Size Effect in Finite-Life Fatigue of Metals

Andrea Carpinteri, Andrea Spagnoli, Sabrina Vantadori
Department of Civil-Environmental Engineering & Architecture, University of Parma, Parma, Italy

Abstract
Many experimental observations show that the finite-life fatigue strength of metallic materials in the high-cycle regime decreases with increasing the specimen size. Such a decrease can be explained by considering the fractal nature of the reacting cross-sections of structures. Accordingly, the so-called fractal fatigue strength is represented by a force amplitude acting on a surface with a fractal dimension lower than 2, where such a dimensional decrement depends on the presence of damage (cracks, voids) and heterogeneity in the material ligament. A monofractal scaling law for the finite-life fatigue strength of metals is herein proposed, and some experimental results are examined to show how to apply such a theoretical approach.

1. Introduction

Many experimental observations show that static strength and fatigue strength of materials decrease with increasing the specimen size (size effect), and this decrease is more significant for comparatively heterogeneous and/or damaged materials. Such a phenomenon was analysed by Griffith [1] for glass filaments by assuming the presence of microcracks, whereas Peterson [2] examined size effect for fatigue fracture. Weibull [3] introduced the statistical concept of the weakest link in a chain, that is, by increasing the volume of the structural component (or specimen), the probability of failure increases due to the higher probability of finding a critical microcrack provoking macroscopic fracture. Then a scaling law for fracture failure of structures has been proposed in Ref. [4], by showing that the size of the most dangerous defect increases with increasing the size of the structure.

An usual engineering approach to explain size effect in fatigue is based on the stress gradient concept [5]. Accordingly, fatigue cracks can initiate and grow in a structure only if a finite region of this structure is subjected to a cyclic stress greater than a characteristic value, with the finite region size dependent on the characteristics of the material microstructure (e.g. grain size in metals).

Then the fractal nature of the material microstructure [6] and the renormalization group theory [7] have been considered in Ref. [8] (a detailed review on the application of the fractal approach to the mechanics of heterogeneous and disordered materials can be found in Ref. [9]). More precisely, the reacting cross-section of a given structure shows a self-similar weakening due to the material heterogeneity, cracks, defects, etc., and therefore the fractal (noninteger) dimension of such a surface can be assumed to be lower than 2 [10], that is, the damaged ligament of a heterogeneous solid may be modelled through a "lacunar" fractal set, as for instance the celebrated Sierpinski carpet. Through the renormalization procedure [7], new mechanical properties can be defined with physical dimensions dependent on the fractal dimension of the damaged heterogeneous ligament, and these properties are scale-invariant constants [8], such as the fractal (or renormalized) finite-life fatigue strength σ_a^* discussed in the following.

The above fractal concepts have recently been applied to size effect in fatigue [11,12]. In more detail, a monofractal scaling law (related to self-similar fractal topologies) and a multifractal scaling law (related to self-affine fractal topologies) have been deduced for fatigue limit [11]. Then, a size-dependent crack propagation law has been proposed [12]. It is interesting to note that the generalized Frost and Dugdale crack growth model discussed in Ref. [13] follows the law determined in Ref. [12].

In the present paper, a so-called monofractal scaling law for the finite-life fatigue strength of metals is discussed, and some experimental results [14] are examined to show how to apply the theoretical fractal approach proposed.

2. Theoretical Monofractal Approach for Finite-Life Fatigue Strength

Let us consider two geometrically similar cylinders (A and B, with B larger than A), made up of the same material, subjected to cyclic axial loading. The apparent finite-life fatigue strengths for such bodies are equal to (the subscript a stands for amplitude):

$$\sigma_{a,A} = \frac{4\ F_{a,A}}{\pi\ D_A^2} \tag{1a}$$

$$\sigma_{a,B} = \frac{4\ F_{a,B}}{\pi\ D_B^2} \tag{1b}$$

where $F_{a,A}$ and $F_{a,B}$ are the axial force amplitudes (acting on the two cylinders, respectively), which provoke fatigue fracture failure after $\overline{N}$ loading cycles (Fig.1). From experimental tests, it has been deduced that, for cylinder B larger than cylinder A, $\sigma_{a,A}$ is greater than $\sigma_{a,B}$.

As is well-known, the finite-life fatigue strength for metals can be represented in the high-cycle regime by the following Basquin-like power law:

$$C(D) = N\ \left(\sigma_a\right)^{\beta} \tag{2}$$

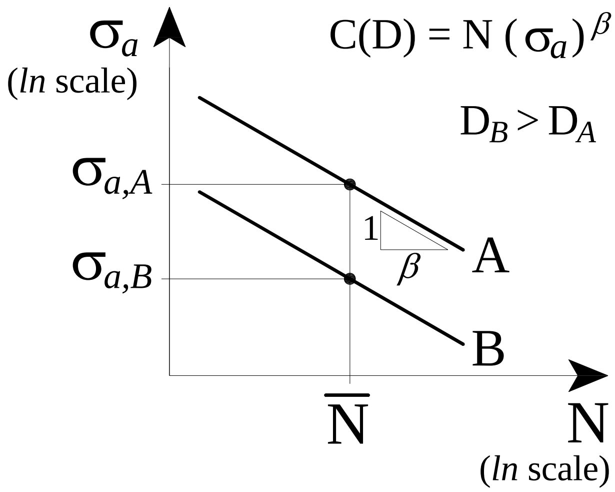

Figure 1. S-N curves in a bilogarithmic diagram.

where β is related to the slope $-(1/\beta)$ of the straight lines in the bilogarithmic diagram in Fig.1 and, for a given material and a fixed value of cylinder size D, the S-N curve parameter $C(D)$ is a constant. Since $\sigma_{a,A} > \sigma_{a,B}$ for $N = \overline{N}$ and cylinder B larger than cylinder A, we get that $C_A > C_B$:

$$C_A = \overline{N} \left(\sigma_{a,A}\right)^{\beta} \tag{3a}$$

$$C_B = \overline{N} \left(\sigma_{a,B}\right)^{\beta} \tag{3b}$$

that is, the parameter $C(D)$ decreases by increasing D.

As is said in the previous Section, the reacting cross-section of a structure can be assumed to present a fractal dimension $\alpha = 2 - d$, with $0 \le d \le 0.5$, where the decrement d depends on a self-similar microstructural (heterogeneity) and micromechanical (damage) weakening [8, 10], and the value of d is higher when such a weakening is more significant. Then, by assuming that the fractal (renormalized) fatigue strength σ_a^* is a material constant with physical dimensions given by $[F] \,/\, [L]^{2-d}$, the following expression holds:

$$\sigma_a^* = \frac{4\ F_{a,A}}{\pi\ D_A^{2-d}} = \frac{4\ F_{a,B}}{\pi\ D_B^{2-d}} \tag{4}$$

that is,

$$F_{a,B} = F_{a,A} \left(\frac{D_B}{D_A}\right)^{2-d} , \tag{5}$$

whereas Equation (2) in a fractal form is given by:

$$C^* = N \left(\sigma_a^*\right)^{\beta} \tag{6}$$

The fractal parameter C^* is a scale-invariant material property, that is, the S-N curve becomes independent of the structure size (Fig.2).

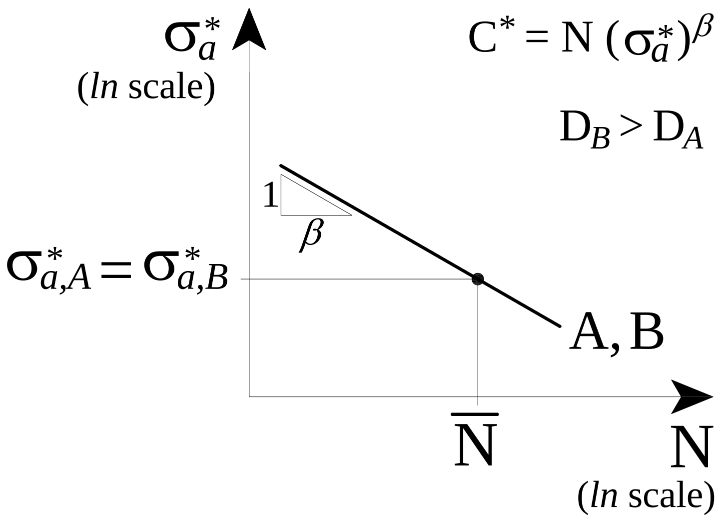

Figure 2. Fractal (or renormalized) S-N curve in a bilogarithmic diagram.

Recalling Eqs (1b), (5), (1a) and (3a) in this order, Equation (3b) becomes:

$$C_B \;=\; C_A \;\left(D_B/D_A\right)^{-d\,\beta} \tag{7a}$$

and in a logarithmic form :

$$\ln C_B \;=\; \ln C_A \;-\; \left(d\,\beta\right)\;\ln\left(D_B/D_A\right) \tag{7b}$$

By assuming $D_A = 1$ and $D_B = D$ and calling with the symbol C_1 the value of C_A for a cylinder with diameter equal to 1, the last two expressions can be written as follows:

$$C(D) \;=\; C_1 \;\left(D\right)^{-d\,\beta} \tag{8a}$$

$$\ln C(D) \;=\; \ln C_1 \;-\; \left(d\,\beta\right)\;\ln D \tag{8b}$$

where the latter equation represents a straight line with a slope equal to $-d$ times β (Fig.3). Note that C_1 is equal to the scale-invariant material property C^*, defined in Eq.(6). As a matter of fact, recalling Eqs (4), (1a) and (3a), Equation (6) for $N = \overline{N}$ becomes:

$$C^* \;=\; \overline{N}\;\left(\frac{4\;F_{a,A}}{\pi\;D_A^{2-d}}\right)^{\beta} \;=\; C_A \;\left(D_A\right)^{d\,\beta} \tag{9}$$

and this expression for $D_A = 1$ gives us $C^* = C_1$, that is, C^* corresponds to the value of the parameter $C(D)$ for a cylinder with $D = 1$.

Equation (2) can be transformed into a logarithmic relationship:

$$\ln\sigma_a \;=\; -\left(1/\beta\right)\;\ln N \;+\; \left(1/\beta\right)\;\ln C(D)\;, \tag{10}$$

and this expression represents parallel S-N curves which depend on the structure size D (Fig.4). Note that each line intersects the horizontal axis of the coordinate system in a point the abscissa of which is equal to $\ln C(D)$ (see also Fig.3).

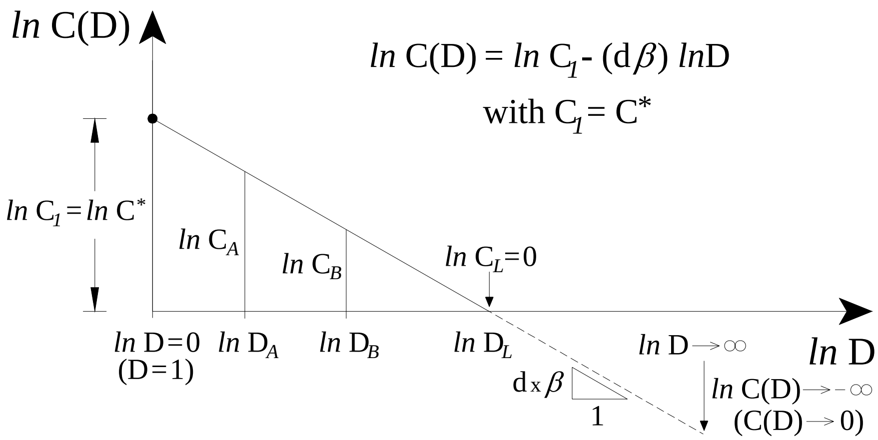

Figure 3. S-N curve parameter $C(D)$ against structure size D.

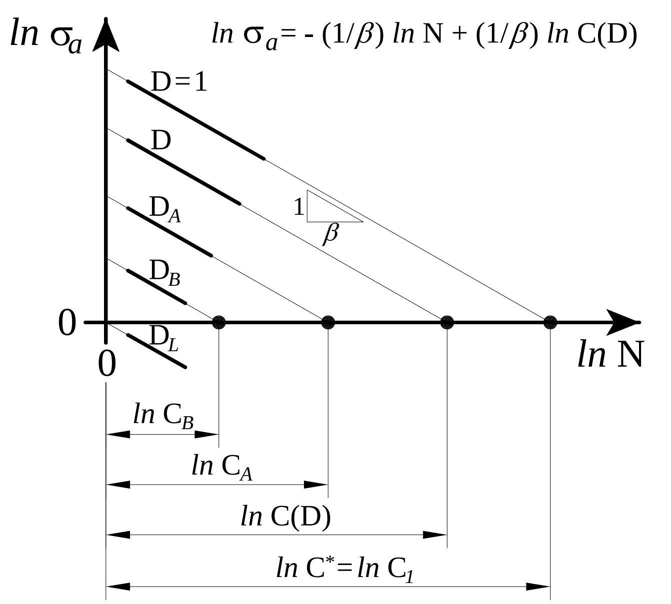

Figure 4. Finite-life fatigue strength σ_a against number N of loading cycles.

Equations analogous to those for push-pull loading (Eqs (1) to (10)) can be deduced for rotary bending.

3. Experimental Application

Now some experimental results are analysed to show how to apply the above equations. Hatanaka et al. [14] performed fatigue tests on smooth specimens made up of two different materials: a cast steel (JIS SCMn 2A) originally including many defects (comparatively disordered material), and a forged steel

(JIS SF 50) with a quite homogeneous microstructure (comparatively ordered material). For SCMn 2A steel, the yield stress is equal to 325MPa, the ultimate tensile stress to 576MPa and the elongation to 18.2%. For SF 50 steel, the yield stress is equal to 283MPa, the ultimate tensile stress to 484MPa and the elongation to 39.1%.

Cylindrical smooth specimens with diameter D equal to 8, 20, 30 and 40 mm, respectively, were tested. The experimental S-N curves for the two steels are displayed in Fig.5 by excluding the run out data. Note that, for both materials, the fatigue strength decreases by increasing the specimen size. In particular, by increasing D from 8 to 40 mm, the amount of decrease in the value of fatigue strength σ_a for $N = 10^6$ cycles is equal to about 24.3% for SCMn 2A steel and about 13.5% for SF 50 steel.

For each S-N curve in Fig.5, the parameter β can be determined from the straight line slope (which is equal to $-(1/\beta)$, see Eq.10). In particular, for the disordered

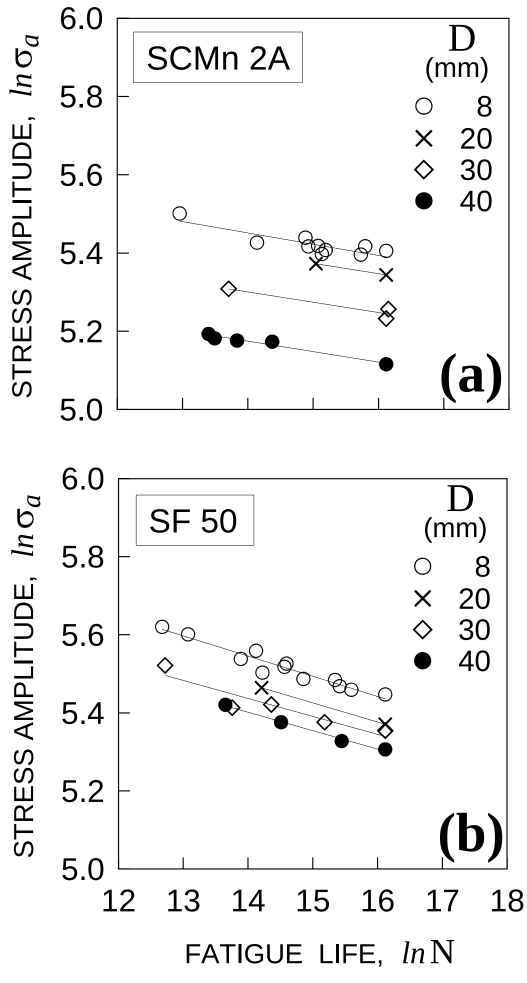

Figure 5. S-N curves related to two steels tested by Hatanaka et al.[14], for different values of specimen diameter D. Fatigue strength σ_a is expressed in MPa.

material (Fig.5a) the best-fit values of β are equal to 34.544, 37.540, 38.102, 37.609 (for D = 8, 20, 30, 40 mm, respectively) and, since there is a little variation between them, the average value $\tilde{\beta}$ (SCMn 2A) = 36.949 can be considered in the following. Then, for each straight line with a slope $-\left(1/\tilde{\beta}\right)$, the value of $\ln C(D)$ can be deduced from the abscissa of the intersection point with the horizontal axis of the coordinate system, as is discussed in the previous Section (Fig.4). For the ordered material (Fig.5b) the best-fit values of β are equal to 19.342, 20.401, 21.719, 21.195 (for D = 8, 20, 30, 40 mm, respectively), the average value is $\tilde{\beta}$ (SF 50) = 20.664, and the experimental values of $\ln C(D)$ can be obtained as is described above.

Finally, by plotting such values of $\ln C(D)$ against $\ln D$ (Fig.6), two straight lines can be obtained through the least squares method: the straight line slope, $-\left(d\,\beta\right)$ (see Eq.8b and Fig.3), for the cast steel is equal to –6.628, whereas that for the forged steel is equal to -2.754. Therefore, by considering the average values $\tilde{\beta}$, the decrement d is equal to 0.18 and 0.13, respectively, and the reacting cross-section presents a fractal dimension $\alpha = 2 - d$ equal to 1.82 and 1.87, respectively, i.e. the material ligament is closer to a two-dimensional Euclidean surface for the comparatively ordered material (forged steel: JIS SF 50).

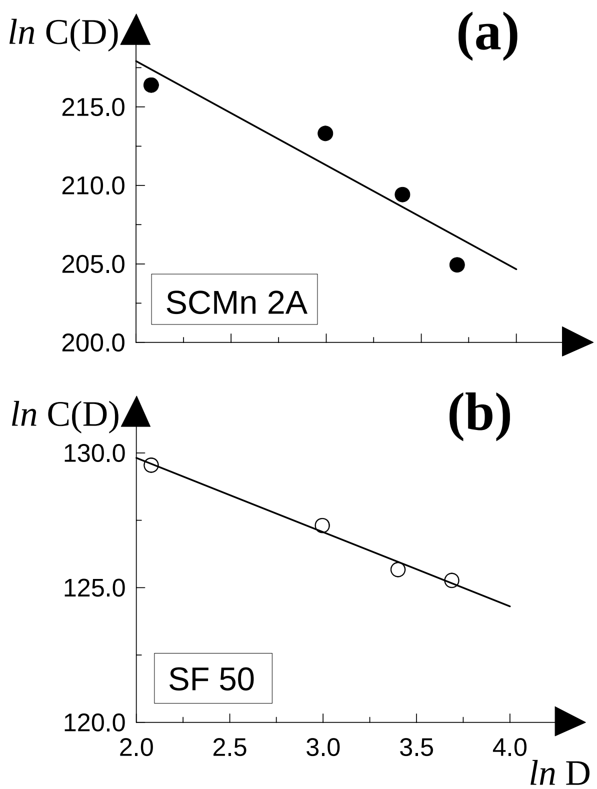

Figure 6. Experimental S-N curve parameter $C(D)$ against structure size D for two steels tested by Hatanaka et al. [14].

4. Conclusions

Many experimental observations show that the finite-life fatigue strength of a given material in the high-cycle regime decreases by increasing the specimen size. Size effect in finite-life fatigue strength has herein been examined through fractal geometry concepts, by assuming a self-similar weakening (monofractal approach) of the reacting cross-sections of structures, due to material heterogeneity and mechanical damage. Experimental data related to two different steels have been examined to discuss how to apply the above theoretical monofractal approach.

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