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A criterion for plastically induced cleavage after general yield in notched bend specimens

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Summary

Charpy V-notch specimens of mild steel with varying root radii were fractured in slow three point bending over a temperature range – 196 to 20°C. An analysis of the fracture data is presented based upon the relationship between the macroscopic deformation represented by the bend angle at fracture, and the local plastic strains around the notch root. This analysis predicts that the critical bend angle at fracture should vary as the square of the root radius at a given temperature, and is in agreement with the experimental observations. The physical significance of these micro- and macroscopic criteria are discussed.

Introduction

Plain strain brittle fracture before general yield can occur in cracked specimens when the stress intensity factor K_I, reaches a critical value, K_{Ic}. This approach allows fracture data obtained from a variety of specimen geometries to be analyzed by one coherent method [1]. In notched specimens, where the defects have a finite root radius ρ, the stress intensity factor is modified and the macroscopic fracture criterion can be written as $K_I(\rho) = K_{Ic}(\rho)$ [2]. In low yield strength materials, such as ferritic pearlitic steels, very high strain rates or low temperatures are required to allow this criterion for elastic-plastic fracture to be satisfied. On a microscopic scale, fracture in this range occurs by cleavage. At very low temperature the microscopic fracture criterion appears to be based on the achievement of a critical tensile stress $\sigma_f{}^*$ in the plastic zone ahead of the notch [3].

At higher temperatures, for example near the lower toe of the Charpy or DWTT curves, fracture can still occur by brittle cleavage, but only after large plastic strains have been produced near the notch tip and, in small specimens, have spread completely across the minimum section of the specimen, causing general yield. The principles of linear elastic fracture mechanics can no longer be used to predict fracture behaviour in terms of fracture load, defect size and, crack tip toughness, and a full plastic analysis is required. This is a problem of some practical importance, as brittle fracture in large steel structures such as pressure vessels often initiates at defects that lie in fully plastic regions (e.g., near nozzle inserts) and then propagates unstably through the nominally elastic portions of the structure.

The present work was undertaken to determine some usable criterion for the onset of the brittle cleavage fracture that occurs after general yield. For these purposes, we have employed standard Charpy-V specimens, having variable root radii and loaded in slow three-point bending, since a great deal of information on plastic behaviour was already available for this type of specimen. Analysis of the data indicates that the microscopic criterion for cleavage fracture after general yield may be described by the attainment of a critical plastic strain, ϵ_f, in the region of highest triaxility ahead of the notch. This 'local' fracture criterion leads to a nominal (macroscopic) fracture criterion in which the critical bend angle at fracture, θ_F, varies linearly with ρ^2 at a given temperature. In mild steel ϵ_f, and hence θ_F, increase with increasing temperature. The physical significance of these microscopic and macroscopic criteria are discussed.

Experimental Procedure and Results

Fracture experiments were performed on rectangular prismatic specimens, 10 × 10 × 60 mm, containing 2 mm deep, 45° notches of varying root radii. They were machined from hot rolled plate of 'high nitrogen steel' containing the following chemical composition (in wt. %)

C	*Mn*	*Si*	*P*	*S*	*Al*	N_2
0·14	0·24	0·05	<0·01	0·02	<0·005	0·013

Following machining, all specimens were annealed in vacuo at 850 °C for one hour, to produce a uniform grain structure, 30-40 μ in diameter. The specimens were then broken in three point loading at a constant deflection rate of 0·1 in/min, at temperatures between 0 and –196 °C.

The fracture loads of the notched bars of variable root radii are presented in Fig. 1. The general yield loads are not shown; they were independent of the root radius and were all very close to the dashed curve. The temperature dependence of the fracture load and general yield curves are similar to those reported previously [4]. As viewed with a low power microscope, fracture occurred completely by cleavage at low temperatures below about –60 °C; above this temperature, cleavage was observed on the fracture surfaces of the sharply notched specimens, ($\rho \leq 0{\cdot}125$ mm) but it was not possible to determine if the fracture *initiated* by unstable cleavage below the root, as compared with stable rupture. Consequently, most of the subsequent discussion deals with the temperature range (–70-110 °C) where it is known that cleavage initiated fracture after general yield.

The bend angles at fracture θ_F were measured from the cross-head displacement and are presented in Fig. 2. It is apparent that toughness decreases with decreasing root radius, at a given test temperature. In fact, as shown in Fig. 3, the nominal fracture criteria can be expressed as

$$\theta_F = \lambda\rho^2 \quad (0{\cdot}05\ \text{mm} < \rho) \quad (-110\,°\text{C} < T < -70\,°\text{C}) \tag{1}$$

where λ is a parameter that increases with increasing temperature.

Between –65 and –80 °C θ_F was found to be identical, within experimental error, for fatigue cracked specimens and specimens containing notches of root radius $\rho = 0{\cdot}05$ mm. This behaviour is consistent with that noted at very low temperature (Fig. 1) where all fractures occur well before general yield (–130 °C < T) and results from the fact that a critical volume of material near the crack tip must be under high triaxial stress before an unstable microcrack can develop ahead of it [2]. The abrupt increase in θ_F (c.f. Fig. 1) for $\rho = 0{\cdot}05$ and $\rho = 0{\cdot}125$ mm specimens as general yield is approached. (–130 °C < T < –110 °C) is not well understood at present, but is probably related to the change in flow pattern from logarithm spiral to hinge type deformation.

Calculations and Discussion

Strain distribution near the notch root

The results presented above indicate that when cleavage fracture initiates after general yield in notched bars loaded in plane strain bending, the bend angle at fracture θ_F is proportional to ρ^2 and increases with increasing temperature. It is of interest to relate this macroscopic criterion for fracture to microscopic processes that occur in the plastic zone near the notch root. In order to develop a fracture criterion, it is first necessary to relate the macroscopic strains (bend angles) to local strains near the root.

Plastic deformation of a notched bar after general yield occurs along two circular arcs ('hinges') which pass through the notch root and have a radius r but no common centre. Wells [5] has shown that the notch tip opening displacement $2V(c)$ is given by

$$2V(c) = a\theta$$

where θ is the bend angle (in radians) and a is the depth of the axis of rotation below the notch root (approximately one-half the ligament depth). For the Charpy specimen employed in this investigation $a = 3{\cdot}5$ mm so that

$$V(c) = 0{\cdot}03\theta^{\circ}\ \text{mm} \tag{2}$$

where θ is expressed in degrees.

If we assume that when the notch tip strains $\epsilon(c)$ are small, the region at the notch tip can be treated as a 'miniature tensile specimen' of gauge length 2ρ, and that all of the tip opening displacement is concentrated in this specimen, then

$$2V(c) = 2\rho\epsilon(c) \qquad (3)$$

and hence

$$\epsilon(c) = \frac{a\theta}{2\rho} \qquad (4)$$

For the standard Charpy V-notch specimen, $a = 3{\cdot}5$ mm [6], so that for $\rho = 0{\cdot}25$ mm equation 4 becomes simply,

$$\epsilon(c) = 0{\cdot}125\theta^{o} \qquad (5)$$

This relation is in excellent agreement with independent measurements of the relation between notch tip strain and bend angle, as reported by Lequear and Lubahn [7] for $\epsilon(c) < 0{\cdot}25$, and provides support for the assumption used in setting up equation (3). For $\epsilon(c) > 0{\cdot}25$, $\rho = 0{\cdot}25$ mm, Lequear and Lubahn noted that the notch tip strains increased more slowly than predicted by equation (5). This implies that a significant fraction of the tip opening displacement is accommodated by deformation that occurs within regions well removed from the tip (e.g. along wings that spread from the tension surface of the specimen to the ligament) and that equation (3) is less satisfactory.

A third set of independent measurements, also made on mild steel, by Wilshaw [8], indicate that the strain distribution a distance x *ahead* of the Charpy V-notch and in the notch plane is given by

$$\epsilon(X) = \frac{F\theta}{x} \qquad x > 0{\cdot}1 \text{ mm} \qquad (6)$$

Equation (6) predicts the existence of a strain singularity at the notch tip. We do not believe that such a singularity exists for notched specimens, first because continuum variations in strain have little if any physical significance over distances of the order of the dislocation cell size or grain size and second, because the experimental data (equation (5)) indicate that around notches having root radii of 0·25 mm, for example, the root strains $\epsilon(c)$ are finite at a given value of θ.

In order to match up the experimental relations given by equations (5) and (6), we assume that over some finite distance x^* ahead of the notch the root strains are constant as shown schematically in Fig. 4, and that

$$\epsilon(c) = \epsilon(x) \text{ at } x = x^* \qquad (7)$$

Combining relation (5-7) gives

$$F = \frac{\epsilon(x)x}{\theta} = \frac{\epsilon(c)}{\theta}x^* \qquad (8)$$

and introducing (4)

$$\epsilon(x) = \frac{a\theta}{2\rho}\,\frac{x^*}{x} \qquad x > x^* \qquad (9)$$

so that

$$F = F(\rho) = \frac{ax^*}{2\rho}$$

The dependence of F on $(1/\rho)$ is not surprising, since recent calculations of Mode III deformation [9] by Rice and Rosengren predict a strain concentration factor that varies as $(1/\rho)\ 1/1+n$, where n, the strain hardening exponent, is approximately equal to 0·2-0·3 for the present material. To evaluate x^* we have the experimental value of $F = 0{\cdot}005$ for $\rho = 0{\cdot}25$ mm; with $a = 0{\cdot}06$ mm/degree, we find that $x^* = 0{\cdot}04$ mm, which is equivalent to the grain diameter for this material. Introducing this value into equation (9) gives

$$\epsilon(x) = \frac{a\theta}{\rho x}(0{\cdot}02) \qquad x > x^*$$

$$\epsilon(x) = \frac{1{\cdot}20 \times 10^{-3}}{\rho x}\theta \quad \text{(For Charpy)} \quad x > 0{\cdot}1 \text{ mm} \qquad (10)$$

or

$$\theta = 50\rho \times \epsilon(x)/a$$

$$\theta \cong 8{\cdot}3 \times 10^{2}\,\rho x\,\epsilon(x) \quad \text{(For Charpy)} \quad x > 0{\cdot}1 \text{ mm} \qquad (11)$$

Fracture Criteria

Various local (microscopic) criteria have been proposed to account for the onset of unstable crack propagation; the appropriate criterion will, of course, depend on the microscopic mode of fracture that is actually taking place. For the plastically induced cleavage that occurs between −70 and 110 °C a criterion based on a critical strain at *the notch tip* $\epsilon(c)$ is *not* appropriate because unstable fracture does not initiate there and because equation (4) would predict that the bend angle at fracture θ_F is proportional to ρ, whereas Fig. 3 indicates that θ_F is in fact proportional to ρ^2. The ρ^2 dependence of θ_F can, however, be accounted for with a model based on the achievement of a critical tensile strain $\epsilon_f(R_\beta)$ at *some point* R_β *below the root,* where maximum triaxial stress is first achieved, according to plane strain plasticity theory for a perfectly plastic solid.

When a notched bar is loaded in plane strain bending, plastic zones form at the notch root and spread a distance R into the ligament. For perfectly plastic solids, the longitudinal stress within the plastic zone ($x<R$, Fig. 5) is given by

$$\sigma_{yy} = \sigma_Y\,[1 + \ln(1 + x/\rho)] \qquad (12)$$

where σ_Y is the tensile yield strength, assuming a Tresca yield criterion [10]. The maximum value of σ_{yy}, $(\sigma_{yy}{}^{max})$, occurs at the elastic-plastic interface ($x = R$)

$$(\sigma_{yy}{}^{max}) = \sigma_Y\,[1 + \ln(1 + R/\rho)] = \sigma_Y\, K_{\sigma(\rho)} \qquad (13)$$
$$0 < R < R_\beta$$

$K_{\sigma(\rho)}$ is the plastic constraint factor and is a measure of the degree of triaxiality.

Equations (12) and (13) are only valid for $R<R_\beta$. For $R = R_\beta$, and for subsequent plastic flow along hinges, $\sigma_{yy}{}^{max}$ remains constant, at its maximum possible value (Fig. 5), $(\sigma_{yy}{}^{max})^{max}$, which depends only on the flank angle of the notch ω

$$(\sigma_{yy}{}^{max})^{max} = \sigma_Y\left[1 + \frac{\pi}{2} - \frac{\omega}{2}\right] = \sigma_Y\, K^{max}_{\sigma(\rho)} \qquad (14)$$

Since equations (13) and (14) are equivalent for $R = R_\beta$, we have

$$R_\beta = \rho\left[\exp\left(\frac{\pi - \omega}{2}\right) - 1\right] \qquad (15)$$
$$= 2{\cdot}25\rho \text{ for Charpy V Geometry } \omega = 45^\circ.$$

In a strain hardening solid the situation is much more complicated and, in fact, unresolved at present. Since the flow stress is no longer a constant but varies through the plastic zone, it is necessary to introduce the variation of σ_Y with position into the characteristic equations which form the basis for determining the hydrostatic stress elevation in the plastic zone, due to the curvature of the slip line fields. This leads to the result [11] that

$$\sigma_{yy} = \left[\frac{\sigma_s + \sigma_i}{2}\right] - \int_\beta \sigma_Y(x, y)\, d\phi - \int_\beta \frac{\partial\sigma_Y/2}{\partial s_\alpha}\, ds_\beta \qquad (16)$$

The first term in equation (16) is the contribution from the unconstrained flow stress at the surface of the notch $\sigma_s/2$ and some interior point $\sigma_i/2$, and corresponds to σ_Y in equation (12) for the perfectly plastic solid. The second and third terms are the hydrostatic contributions that result from the curvature of the α and β slip lines in the plastic zone when σ_y is a function of (x, y) because of the variation of plastic strain and hence strain hardening with position. It corresponds to the log term in equation (12).

At the present time, there are insufficient data or analyses to adequately evaluate the value of σ_{yy} according to equation (16) and consequently it is necessary to extrapolate from the perfectly plastic analysis to interpret the variation of fracture bend angle with root radius.

Microscopic observations on standard Charpy V specimens that broke after general yield indicate that stable microcracks initiate approximately 0·5 mm ahead of the notch root and trigger unstable fracture of the entire specimen. This distance corresponds to the value of R_β = 2·25(0·25) = 0·55 mm where maximum constraint would be achieved in a perfectly plastic solid (equation 15). If we suppose that unstable fracture occurs when the strains $\epsilon_f(R_\beta)$ at a position $x = R_\beta$ achieve some critical value, $\epsilon_f(R_\beta)$, then since $R_\beta = 2{\cdot}25\rho$ for the 45° notches, equation (11) gives

$$\theta_f = (50\rho)(2{\cdot}25\rho)\epsilon_f(R_\beta)/a$$
$$= 1{\cdot}8 \times 10^3\, \rho^2 \epsilon_f(R_\beta)$$

The predicted variation of θ_f with ρ^2 is in excellent agreement with the data shown in Fig. 3. From the slopes of the curves, we are then able to determine the temperature dependence of $\epsilon_f(R_\beta)$; the results are shown in Fig. 6.

The increase in 'local ductility' with temperature probably arises from one of two processes. At very low temperatures (−196 to −150 °C) cleavage fracture initiates below the notch root when a critical tensile stress $\sigma_{yy} = \sigma_f{}^*$ is achieved in the plastic zone ahead of the notch. As the temperature increases and the yield strength decreases, a greater degree of triaxiality $K_{\sigma(\rho)}$ must be achieved to satisfy the fracture criterion, and the critical plastic zone size for fracture increases. Since $K_{\sigma(\rho)}$ has a maximum value of 2·18, a constant value of tensile stress $\sigma_f{}^* = 2{\cdot}18\sigma_{flow}$ can only be achieved by increasing the degree of strain hardening in the plastic zone, since the yield stress continues to decrease with increasing temperature. Consequently, an increase in $\epsilon_f(R_\beta)$ with increasing temperature would be expected if the local criterion for fracture was based upon the achievement of a critical tensile stress in the temperature range (−110° to −70 °C). The critical tensile stress would be that which is required to allow unstable microcrack propagation, as discussed elsewhere.

Alternatively, if unstable fracture in this temperature range results from the plastic growth and coalescence of microcracks lying in the plastic zone near R_β, to form a dynamic crack of critical size, then the local fracture criterion would be expected to be similar to that derived by McClintock [12] for hole growth. He showed that the ductility of a deforming region was greatly reduced by the presence of a high bi-axial or triaxial stress field (e.g. at $x = R_\beta$) and by a high density of holes. Since the microcrack density increases sharply with tensile stress level, the initial microcrack density is also highest at $x = R_\beta$, and rupture by microcrack coalescence would therefore also be expected to initiate at this point. The increase in ductility with increasing temperature would then arise from the fact that since the flow stress decreases with increasing temperature, the microcrack density at $x = R_\beta$ would also decrease, at a given strain $\epsilon(R_\beta)$, and hence a greater amount of strain would be required for coalescence to the point of instability. The sharp increase in $\epsilon_f(R_\beta)$ at and above $-90\,^\circ C$ probably results from a breakdown in plane strain deformation, which is accompanied by a loss of triaxiality.

It is very difficult to decide which of these two local processes (unstable growth of one microcrack vs. coalescence of several stopped ones followed by instability due to microcrack growth) is responsible for brittle cleavage that occurs in this temperature range. Both processes are favoured by the same factors, particularly a high degree of triaxiality (i.e. thick specimen, sharp crack) and a high flow stress, and further work needs to be done to examine this question. We believe that the significance of the work reported here is that whatever microscopic process is ultimately responsible for the instability, this process can be characterised by a critical tensile strain in a local region that is a function of the triaxiality there. In notched bend specimens this strain can be directly related to a macroscopic criterion for fracture (θ_F) through a knowledge of the strain concentration effect of the notch. This suggests that other problems in plastic fracture mechanics might benefit from a similar approach, based on relating the nominal strains to the local strains by experimental and analytic methods. If any one macroscopic parameter could be used to characterise fully plastic fracture, and this is by no means certain, it will clearly be some form of critical strain intensity factor related to the degree of triaxiality rather than some critical stress intensity.

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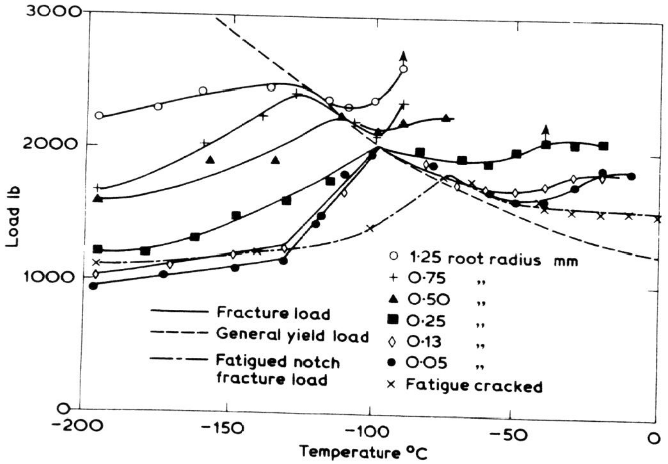

Fig. 1. Fracture load vs temperature for Charpy specimens with varying root radii loaded in three point bending at a rate of 0·1 in/min..

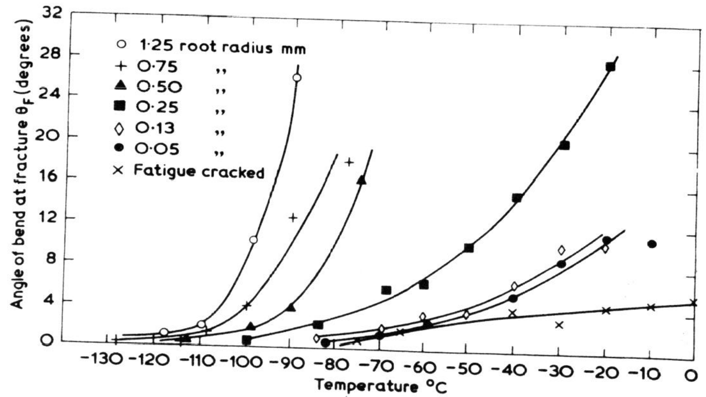

Fig. 2. Fracture bend angle vs temperature.

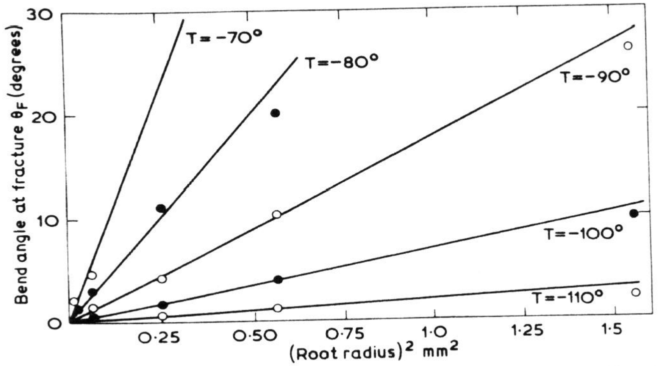

Fig. 3. Fracture bend angle vs the square of the root radius for various temperatures.

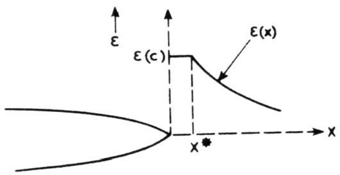

Fig. 4. Schematic diagram showing the distribution of plastic strain ahead of the notch root.

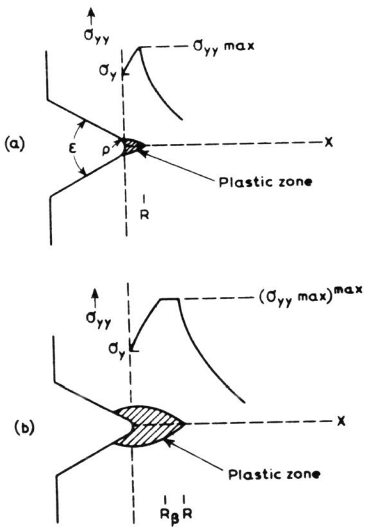

Fig. 5. The elastic-plastic stress distribution ahead of a notch.

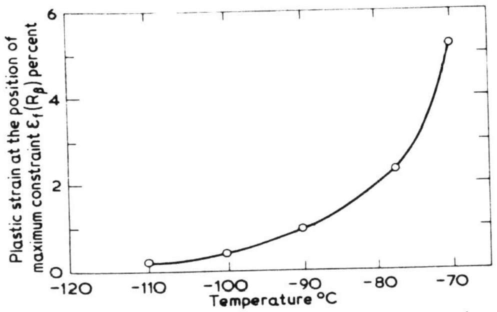

Fig. 6. Plastic strain at the position of maximum constraint required to cause fracture at various temperatures.