Hybrid Moiré Method for Three-dimensional Stress Analysis

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Abstract For evaluating reliability of structures, it is important to know the distributions of displacement, strain and stress. In this paper, a hybrid moiré method, which combines full-field measurement technique, i.e. sampling moiré method, and finite element (FE) analysis for determining the boundary conditions of loading force and position is proposed. The distribution of displacement measured by sampling moiré method is compared with the analyzed result based on elastic FE analysis at assumptive boundary conditions. The error between the displacement distributions of experimentally obtained and FE analysis is corrected so as to make it minimum by changing the boundary conditions of loading force and position. Experimental results show that the loading force and position can be evaluated accurately using the proposed method. Once the appropriate boundary conditions are determined, three-dimensional distributions of strain and stress are easily obtained by FE analysis for evaluating reliability of structures.

1. Introduction

For evaluating reliability of structures, it is important to know the distributions of displacement, strain and stress [1]. Owing to the recent development of advanced computer, finite element method (FEM) is a powerful tool to analyze the complete state of displacement, strain and stress of complex structures for the given constitutive model and boundary conditions. The constitutive model and material properties such as Young's modulus and Poisson's ratio can be easily known. On the other hand, the proper boundary conditions such as loading force and position are not easily determined for many reasons in practical applications. The uncertainty of knowledge in these given boundary conditions, however, limited the accuracy in real phenomenon. If the boundary conditions can be determined with high accuracy, more reliable three-dimensional distributions of stresses can be evaluated. Therefore, experimental results are needed to support the simulation predictions.

Strain gages and piezoelectric film sensor are mostly used for measurement of displacement and strain. However, it required many sensors and measurement locations to obtain a displacement distribution because these methods are point-by-point measurement. The installation of these sensors is time-consuming and high cost. Another approach is the use of noncontact measurement methods such as digital image correlation (DIC) [3], speckle interferometry [4], and digital holographic interferometry [5,6]. Sampling moiré method is a promising noncontact full-field deformation measurement technique [2] proposed by the authors. This method has several advantages over other full-field deformation measurement technique: (1) accurate, (2) fast, (3) easy procedure, and (4) low cost.

In this study, a hybrid moiré method, which combines the sampling moiré method and FE analysis to determine the boundary conditions of loading force and position is proposed. The distribution of displacement measured by sampling moiré method is compared with the analyzed result based on elastic FE analysis at assumptive boundary conditions. The error between the displacement distributions of experimentally obtained and FE analysis is corrected so as to make it minimum by changing the boundary conditions of loading force and position. Once the appropriate boundary conditions are determined, three-dimensional distributions of strain and stress are easily obtained by FE analysis for evaluating reliability of structures.

2. Hybrid Moiré Method

2.1 Full-field displacement distribution using the sampling moiré method

Recently, we proposed a novel phase analysis technique, i. e., sampling moiré method for measuring the deformation distribution of structures. When one image of a grating pattern is sampled at constant pixels, a moiré fringe pattern is obtained. If the sampling phase is shifted, the phase of the moiré fringe is also shifted. From the phase shifted moiré patterns, the phase of the moiré fringe is analyzed using the phase-shifting method.

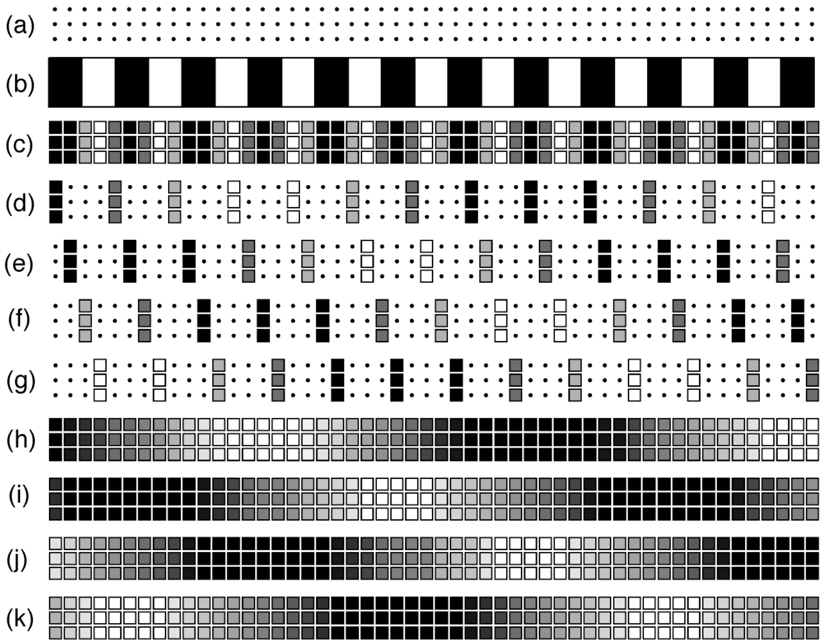

Fig. 1 Principle of the sampling moiré method

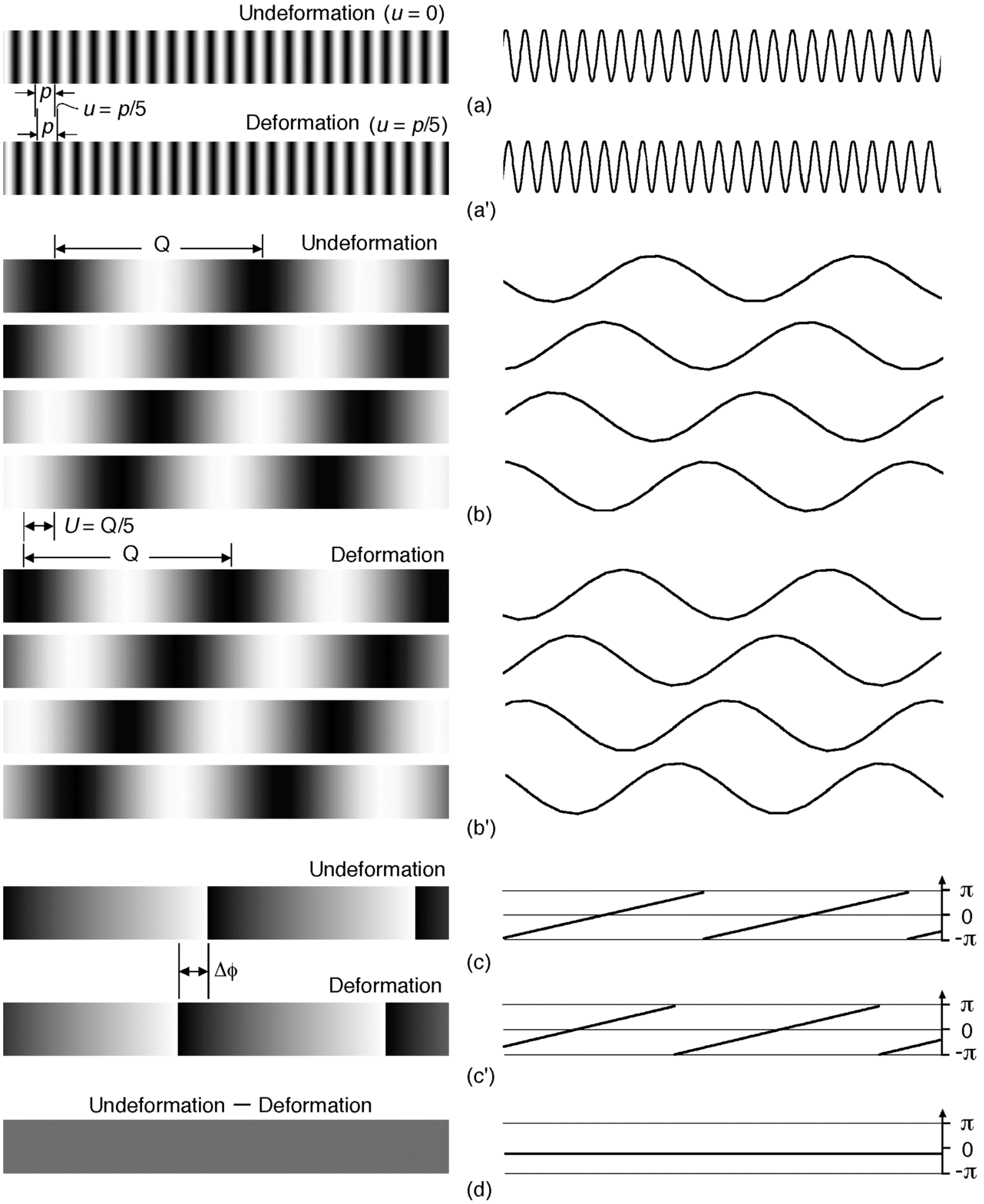

Fig. 2 Principle of the full-filed deformation measurement by using the sampling moiré method

Figure 1 illustrates the appearance of a moiré fringe pattern as visualized by the sampling moiré method. Figure 1(a) shows the sampling points of a digital camera as black dots. Figure 1(b) shows a grating pattern with a horizontally periodic line grating attached on the specimen surface. In Fig. 1(b), the pitch of the grating is 4.5 times larger than the sampling pitch shown in Fig. 1(a). Then, the image recorded by the camera is shown in Fig. 1(c), in which no moiré fringe pattern can be discerned. However, if every *N*-pixel (in this case, $N = 4$) from the first sampling point is picked up from Fig. 1(c), a moiré fringe pattern shown in Fig. 1(d) is obtained. We called this image processing "thinned-out" and *N* is called "thinned-out index". If instead of the second sampling point is selected and every *N*-th pixel is sampled, the image of the $2\pi/N$ phase-shifted moiré fringe is obtained as shown in Fig. 1(e). Figures 1(f) and (g) show the images of the moiré fringe obtained by selecting the third and the fourth sampling points, respectively, by the same manner. This process corresponds to the phase shifting of the moiré pattern. If all the sampled images shown in Figs. 1(d)-(g) are interpolated using neighboring sampled data, the image becomes clearer and keep the original image resolution. Figures 1(h)-(k) show the images after linear interpolated from the neighboring two sampled data. Then, multiple phase-shifted images of the moiré pattern can be obtained from a single image. In Fig. 1(h)-(k), the *k*-th phase-shifted images can be express as follows:

$$I_k(x,y) = I_a(x,y)\cos[\phi_m(x,y) - k\frac{2\pi}{N}] + I_b(x,y), \quad (k = 0,1,\cdots,N-1) \qquad (1)$$

where I_b represents background intensity in the image, which are insensitive to the change in phase; I_a represents the amplitude of the grating intensity and are sensitive to the phase change of the grating; and ϕ_m is the initial phase value of the moiré fringe. The phase distribution of the moiré fringe pattern can be obtained by Discrete Fourier Transform (DFT) algorithm [7] using Eq. (2).

$$\phi_m(x,y) = \tan^{-1}\frac{\Sigma_{k=0}^{N-1} I_k(x,y)\sin(k\frac{2\pi}{N})}{\Sigma_{k=0}^{N-1} I_k(x,y)\cos(k\frac{2\pi}{N})} \qquad (2)$$

The principle of deformation measurement using the sampling moiré method is shown in Fig. 2. Figures 2(a) and (a') show the specimen grating before and after deformation. For instance, the specimen grating of Fig. 2(a') is moved horizontally by a distance $u = p/5$ from the initial position of Fig. 2(a). The grating pitch is defined as p. Figures 2(b) and (b') show the phase-shifted moiré patterns obtained by sampling moiré method before and after deformation. The pitch of the moiré fringe is defined as *Q*. If the specimen with an attached grating is deformed by $u = p/5$ in horizontal direction, the phase value of the moiré fringe will change by $\Delta\phi = 2\pi/5$ according to the displacement of the specimen. Thus, the displacement u can be obtained from the phase difference $\Delta\phi = \phi_m - \phi'_m$ of the moiré fringe before and after deformation.

$$u(x,y) = p\frac{\Delta\phi(x,y)}{2\pi} \qquad (3)$$

2.2 Principle of the hybrid moiré method

In hybrid moiré method, the displacement data are generated through both experimental and numerical procedures. The principle of the hybrid moiré method is shown in Fig. 3. Loading force and position are two unknowns of boundary conditions. The measured displacement distribution by sampling moiré method is compared with the analyzed result of displacement distribution by FE analysis at assumptive boundary conditions. In order to compare the experimental results and FE analysis at same global coordinate, the local coordinates of FEM model is transformed to the coordinates of experiment so as to match the four corners of the specimen. Then, the experimental displacement data are interpolated to equal each nodes of FEM in analyzed area. The average of the sum of absolute difference between the experimental result and the FE analysis is defined as the evaluation error E_{err}, i.e. objective function, for determining the optimum boundary conditions as follows:

$$E_{err} = \frac{1}{n} \sum_{i=1}^{n} | u_{i(\mathrm{EXP})} - u_{i(\mathrm{FEM})} | \qquad (4)$$

where i is the evaluation point in analyzed area. The accuracy of evaluation can be improved if the number of n becomes larger. Therefore, full-field measurement technique is suitable than point-by-point measurement such as using strain gage and piezoelectric film sensor since a large number can be obtained simultaneously.

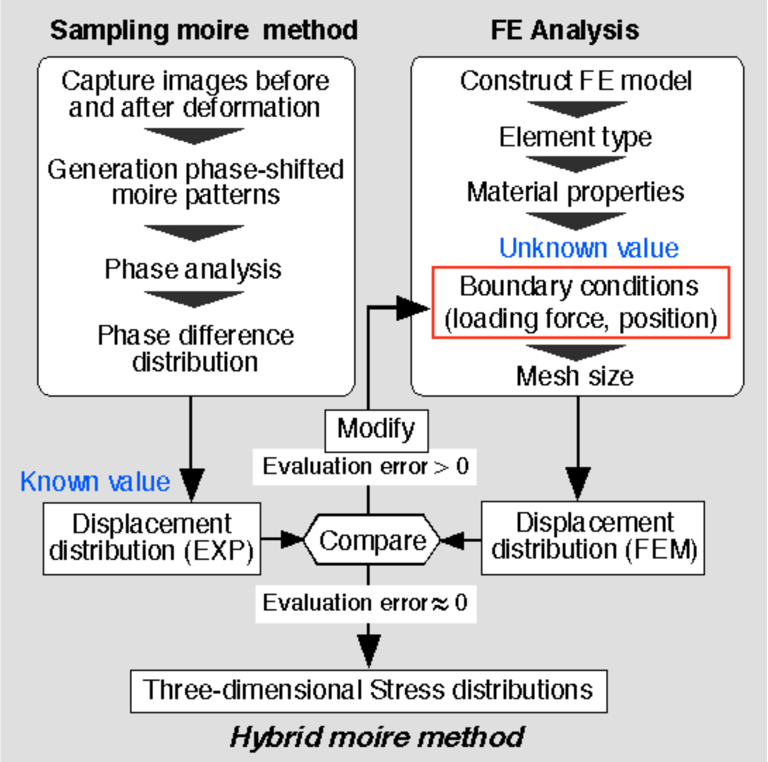

Fig. 3 Diagram of the proposed hybrid moiré method

The evaluation error E_{err} is corrected so as to become minimum by changing the boundary conditions. Once the appropriate boundary conditions are determined, three-dimensional distribution of strain and stress can be estimated by FE analysis in details.

3. Experiment

In this experiment, three-point bending test was conducted on a hollow specimen to verify the effectiveness of the proposed method. Loading force P and loading position L from the center of the specimen are two unknowns of boundary conditions in this study. In order to determine these unknowns, both experiment and FE analysis are performed.

3.1 Experimental Setup

A hollow aluminum specimen with the size of 304 mm in length, 20 mm in width, depth in 30 mm, and 2 mm thickness is measured. A tape of a cross grating with 2.0 mm pitch is pasted on the surface of the specimen. The experimental setup is shown in Fig. 4. The loading to specimen is performed by using a versatile examination machine (Autograph, AG-I 50 kN). 610 N force by edge load is applied straightly about to the center position of the specimen. A digital interface CCD camera (SONY, XCD-SX90) with 1280 vertical pixels and 960 horizontal pixels is used to record the images before and after deformation. After capturing images, the displacement distribution was analyzed by using the sampling moiré method as mentioned in Sec. 2.1.

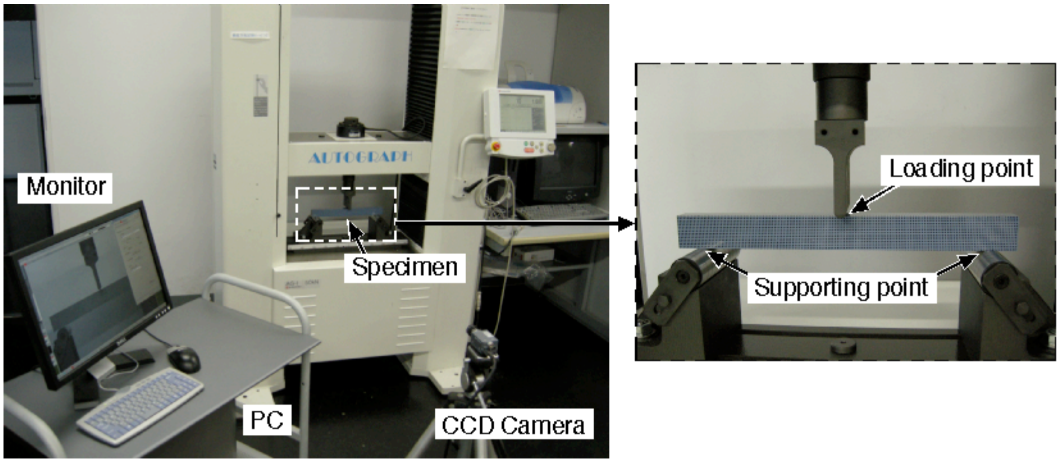

Fig. 4 Experimental setup

3.2 Measurement Results

Figure 5 shows the measured results. In this image, the region of interest with 1120 pixels ×50 pixels was analyzed and one pixel length corresponds to 0.262 mm. Figures 5(a) and (a’) show the captured images of the specimen before and after deformation, respectively. Figures 5(b) and (b’) show the phase-shifted images obtained by the sampling moiré method (the thinned-out number N is seven in horizontal direction) before and after deformation. Figures 5(c) and (c’)

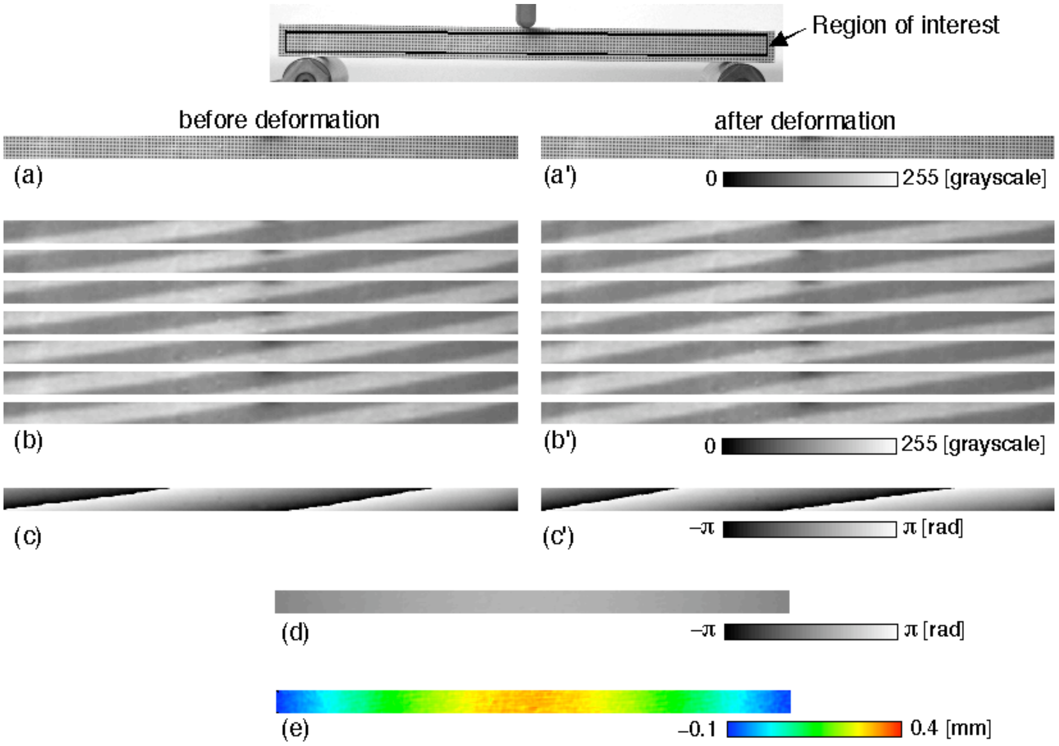

Fig. 5 Measured results: recorded images (a) before deformation and (a') after deformation, thinned-out moiré fringe images (b) before deformation and (b') after deformation, wrapped phase distributions (c) before deformation and (c') after deformation, (d) phase difference between (c) and (c'), (e) displacement distribution

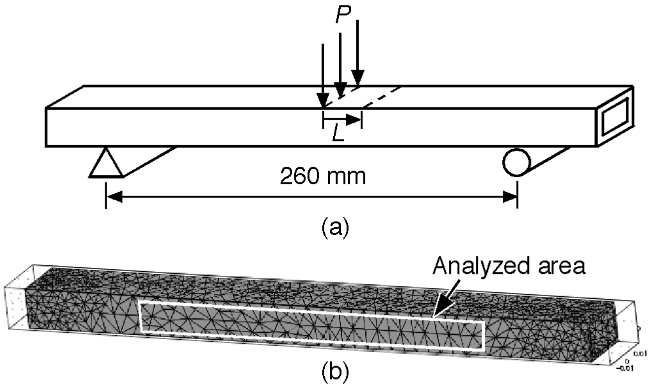

Fig. 6 FE model: (a) geometry and boundary conditions, (b) finite element mesh

show the wrapped phase distributions of Figs. 5(b) and (b') before and after deformation, respectively. Compared to Figs. 5(c) and (c'), the phase value is slightly changed after deformation. Figure 5(d) shows the phase different distribution of Figs. 5(c) and (c'), and Fig. 5(e) shows the displacement distribution in y direction obtained from Eq. (3). It was found that 0.31 mm is deflected in the loading position.

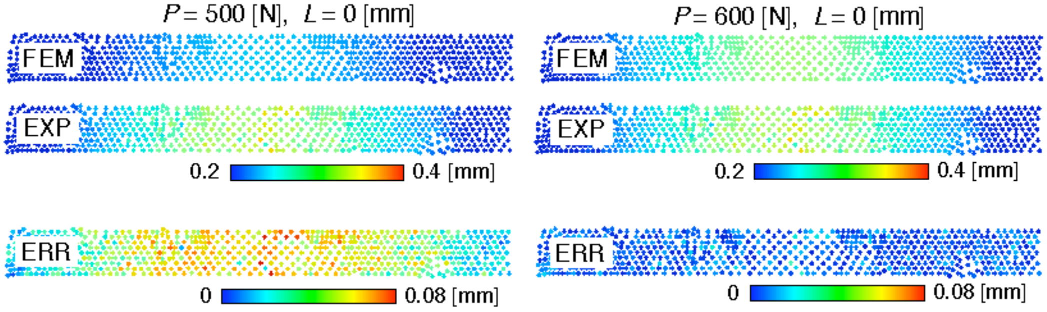

Fig. 7 Results of hybrid moiré method

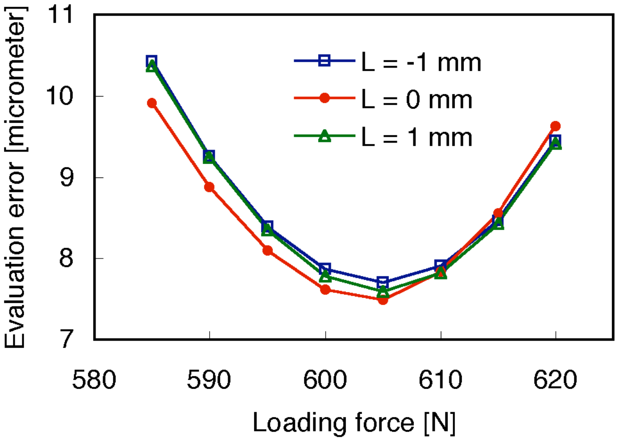

Fig. 8 Graph of evaluation error in various boundary conditions

3.3 Finite Element Analysis

The three-point bending test is simulated by elastic FE analysis using a three-dimensional model. The commercial FEM software COMSOL MultiPhysics was used in all the modeling. In FEM simulation, the material of the aluminum specimen is isotropic and the material properties of Young's modulus E and Poisson's ratio ν are set to 70 GPa and 0.33, respectively. The geometry and boundary conditions are shown in Fig. 6(a), and the finite element mesh is shown in Fig. 6(b). It contains 29,380 constant-strain tetrahedral elements.

4. Experimental results and discussion

Firstly, we changed the loading force in FEM from 500 N to 700 N by100 N steps at the position L = 0 mm. Figure 7 shows two results of the hybrid moiré method when the applied force are 500 N and 600 N in position L = 0 mm. In case of loading force P is 500 N, the evaluated error distribution is much larger than loading force P is 600 N. We found that the proper loading force is nearly 600 N, then we analyzed the loading force from 585 N to 620 N by 5 N steps, and also changed the position from L = -1 mm to L = 1 mm by 1 mm steps. Figure 8 shows the relationship between the loading force P and position L in FE analysis and the evaluation error E_{err}. A large number of n = 682 in the analyzed area is used to calculate the evaluation error. It was experimentally found that the loading force P = 605 N and loading position L = 0 mm are the optimum boundary conditions. From Fig. 8, it is found that the proposed technique enables to determine the loading force and loading position from the displacement field including experimental error. Thus, the minimum evaluation error cannot be zero (7.5 μm) because it includes imperceptible experimental error. In this experiment, the loading force was 610 N by measured a load cell. Analyzed result shows that the loading force can be evaluated accurately using the proposed method. The estimation error of loading force is less than 1 percent and the position can be estimated in mm accuracy. Once the proper boundary conditions are determined, three-dimensional distributions of stresses, including the occlusion and inner parts, are easily obtained by FE analysis for evaluating reliability of structures.

5. Conclusions

In this paper, a hybrid moiré method combining the full-field measurement technique and FE analysis is explained in order to determine the boundary conditions of loading force and position for the analysis of three-dimensional distribution of strain and stress. The characteristics of the proposed technique are given as follows:

1. The displacement distribution with high resolution can be obtained from single image before and after deformation by using the sampling moiré method. Thus, the accuracy of evaluation can be improved by using a large number of measurement points.
2. The proposed hybrid moiré method enables us to determine the boundary conditions of loading force and position with high accuracy. The estimation error of loading force is less than 1 percent and the position can be estimated in mm accuracy.

References

[1] Z. Wu, M. Abe, Structural Health Monitoring and Intelligent Infrastructure, Swets & Zeitlinger, Lisse, 2003

[2] S. Ri, M. Fujigaki, Y. Morimoto, Sampling moiré method for accurate small deformation distribution measurement, Exp Mech (2009) in press

[3] M. A. Sutton, M. Cheng, W. H. Peters, Y. J. Chao, S. R. McNei, Application of an optimized digital correlation method to planar deformation analysis, Image and Vision Computing 4 (3) (1986) 143-150

[4] C. Jeonathan, B. Franze, P. Haible, H. J. Tiziani, Speckle interferometry with temporal phase evaluation for measuring large-object deformation, Appl Opt 37 (13) (1998) 2608-2614

[5] I. Yamaguchi, T. Zhang, Phase-shifting digital holography, Opt Lett 22 (16) (1997) 1268-1270

[6] I. Yamaguchi, J. Kato, H. Matsuzaki, Measurement of surface shape and deformation measurement by phase-shifting image digital holography, Opt Eng 42 (5) (2003) 1267-1271

[7] K. Kinnstaetter, W. Lohmann, J. Schwider, N. Streib, Accuracy of phase shifting interferometry, Appl Opt 27 (24) (1988) 5082-5089