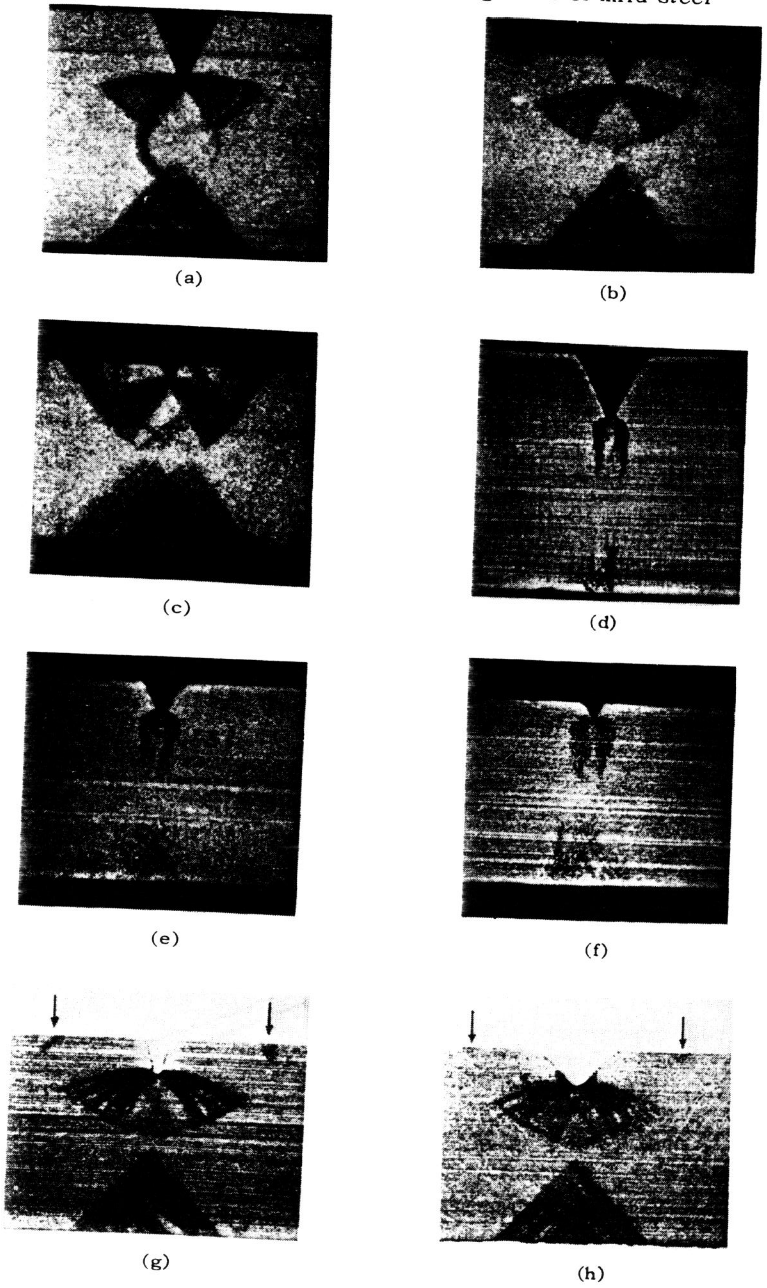


Fig. 1. General yield load as a function of notch depth (0·5 in thick bars).

Fig. 2. Slip-line fields as revealed by etching in Fry's reagent.
(a) notch depth 0·15 in, thickness 0·5 in.
(b) notch depth 0·08 in, thickness 0·5 in.
(c) notch depth 0·04 in, thickness 0·5 in.
(d) notch depth 0·15 in, thickness 0·05 in.
(e) notch depth 0·08 in, thickness 0·05 in.
(f) notch depth 0·04 in, thickness 0·05 in.
(g) notch angle approx. 20°, depth 0·08 in, thickness 0·5 in.
(h) notch angle approx. 90°, depth 0·08 in, thickness 0·5 in.
All × 2·5 approx.


(a) (b)
(c) (d)
(e) (f)
(g) (h)

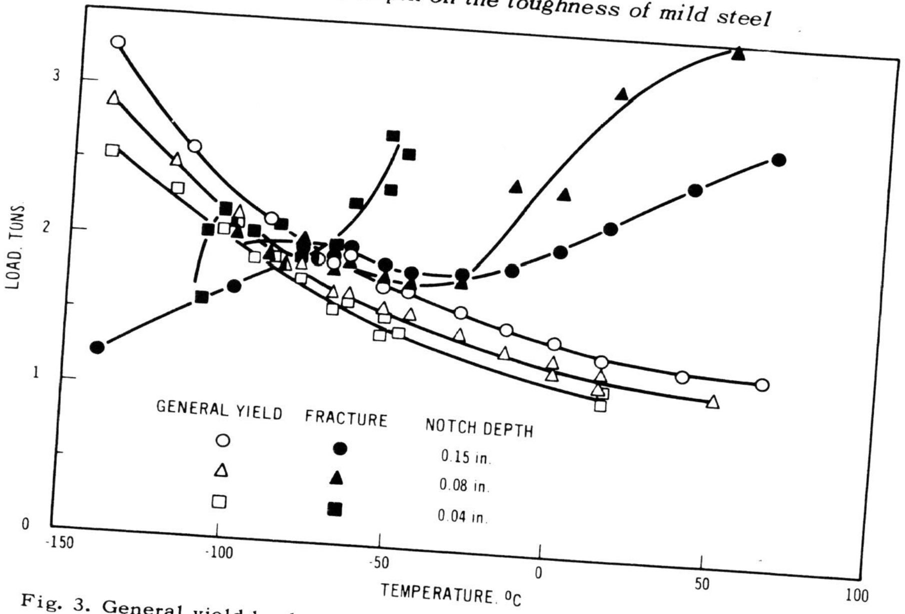


Fig. 3. General yield load and fracture load as a function of temperature and notch depth (0·5 in thick specimens).

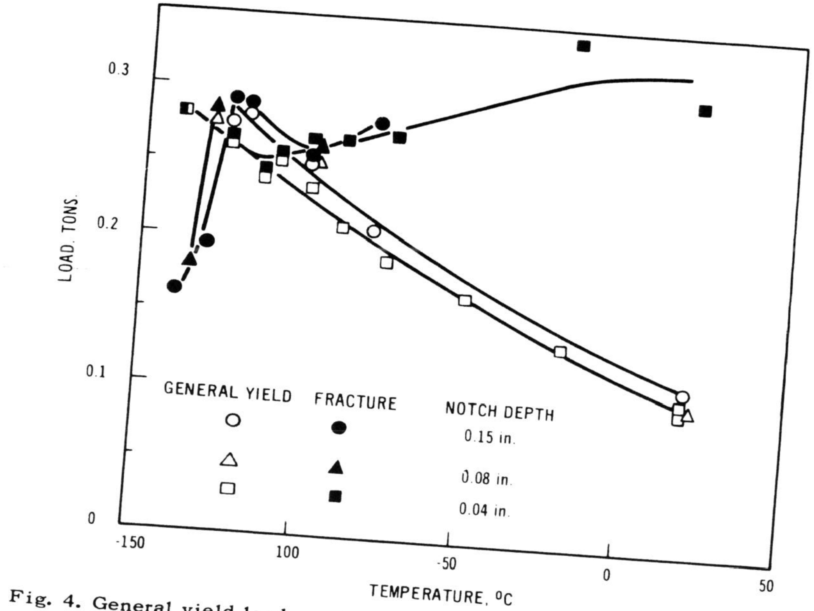


Fig. 4. General yield load and fracture load as a function of temperature and notch depth (0·05 in thick specimens).

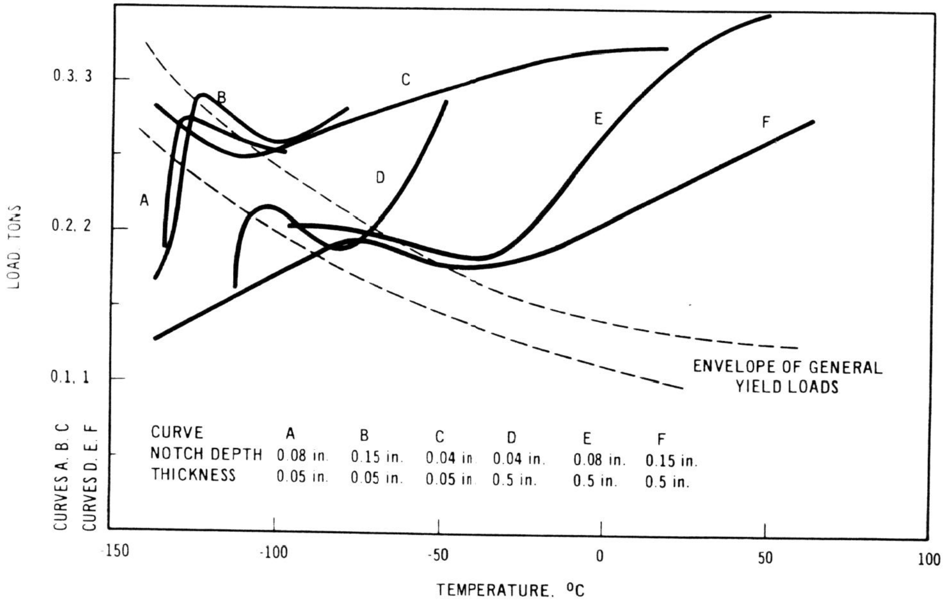


Fig. 5. Summary of results in Figs. 3 and 4.

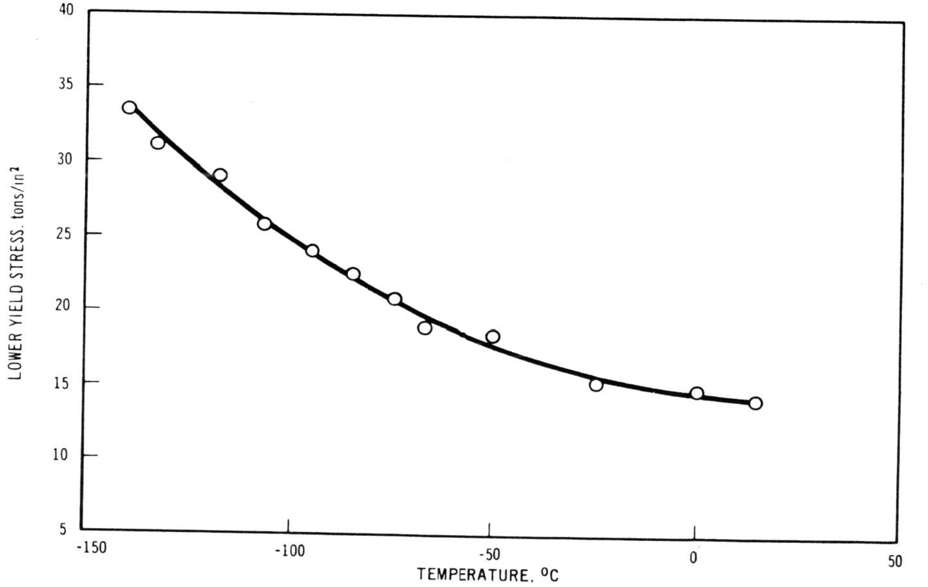


Fig. 6. Uniaxial lower yield stress as a function of temperature.

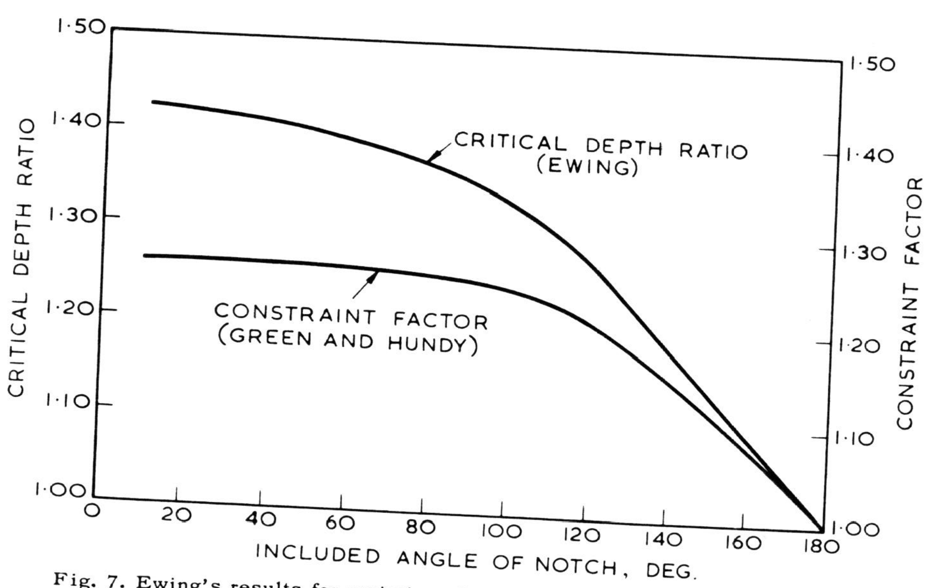


Fig. 7. Ewing's results for variation of critical depth ratio and notch angle.

Fig. 8. General yield slip-line field pattern for Charpy specimen as revealed by etching in Fry's reagent (r_d = 1·25, but three-point bending).

Table 1

Notch Depth in	0·15	0·08	0·04	0·15	0·08	0·04
Thickness in	0·5	0·5	0·5	0·05	0·05	0·05
T_{GY} °C	−75	−85	−107	−128	−133	−140
σ_y tons/in^2	21·2	22·5	25·7	30·5	31·0	33·0
$R = \sigma_F / \sigma_y$	2·51	2·36	2·07	1·74	1·72	1·60
L exptl.	1·26	1·17	1·08	~1·08	~1·02	~1·00
R theor.	2·51	2·40	1·96	–	–	–
L theor.	1·26	1·21	1·14	–	–	–