Incubation Time Based Description of Dynamic Fracture Processes

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The fundamentals of incubation time approach in fracture dynamic problems will be outlined. The continual (kinetic) formulation of incubation time based fracture dynamics will be discussed. It provides the investigation of microscale incubation processes anticipating the macrofracture event as well as the dynamic-strength criterion at the macroscale. The abilities of proposed approach in description of dynamic crack propagation process will be considered.

1. Structural-Temporal Criterion of Fracture

Nowadays, the nonoperability of traditional quasistatic fracture models in the case when a fracture happens in rather short time intervals after the beginning of exterior pulse application (which corresponds to high loading rates) became apparent. At the modeling of dynamic fracture processes one should consider an effect of high local deformations together with elastic resistance of the material. The structural-temporal approach *integrally* considering this phenomenon was proposed by Morozov, Petrov and Utkin [1]. The brief exposition of its main statements is provided further (full description one can find in [2, 3]).

Firstly, let consider the static fracture of *quasi-brittle materials*. For simplicity we will assume some arbitrary two-dimensional stress field (e.g. the plane strain) and suppose that fracture will happen along some direction Ox being the symmetry line. It is known, that in "non-simple" static problems a classical critical strength (as well as critical fracture toughness) approach does not work (e.g., for the crack growing from an angular vertex, when the stress field has a non-root singularity [4]). But there is the approach working well in the most complicated static problems – namely, Neuber-Novozhilov's approach, known also as *nonlocal fracture mechanics*. Referring to the classical force approach of fracture mechanics, in the case of *quasistatic* loading a fracture occurs when the instantaneous local force acting in the supposed place of rupture attains a critical value: $F \leq F_C$. In the terms of continual stress field σ it can be written in the form

$$\int_{x-d}^{x} \sigma(x')dx' \leq \sigma_* d, \qquad (1)$$

where d is some still undefined linear size describing the spatial structure of solid and σ_* could be considered as some critical stress introduced instead of critical force F_C. The criterion (1) is known as *Neuber-Novozhilov criterion*. The basic principles of Neuber-Novozhilov's approach could be reduced to the following statements (similar to the basic principles of quantum mechanics):

1. all solids consists of spatial-structural elements of finite size;
2. an elementary act of fracture is a fracture of one structural element;
3. criterion parameters should be chosen to preserve the results of classical fracture theory in the limit of low load rates.

Regarding the case of intact fracture and corresponding fracture criterion of critical stress $\sigma \leq \sigma_C$ (where σ_C is the static strength of the material) and applying the third basic principle one will obtain $\sigma_* = \sigma_C$. On the other hand, considering the classical Griffith crack problem and corresponding Irwin fracture criterion $K_I \leq K_{IC}$ (where K_I is the stress intensity factor and K_{IC} is the static fracture toughness), after substitution of the tensile stress in the crack tip $x = 0$

$$\sigma(x) = \frac{K_I}{\sqrt{2\pi x}} + O(1), \quad x \to 0 \qquad (2)$$

into criterion (1) we obtain $d = 2/\pi \ (K_{IC}/\sigma_C)^2$.

Outlined Neuber-Novozhilov's approach, which permits efficacious fracture forecast for quasi-static loading of brittle materials, gives us the background to describe the dynamic fracture phenomenon. Formally speaking, nonlocal fracture mechanics postulate the trivial fact: the fracture process has to be considered not in the point but in some volume (structural element) and the characteristic size of this volume is the structural size d. This postulate discovers the kinetic nature of fracture process: when we consider a fracture of some volume we have to admit that fracture does not occur instantaneously but material needs some time to release stresses and form a fracture surface. In static case, when this time is much less then time of load action, it can be neglected; but in the case of *dynamic fracture*, when these times are comparable, we have to take into consideration not only the instantaneous components of the force field but also the time of its action. And the criterion of fracture could be formulated in the following manner: the force pulse acting during some time period attains its critical value

$$J(t) \leq J_C, \quad J(t) = \int_{t-\tau}^{t} \int_{x-d}^{x} \sigma(x', t') dx' dt', \quad J_C = \sigma_C \tau d. \qquad (3)$$

The meaning of temporal scale τ (called *incubation time of fracture*) will be soon discussed. Now, we just note that it could be considered as a characteristic time of fracture of one structural element. Thereby, the *structural-temporal criterion* (3) operates with two material scales: the temporal scale τ and spatial scale d.

Numerous investigations (see, e.g. [2,3]) shown that structural-temporal approach is able to catch all the variety of experimentally observed effects of dynamic fracture, using just the triple of experimentally defined material parameters σ_C, K_{IC} and τ. But one has to keep in mind that, strictly speaking, they are not material constants but depend on sample size (or geometrical scale). Indeed, it is well known that material parameters like static strength σ_C and static fracture toughness K_{IC} are size-dependent (see, e.g. [5]). Therefore, the spatial fracture scale d is size-dependent as well. And then, the incubation time τ being the characteristic time of fracture of one structural element depends on sample size too. That is, all the parameters entering into criterion (3) have to be considered as material constants at the given geometrical scale. Finalizing the brief exposition of structural-temporal approach, let us analyze two important partial cases. The first one is the case, when the stress field does not depend on time and the criterion (3) coincides with Neuber-Novozhilov criterion. Another one, when the stress field near supposed rupture point depends only on time (e.g., in spalling fracture) and the criterion (3) turns into

$$\frac{1}{\tau}\int_{t-\tau}^{t}\sigma(t')dt' \leq \sigma_C. \tag{4}$$

In particular case of macrocrack presence, resequencing the procedure shown above, we obtain

$$\frac{1}{\tau}\int_{t-\tau}^{t}K_I(t')dt' \leq K_{IC}. \tag{5}$$

Relying on considerations used to introduce criterion (3), one can expect that eq. (4), called *the incubation time criterion*, has to be a generalization of classical quasistatic approaches. Indeed, when we can neglect by incubation time (in static case it is much less than the time of load action) then the criterion (4) becomes the classical critical stress (or (5) – Irwin's critical fracture toughness) criterion.

2. Nature of Incubation Time

Now, let try to understand deeper the physical meaning of incubation time. Firstly, we are going to consider the tensile fracture test of uniform bar subjected to external load producing the uniform stress field $\sigma(t) = PH(t)$, where P is some constant stress and $H(t)$ is the unit step function. Applying the incubation time criterion (4) we can define the dependence of time to fracture t_* (time period from the moment of loading application till the moment of sample dividing into two parts) from external load P. There are three possibilities: $P = \sigma_C$, then $t_* = \tau$; $P > \sigma_C$, then $t_* = \frac{\sigma_C}{P}\tau < \tau$; or $P < \sigma_C$, and then $t_* = \infty$. So, it is clear that τ is the time needed to produce a rupture by stress equaled to static material strength. By the way, it means that incubation time can be measured directly from quasistatic tensile fracture test as the temporal characteristic of the front of unloading wave propagating inside the sample after rupture.

From another point, it can be demonstrated that τ is the minimal time needed to produce the rupture in cleavage problem by threshold loading pulse (see, e.g., [2]). That is τ defines the time asymptote (so called *dynamic branch*) of *the temporal dependence of strength* for considered material. Indeed, let us consider the classical one-dimensional cleavage problem: the reflection of a triangular compressive loading pulse from the free end of a semi-infinite bar located along x-axis at $x > 0$. The incident pulse σ_- and reflected pulse σ_+ are defined by

$$\sigma_\mp = \mp P\left(1 - \frac{ct \pm x}{ct_i}\right)\left(H(ct \pm x) - H(ct \pm x - ct_i)\right). \quad (6)$$

Here P is the loading pulse amplitude, t_i is its duration and c is the velocity of elastic waves in considered material. The combined stress $\sigma = \sigma_- + \sigma_+$ attains its maximum (***tensile***) value for the first time at the point $x_* = ct_i/2$. To determine the temporal dependence of strength (the dependence between threshold loading amplitude and time to fracture), we have to calculate the rupture amplitude P_*, minimal for every given pulse duration t_i:

$$\max_T \int_{T-1}^{T} \sigma(T')dT' = \sigma_C. \quad (7)$$

Here the normalized (dimensionless) time $T = t/\tau$ is introduced. It easy to see from (6, 7) that the required dependence has the form

$$T_* = \begin{cases} 1 + \dfrac{1/4}{1 - \sigma_C/P_*}, & 1 \le P_*/\sigma_C \le 2 \\ 1 + \sigma_C/P_*, & P_*/\sigma_C \ge 2 \end{cases}. \quad (8)$$

Here $T_* = t_*/\tau$ is the time to fracture, defined as the time moment when integral in (4) attains its critical value. The temporal dependence of strength (8) for aluminum is plotted at Fig. 1, together with experimental data of Regel et. al. [6].

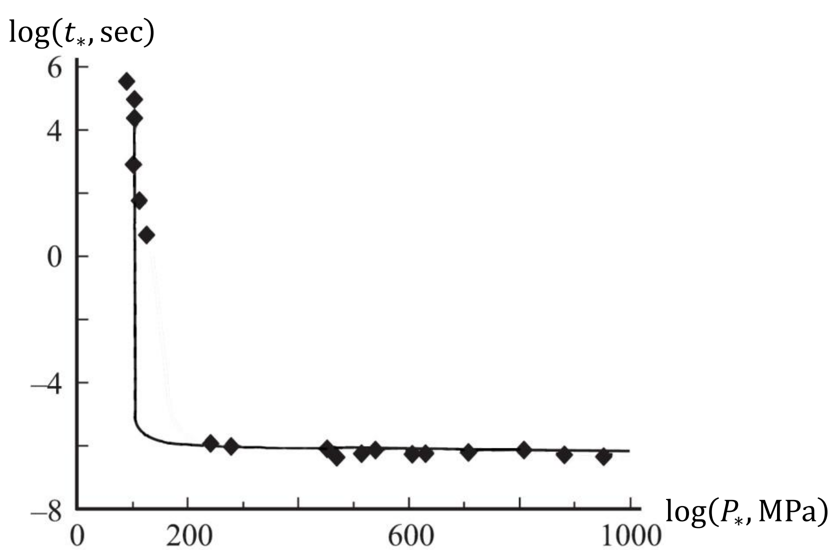

Fig. 1. Temporal dependence of strength for alluminium

Finally, having the instruments to measure the incubation time for given materials, we will show at the simplest example that the incubation time is closely connected to the relaxation processes, accompanying rupture development. Let consider a spatially isotropic quasi-static fracture problem and accept the simplest deformation based fracture criterion at the given point (supposed point of rupture)

$$E\varepsilon(t) \leq \sigma_C. \tag{9}$$

Here E (the volumetric elasticity modulus) and σ_C (the static strength) are material constants, and $\varepsilon(t)$ is a volume deformation (a relative volume variation, caused by deformation and microdamage accumulation) at the given point under some stress field $\sigma(t)$. In the case of linear-elastic solid behavior $\sigma(t) = E\varepsilon(t)$ and inequality (9) coincides with the critical stress criterion of fracture $\sigma(t) \leq \sigma_C$. But in the case of viscous deformation the material behavior could be described by the rheological law (Kelvin-Voigt model)

$$\sigma(t) = E\varepsilon(t) + \eta \frac{d\varepsilon(t)}{dt}, \tag{10}$$

where η is the viscosity factor. Then

$$\begin{aligned} \varepsilon(t) &= \frac{1}{\eta} \int_{-\infty}^{t} \exp\left(-\frac{E}{\eta}(t-s)\right) \sigma(s) ds = \\ &= -\frac{1}{\eta} \int_{0}^{\infty} \exp\left(-\frac{E}{\eta} s\right) \sigma(t-s) ds. \end{aligned} \tag{11}$$

The kernel of integrand in eq. (11) is the function $\exp(-s\,E/\eta)$, rapidly decreasing with time and $\int_0^\infty \exp(-s\,E/\eta)\,ds = \eta/E$. Replacing it by the step function

$$Q(t) = \begin{cases} 1, & t \leq \eta/E \\ 0, & t > \eta/E \end{cases}, \tag{12}$$

to satisfy $\int_0^\infty \exp(-s\,E/\eta)\,ds = \int_0^\infty Q(s)\,ds$, one will obtain $\varepsilon(t) = -\frac{1}{\eta}\int_0^{\eta/E} \sigma(t-s) ds$. Then, criterion (9) yields

$$\frac{1}{\eta/E} \int_{t-\eta/E}^{t} \sigma(s) ds \leq \sigma_C, \tag{13}$$

which coincides exactly with the incubation time criterion (4) if $\tau = \eta/E$. This relation defines the incubation time as the characteristic time of relaxation processes. It should be kept in mind that the real relaxation is caused not only by viscous deformation, but mostly by microfracture, accompanying the rupture development. However, the microfracture accumulation in brittle material is described by equation having the form of eq. (10) [7, 8]. So, the incubation time has the physical meaning of characteristic time of relaxation processes caused by microfracture accumulation accompanying the macrofracture in solids.

3. Continual-Temporal Approach

We have revealed the fundamental role played by incubation time regarding to dynamic fracture processes. But incubation time criterion (4) allows an integral consideration of relaxation processes and does not provide a continual description of fracture evolution and corresponding incubation processes at the microscale. Here we would like to present the new *kinetic description of dynamic fracture* based on incubation time approach. It operates with a function corresponding to instant local microfracture state (*the damage function*) to describe the microfracture evolution (including the processes of nucleation, interaction and following coalescence of microfracture – microcracks, microdamage and so on).

Let us consider a *spatially isotropic* process of microfracture evolution and fix an arbitrary small solid volume. Its mass is denoted as m, its volume before deformation is V_0, whereas the total volume of microfracture (damage) accumulated inside the chosen portion is V_*. Thus, during the damage accumulation process its volume changes as $V = V_0 + V_*$. The change of volume is obviously accompanied by a variation of local density $\rho(t)$, described by the mass conservation law $d\rho/dt = -\rho \operatorname{div} \overline{v}$, where $\overline{v}$ is a local velocity of material particles. We can express the local density as $\rho = \frac{dm}{dV} = \frac{dm}{dV_0}\frac{dV_0}{dV} = \frac{dm}{dV_0}\left(1 - \frac{dV_*}{dV}\right)$. Introducing the damage function $\theta = \frac{dV_*}{dV}$ and setting $\rho_0 = \frac{dm}{dV_0}$ we obtain $\rho = \rho_0(1-\theta)$. Substitution of this expression into the mass conservation law yields

$$\frac{d\theta}{dt} = (1-\theta) \operatorname{div} \overline{v}. \tag{14}$$

It is clear that damage function θ can takes the values from $\theta \in (-\infty, 1]$ and the local state of macroscopic fracture is referred to $\theta = 1$. Eq. (14) has the form of kinetic equation describing the creepage. Its right part represents the source of microfracture and, then, it has to depend on time indirectly, through the local force field and current damage level. It is natural to believe that fracture intensifies with increasing of damage level (in the most common form $\operatorname{div} \overline{v} \sim \theta^{\alpha_1}(1-\theta)^{\alpha_2}$, $\alpha_1 \geq 0$, $\alpha_2 \leq 0$). Besides that, fracture process has to be intensified with increasing of local stress. Formally, we could accept $\operatorname{div} \overline{v} \sim \sigma^{\beta}(t)$, $\beta > 0$, but such representation does not distinguish tensile and compressive stresses for non odd β. To correct it, we will suppose $\operatorname{div} \overline{v} \sim \sigma_s^{\beta}(t)$, where $\sigma_s^{\beta}(t) = \operatorname{sign} \sigma(t)\ \sigma^{\beta}(t)$. Then, we have to take into account the changing of stress field during incubation period which finally yields $\operatorname{div} \overline{v} \sim \sigma_s^{\beta}(t) -$

$\sigma_s^\beta(t-\tau)$. So, from dimensional analysis we will have (further $\zeta > 0$ denotes a dimensionless proportionality constant)

$$\frac{d\theta}{dt} = \frac{1}{\tau\zeta}\frac{\sigma_s^\beta(t) - \sigma_s^\beta(t-\tau)}{\sigma_C^\beta}\theta^{\alpha_1}(1-\theta)^{1+\alpha_2}. \tag{15}$$

Here ζ, α_1, α_2 and β are some dimensionless parameters which will be defined to obtain the known criteria in particular cases. Thus, let us consider the case when the force field near the rupture point fulfills the condition $\sigma_s^\beta(t) = \sigma^\beta(t) \geq 0$. The initial condition corresponding to intact material is $\theta(0) = 0$ if the time “starts” in the moment of loading application, and the criterion of macrofracture is $\theta(t_*) = 1$ if t_* is the time to fracture. Integration of eq. (15) on $[0, t_*]$ yields

$$\int_0^1 \frac{d\theta'}{\theta'^{\alpha_1}(1-\theta')^{1+\alpha_2}} = \frac{1}{\tau\zeta}\int_0^{t_*}\frac{\sigma^\beta(t') - \sigma^\beta(t'-\tau)}{\sigma_C^\beta}dt'. \tag{16}$$

The integrand in the left side of eq. (16) grows unboundedly when θ' comes to 0 (if $\alpha_1 > 0$) as well as when θ' comes to 1 (if $\alpha_2 = 0$). So, the only possibility for the integral to converge is $\alpha_1 = 0$ and $\alpha_2 < 0$. From the other hand, considering the static loading $\tau \approx 0$ (i.e. $\sigma^\beta(t) - \sigma^\beta(t-\tau) \approx \tau\beta\sigma^{\beta-1}(t)\ d\sigma/dt$) we obtain

$$\frac{d\theta}{(1-\theta)^{1+\alpha_2}} = \frac{\beta}{\zeta\sigma_C^\beta}\sigma^{\beta-1}(t)\ d\sigma. \tag{17}$$

Integrating eq. (17) on $[0, t]$ (where $t \leq t_*$) and taking into account the initial conditions $\theta(0) = 0$ and $\sigma(0) = 0$ we have

$$\frac{\left(1-\theta(t)\right)^{-\alpha_2} - 1}{\alpha_2} = \frac{1}{\zeta\sigma_C^\beta}\sigma^\beta(t). \tag{18}$$

When $t = t_*$, eq. (18) gives the fracture criterion in the form

$$-\frac{1}{\alpha_2} = \frac{1}{\zeta\sigma_C^\beta}\sigma^\beta(t_*). \tag{19}$$

Demanding the coincidence of obtained relation with the criterion of critical stress ($\sigma(t_*) = \sigma_C$) we have to accept $\alpha_2 = -\zeta$. Now, integrating eq. (15) on $[0, t]$ (where $\tau \leq t \leq t_*$) with initial condition $\theta(0) = 0$ and $\sigma(0) = 0$

$$1 - \left(1-\theta(t)\right)^\zeta = \frac{1}{\sigma_C^\beta\tau}\int_{t-\tau}^{t}\sigma_s^\beta(t')dt' \tag{20}$$

and taking into account that $\max\left(1-\left(1-\theta(t)\right)^{\zeta}\right)=1$ (it is reached when $\theta=1$), we have to suppose $\beta=1$ to obtain the incubation time criterion (4). Finally, we derive the explicit solution of eq. (20) as

$$\theta(t)=1-\left(1-\frac{1}{\sigma_C^{\beta}\tau}\int\limits_{t-\tau}^{t}\sigma_s(t')dt'\right)^{1/\zeta}. \quad (21)$$

The physical meaning of parameter ζ is clarified by the form of solution (21) – it is the rate of damage accumulation under external loading and, as expected, it has to be defined by material constants.

So, we have constructed the continual description of dynamic fracture based on incubation time approach. The corresponding equation has the form

$$\frac{d\theta}{dt}=\frac{1}{\tau\zeta}\frac{\sigma_s(t)-\sigma_s(t-\tau)}{\sigma_C}(1-\theta)^{1-\zeta}, \quad (22)$$

which solution, e.g. given by relation (21), describes the microfracture evolution during dynamic fracture process. Let us to emphasize one more time that obtained approach has the form of kinetic equation and it gives known fracture criteria in limit cases. It is very important fact allowing to stop the "eternal argument" between supporters of kinetical and critical approaches. In fact, both of them could be obtained on common basis as it was shown above. To demonstrate the abilities of proposed continual-temporal approach the solution of previously discussed classical cleavage problem is shown at Fig. 2 ($\zeta=1$ is assumed).

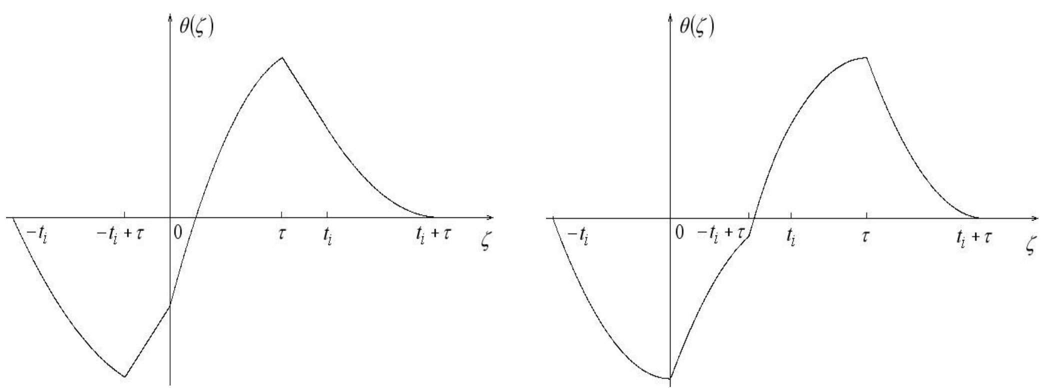

Fig. 2. Changing of damage function with time for long and short loading pulses.

The fracture will occur at the time moment t_* when the damage function attains the value $\theta(t_*)=1$. Let us note, that the "suppressed" zone (corresponding to the negative values of θ) appears at Fig. 2 because of compression wave going through the bar. Namely the accounting of time needed to form and following strain of this "suppressed" zone allows to define correctly the total time needed to fracture. So, the continual-temporal approach is able to give the proper description of microfracture evolution during dynamic fracture process.

4. Dynamic Crack Propagation Process

The abilities of developed approach in simulation of dynamic crack initiation process are obvious. But to construct the model of dynamic crack propagation we have to use the technique discussed in [9]. Referring to that paper we can write the equation of propagating one-dimensional macrocrack as a nonlinear microfracture wave in the form

$$\frac{\partial\theta}{\partial t} = D(t)\frac{\partial^2\theta}{\partial x^2} + \frac{1}{\tau\zeta}\frac{\sigma_s(x,t) - \sigma_s(x,t-\tau)}{\sigma_C}(1-\theta)^{1-\zeta}, \tag{23}$$

where $D(t)$ is the relaxation factor – the rate of microfracture relaxation process anticipating the phenomenon of crack propagation [9]. To be convinced that eq. (23) is able to provide the description of microfracture wave propagation let us to consider the trivial example $D = const$ and $\left(\sigma_s(x,t) - \sigma_s(x,t-\tau)\right)/(\zeta\sigma_C) = \alpha = const$. Introducing new dimensionless variables $X = x/\sqrt{\tau D}$ and $T = t/\tau$ we will have

$$\frac{\partial\theta}{\partial T} = \frac{\partial^2\theta}{\partial X^2} + \alpha(1-\theta)^{1-\zeta}. \tag{24}$$

It is easy to see that this equation admits the solutions in the form of a kink-type autowave. Indeed, eq. (24) is invariant with respect to translation by X and T. It means that after some time period the solution "forgets" the initial condition and goes in steady-state when the wave front remains the same with time and the front profiles are self-similar. Supposing that the wave front moves with constant velocity λ from right to left and interesting in the autowave solution $\theta = \theta(X - \lambda T)$, we can reduce eq. (24) to the ordinary differential equation:

$$\varphi\left(\lambda + \frac{d\varphi}{d\theta}\right) = -\alpha(1-\theta)^{1-\zeta}, \qquad \text{where } \varphi(\theta) = \frac{d\theta}{d(X-\lambda T)}. \tag{25}$$

Accordingly, we have verified that, even in the considered trivial case, when stress field increases at the constant velocity, the equation (23) can be used to describe the propagation of macrocrack as a nonlinear microfracture wave.

1. Conclusions

Still the moment when the scientific community firstly faced with dynamic fracture problems it concentrated on definition of the critical strength properties as material functions by analogy with static case. Here we presented the alternative approach providing the reasonable description of many observed dynamic effects. Namely, the continual (kinetic) formulation of incubation time based fracture dynamics has been provided. It was shown that proposed approach describes the fracture evolution and corresponding incubation processes at the microscopic scale level simulating properly the cleavage phenomenon. The abilities of proposed approach in description of dynamic crack propagation process were also discussed.

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