Numerical Simulations of Dynamic Fracture

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On the basis of the incubation time approach in dynamic fracture introduced into the finite element method different classes of dynamic fracture experiments are simulated and analyzed. It will be shown that all the variety of phenomena observed in these experiments can be obtained numerically using the incubation time fracture criterion. It will be shown that computed crack extension histories are coinciding with ones observed experimentally. It will be demonstrated that the dynamic stress intensity factor can be unambiguously coupled with the crack speed in some experiments (when dynamic crack is initiated by quasistatic loads ex. Shockey, Kalthoff). At the same time in other experiments, when the crack is initiated by dynamic high rate loads (ex. Ravi-Chandar and Knauss), SIF can change independently from the crack speed. This connection between the SIF in the tip of a moving crack and the speed of the crack will be extensively discussed.

1. **Application of the Incubation Time Approach in Numerical Simulations of Dynamic Fracture**

As shown in [1] the incubation time criterion [2], is able to describe crack initiation in dynamic conditions. General form of the criterion for rupture at a point *x* at time *t* reads:

$$\frac{1}{\tau}\int_{t-\tau}^{t}\frac{1}{d}\int_{x-d}^{x}\sigma(x\ \ ,t\ \ )dx\ dt\ \ <\sigma_c\ , \tag{1}$$

where τ is the microstructural time of a fracture process (or fracture incubation time) – a parameter characterizing the response of the material on applied dynamical loads (i.e. τ is constant for a given material and does not depend on problem geometry, the way a load is applied, the shape of a load pulse and its amplitude). *d* is the characteristic size of a fracture process zone and is constant for the given material and chosen scale. σ is stress at a point, changing with time and σ_c is its critical value (ultimate stress or critical tensile stress found in quasistatic conditions). *x** and *t** are local coordinate and time.

Assuming

$$d = \frac{2}{\pi} \frac{K_{IC}^2}{\sigma_c^2}, \tag{2}$$

where K_{IC} is a critical stress intensity factor for mode I loading (mode I fracture toughness), measured in quasistatic experimental conditions. It can be shown that within the framework of linear fracture mechanics, for case of fracture initiation in th tip of an existing crack, loaded by mode I, (1) is equivalent to:

$$\frac{1}{t} \int_{t-t}^{t} K_{\mathrm{I}}(t')dt' \leq K_{\mathrm{I}C} \ . \tag{3}$$

Condition Eq. 2 arises from the requirement that Eq. 1 is equivalent to Irwin's criterion ($K_{\mathrm{I}} \leq K_{\mathrm{I}C}$), in case of $t \to \infty$.

As it was shown in many previous publications, criterion (Eq.3) can be successfully used to predict fracture initiation for brittle solids (ex. [3,4]). Along with prediction of initiation of dynamically loaded cracks incubation time criterion is able to predict dynamic crack propagation, arrest, reinitiation and even fracture of initially intact media. The criterion (Eq.3), while able to predict dynamic crack initiation, cannot be used to predict crack or fracture development in dynamic conditions. The main reason for this is that time dependency of a stress intensity factor in the tip of a crack moving at high speeds does not directly reflect the history of stress-strain fields in the vicinity of a current crack tip location as, at preceding times, crack tip was located at distant (and usually very distant) points of a body. This was also discussed by Ma and Freund [5] and Ravi-Chandar and Knauss [6].

Though criterion using stress intensity factor (Eq.3) is easier to use when simply describing crack initiation, general form of the incubation time criterion (Eq.1) was used even to assess early stages of fracture development. In this work examples on how the incubation time approach, being incorporated into finite element computational codes, can be used to predict fracture initiation, propagation and arrest in real experimental conditions are given.

2. Classical experiments of Ravi-Chandar and Knauss

Incubation time criterion was used to predict dynamic crack development in the classical fracture dynamics experiments reported by Ravi-Chandar and Knauss in 1984 [7]. In these experiments a rectangular sample with a cut simulating a crack is loaded by applying an intense load pulse to the crack faces. Fig. 1 gives an approximation of the load applied to the crack faces.

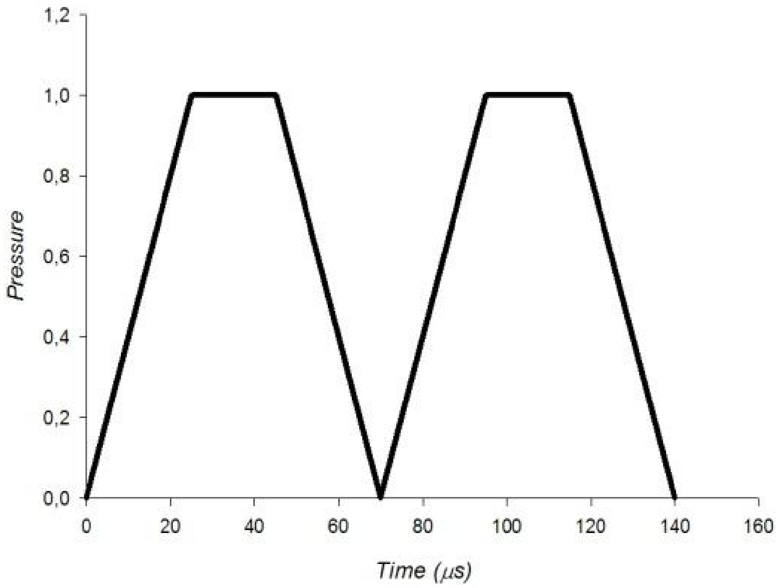

Figure 1

Temporal shape of pressure pulse released at experiments by Ravi-Chandar and Knauss [7]

Behavior of the loaded sample is described by the Lame equations:

$$\rho u_{i,tt} = (\lambda + \mu) u_{j,ji} + \mu u_{i,jj} \; , \tag{4}$$

where "," refers to the partial derivative with respect to time and spatial coordinates. ρ is the mass density, and the indices i and j assume the values 1 and 2. Displacements are given by u_i in the directions x_i respectively. t stands for time, λ and μ are Lame constants. Stresses are coupled with strains by Hooke's law:

$$\sigma_{ij} = \lambda \delta_{ij} u_{k,k} + \mu (u_{i,j} + u_{j,i}) . \tag{5}$$

where σ_{ij} represents components of the stress tensor, δ_{ij} is the Kronecker delta assuming value of 1 for i=j and 0 otherwise. At $t = 0$ the sample is stress free and velocity field is zero everywhere in the body:

$$\sigma_{ij}\big|_{t=0} = u_{,t}\big|_{t=0} = 0\,. \qquad (6)$$

Crack faces are free from tractions:

$$\sigma_{21}\big|_{x_1<0,x_2=0} = 0\,. \qquad (7)$$

The load applied to the crack faces is given by:

$$\sigma_{22}\big|_{x_1<0,x_2=0} = Af(t)\,. \qquad (8)$$

Where *f(t)* is given graphically in Fig. 1 and *A* is the amplitude of the load. The authors create a pressure pulse, constant over the cut.

Unfortunately, in the article by Ravi-Chandar and Knauss there is no information about the amplitude of pressure created in the presented experiments [7].

To check applicability of Eq.1 to describe dynamic crack propagation experimental conditions of [7] were modeled utilizing the finite element method.

3. **Finite element formulation**

Here we do not provide details for the finite element model as this could be found in [16].

4. **Solution Results**

After the stated problem is solved by the FEM package, together with an external program controlling crack propagation, information about K_I time dependency and the crack extension history is provided for further analysis. $K_I(t)$ is computed using the asymptotic behavior of the stress field surrounding the crack tip.

It was observed that, depending on the amplitude of the applied pressure pulse A, three different modes of crack propagation are possible. The first one is trivial – amplitude that is too low results in no crack extension. The second one is the mode observed by Ravi-Chandar and Knauss [7]. The crack starts propagating at a constant speed. Then it arrests, due to the energy flow into the crack tip which is no longer sufficient for its propagation. When the energy from the second trapezoid of the loading pulse approaches the crack tip region, the crack reinitiates and starts propagating at approximately the same speed as in the first stage of its extension.

Further increase of load amplitude *A* results in a propagation mode change. Now the crack is initiated, propagates at some constant speed, and when the energy from the second part of the loading pulse is delivered to the crack tip region the crack is accelerated and continues propagation at a higher speed.

By adjusting the pressure amplitude *A*, it was found that amplitudes around 5 *MPa* result in crack extension histories very close to those observed by Ravi-Chandar and Knauss [7]. In Fig. 2, the computational result for *A*=5.1 *MPa* is compared to one of the experiments [7].

5. **Conclusions on dynamic crack simulations**

It has been shown that, solving the dynamic problem of linear elasticity by FEM and criterion Eq.1 being used to assess critical conditions for crack advancement, the propagation of dynamically loaded cracks can be predicted. It has also been shown that criterion (Eq.1) with ***d***, chosen from the condition of coincidence of Eq.1 with Irwin's criterion in static conditions can be used to describe dynamic crack initiation, propagation and arrest.

Criterion Eq.1, unlike Eq.3, which is applicable only to crack initiation, can also be used as the condition for crack propagation and arrest. In the presented model Eq.1 is used as a condition for node release. This criterion does not even require the presence of a crack. Thus, the condition for crack propagation and arrest appears automatically. The crack propagates whilst Eq.1 is fulfilled for nodes ahead of the moving crack tip; otherwise the crack arrests.

Using a similar method one can model cracks that change their direction of propagation and even branch. In this case Eq.1 should be applied not only to stresses acting perpendicular to the x_1 direction, as is done in the presented research, but in all the possible directions surrounding the x^* point.

According to the incubation-time based approach (see [9 or [2]), in combination with a variety of widely known experimental observations, the critical stress intensity factor at the crack initiation moment under high rate loads may, depending on the experimental geometry, loading conditions and history, either be noticeably smaller or greater than K_{IC} .

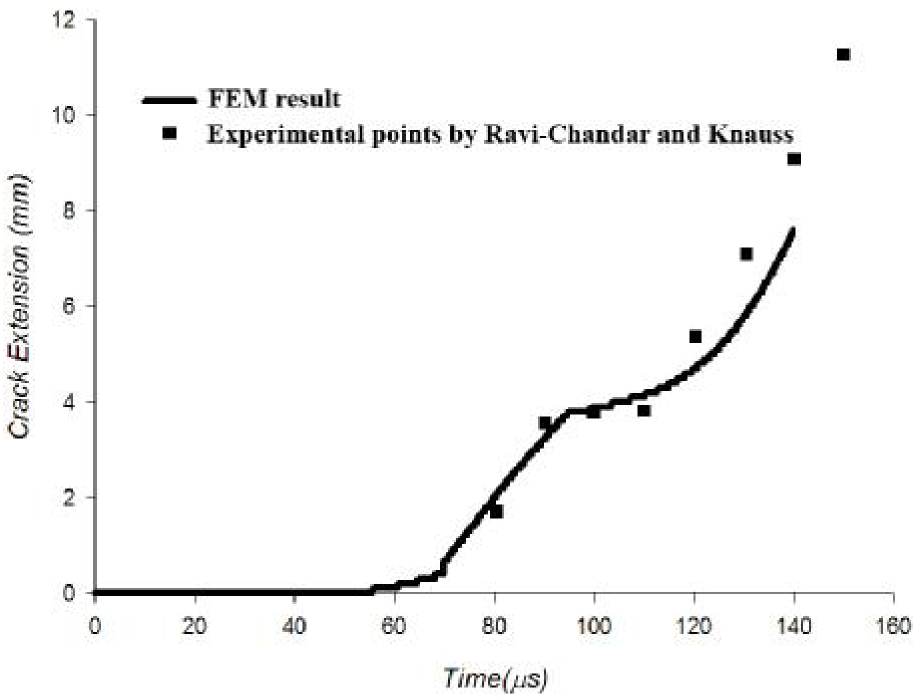

Figure 2

Crack extension history. Comparison of FEM calculation with experimental data points of Ravi-Chandar and Knauss [7]

Application of the incubation-time based approach provides a possibility describe all variety of experimentally observed effects in fracture dynamics. An important consequence of this approach is that it provides an effective way of testing dynamic strength by direct measurement of τ, a parameter intrinsic to the material and not dependent on experimental geometry or the way the load is applied [10]. This provides a tool that can be directly incorporated into practical engineering.

6. **Crack speed - SIF correspondence**

In experiments mentioned above one can plot crack tip speed as a function of the stress intensity factor.

In numerical simulations presented in previous section in compliance with direct measurement results of Ravi-Chandar and Knauss [13], crack speed cannot be unambiguously coupled with the stress intensity factor in the tip of a running crack. In these experimental conditions crack advances at a constant rate, prescribed by conditions at the moment of crack initiation whilst SIF can change significantly.

At the same time other known experimental results (ex. [11],[12],[14],[15]) indicate that there exists an unambiguous connection between crack tip speed and the SIF in the tip of the running crack. Authors of these experimental works see contradiction between results and try to find errors in experimental scheme of their opponent.

At the same time we don see any contradiction between mentioned experimental results. To our opinion the difference in measured SIF-crack tip speed dependency is caused by essential difference in experimental scheme used. In the first case (ex. [13]) the crack is initiated and driven by an intense dynamic load. In the second case (ex. [11],[12],[14],[15]) the crack is initiated and driven by quasistatic load and the crack accelerates as a result of increase in the SIF caused by increase of the crack length.

Numerical modeling of experimental conditions of the second class of experiments (ex. [11],[12],[14],[15]) using the technique described in previous sections does show that in this case there exists an unambiguous connection between SIF and the speed of the crack tip.

Thus, at this point we can conclude that SIF-crack tip speed dependency is not a characteristic inherent in material, but depends on the experimental conditions and can be different for the same material in different experiments. At the same time this dependency, for a given experiment, can be predicted using the incubation time approach combined with numerical simulation of experimental conditions.

Another interesting thing observed while reviewing results of FEM calculations using incubation time fracture criterion (1) is a possible difference in fracture mechanisms responsible for crack propagation in different cases. In experiments when the crack is loaded by an intense dynamic load (ex. [7], [13]) fracture is happening at a bigger scale level as comparing to the second case when the crack is driven by quasistatic load (ex. [11],[12],[14],[15]). To our opinion in the first case there is not enough time for fracture at a smaller scale level to develop and bigger scale fracture is happening directly. In the second case fracture is happening at a smaller scale that is later resulting in macroscopic crack propagation.

7. **Conclusions**

Incubation time fracture criterion has a wide area of applicability. As real dynamic fracture problems rarely can be solved analytically, the majority of applications require numerical simulations. In this connection incubation time approach has a significant advantage – it can be applied for correct description of both quasistatic and dynamic fracture, so one does not have to use separate criteria for different load rates. It is shown that using incubation time criterion incorporated into finite element code a correct description of dynamic fracture initiation, dynamic crack propagation and fracture of initially fractured media is possible. It is remarkable that staying within the framework of linear elastic fracture mechanics, it is possible to predict all the variety of effects inherent in dynamic fracture. And all this is possible while utilizing a rather simple fracture model, not incorporating complicated cohesive laws. The same approach can be used to model dynamic crack arrest, dynamic cleavage, etc.

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