Fractal damage on glaciers of the Italian Alps

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Considering ice material as a metamorphic state of snow, the paper argues about the possible application of fractal geometry to describe the physical properties and the mechanical behaviour of ice. On the basis on the evidence of fractality, the formation of crevasses on the glaciers of the Italian Alps is analysed. Application of the box-counting method permits to analyse the distribution of crevasses network within the glacier continuum. Assuming the glacier as a damaged continuum, the application of Multifractal Scaling Law, originally developed for concrete-like materials [2], on the fracture energy and strength of ice is discussed.

1. INTRODUCTION

Glaciers represent one of the major indicators of past and present climate change of natural systems. But they can represent also the source of hydro-geological risk when collapse of seracs induces snow-ice-rock avalanches or when ice blocks felling may reach populated areas. Therefore, researchers and regional authorities need to know the physical state of glaciers in order to understand their evolution and dynamics, to correctly manage the mountain environment and forecast catastrophic events.
The goal of our research is to describe the formation of ice fractures to understand the evolution of the internal stress state due to the dynamics of the ice mass. This information, coupled with a specific monitoring activity, will help to predict collapse events (i.e., ice falls and seracs collapse) from high-elevation glaciers.
Like in the case of rocks, ice shows quasi-brittle behaviour and fractal patterns can be evidenced in the process of damage (i.e., fracture and faulting). Fractality usually induces scale effects on the mechanical parameters, in particular on fracture energy and strength [1].
Ice dynamic acts at geophysical scale, thus the paper presents the multifractal analysis of fracture networks on one Alpine Glacier of Aosta Valley - Italy.
By means of the application of the box-counting method, the distribution of fractures within the glacier continuum can be analysed. We argue that the scaling behaviour of the fracture energy of ice can be correctly described by the Multi Fractal Scaling Law - MFSL [2].
The final goal is to use the fractal dimension of the crevasses network to evaluate the stress and strain state of glaciers, in order to forecast collapse events.

2. GLACIERS AND CREVASSES IN THE AOSTA VALLEY

The dynamic of glaciers is mostly governed by gravity and ice metamorphism which induce different states of stress and strain influenced by the local geo-morphology of the mountain slopes.
When the stress overcomes the ice strength, ice fracturing starts and develops in different forms. A typical appearance of ice fracturing is represented by crevasses, each one generated by high tensile stress [3], influenced by the presence of adjacent ones and by their depth [4, 5]. As in the case of damage of quasi-brittle material (e.g., concrete), the direction of fracture propagation obeys to the principal tensile stress, i.e. approximately perpendicular to it. For a moving in a valley, glacier different stress states usually induce three kinds of crevasses [5]:

a. shear stress exerted by valley walls only (with an inclination around 45° with respect to the valley walls) - Fig. 1.a;
b. shear stress and tensile flow (perpendicular with respect to the direction of ice flow) - Fig. 1.b;
c. shear stress and compression flow (with an inclination close to or less than 45° with respect to the direction of flow) - Fig. 1.c.

Thanks to the above simple classification, crevasses have usually been studied as simple cracks measuring their regular spacing [6, 7], arguing that a power law describes the scaling of their lengths [7] and after, analysing a population of crevasses on Argentière glacier – French Alps [1].

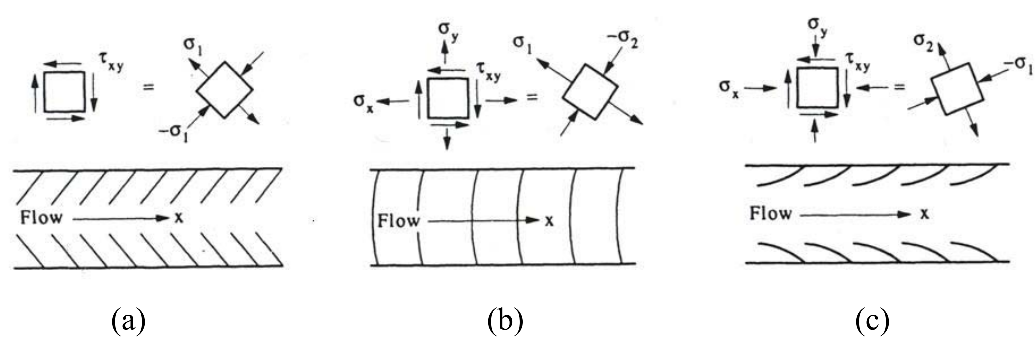


Figure 1. Crevasses patterns in a valley glacier with corresponding stress state [5].

The Italian peninsula presents approximately 500 km^2 of its total surface occupied by glaciers: 135 km^2 are localized in Aosta Valley. Due to its geo-morphology (70% of the Valley has the elevation higher than 1500 m s.l.m.), the glaciers cover about 4% of the total Valley surface (Fig. 2): 75% of the glacier area is contained in only 30 larger glaciers [8].
To investigate the fractal dimension of the crevasses network on glaciers, we have chosen the Cherillon glacier which, together with four other glaciers in the Valley, (Pré de Bar – Ferret Valley; Tzanteleina – Rhêmes Valley, Mont Gelé –

Valpelline and Verra Grande – Ayas Valley), is continuously monitored by the Cabina di Regia dei Ghiacciai Valdostani (a department of Fondazione Montagna Sicura). The Cherillon glacier is localized in Valtournenche, in the South-East sector of the Valley. It is placed between Mont Tabel and Lion Bas glacier, near the Mount Cervino (Fig. 3), not far from to the Breuil – Cervinia (AO) city centre.

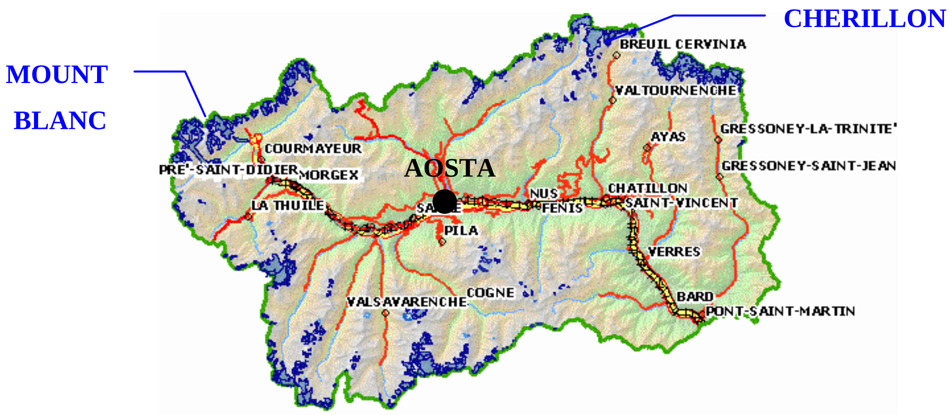


Figure 2. Distribution of glaciers in the Aosta Valley: 25% on Mount Blanc, 18% on Mount Rosa, 18% on Gran Paradiso, 12% on Valpelline, 8% on La Thuile Valley, 8% on Valgrisenche [8].

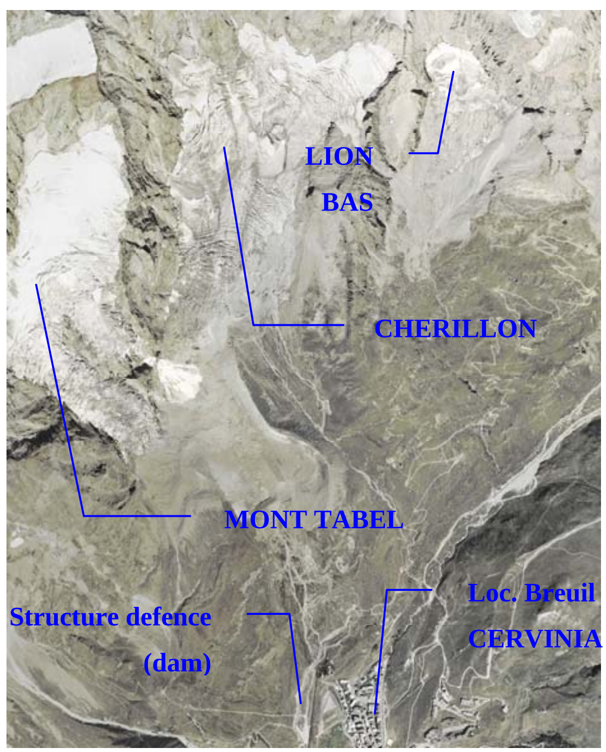


Figure 3. Localization of the Cherillon glacier - Valtournenche (AO) – Italy (Photo by Cabina di Regia Ghiacciai Valdostani - Fondazione Montagna Sicura).

As can be easily seen in Fig. 3, The Cherillon glacier represents a possible risk for the densely populated area of Breuil-Cervinia, where also a wide ski resort area occupies the northern slopes. For example, in 1931, "*during the summer, a huge ice mass fell under the rock-step on which the glacier leans*" (http://www.nimbus.it/glaciorisk/) [9]. Detachment and tilting of large ice masses may occur in the glaciers, and passive protection solutions (e.g. dams) are not feasible to avoid disasters.
Therefore, a continuum monitoring activity is carried by the Cabina di Regia dei Ghiacciai Valdostani. The monitoring activity comprises several analysis [10]:

a. analysis of snow accumulation: periodic survey of snow layers and of their physical and mechanical properties to measure snow melting, evolution and metamorphism to estimate the height of the snow permanent line and the ice mass balance;
b. analysis of photos taken from a fixed points, to have a prompt visual knowledge of the morphological variations;
c. analysis of the GPS survey of the front of the glacier, to determine its spatial variability during time.

Moreover the Cherillon glacier has also been studied by Calmanti et al. [11], with the aim of studying its effects on global warming.
Below the principal characteristics of this glacier are detailed (Table 1).

Cherillon glacier	
Type	Cirque glacier
Latitude (°, cent)	45,98 N
Longitude (°, cent)	7,6 E
Surface [km^2]	1,15
Length [km]	2,2
Maximum altitude [m]	3540
Minimum altitude [m]	2595
Glacier trend	retreat
Front shrinkage since 2004 to 2006 [m]	24
Estimated glacier length [m]	1800
Average slope [°]	19

Table 1. Characteristics of the Cherillon glacier – Valle d'Aosta – Italy [http://www.nimbus.it/glaciorisk/].

3. FRACTAL ANALYSIS OF DAMAGE ON THE CHERILLON GLACIER

To study the spatial distribution of damage on the glacier and to highlight its scaling properties, we have calculated the fractal dimension of the crevasses network on one of glaciers of the Italian Alps, by the box-counting method.

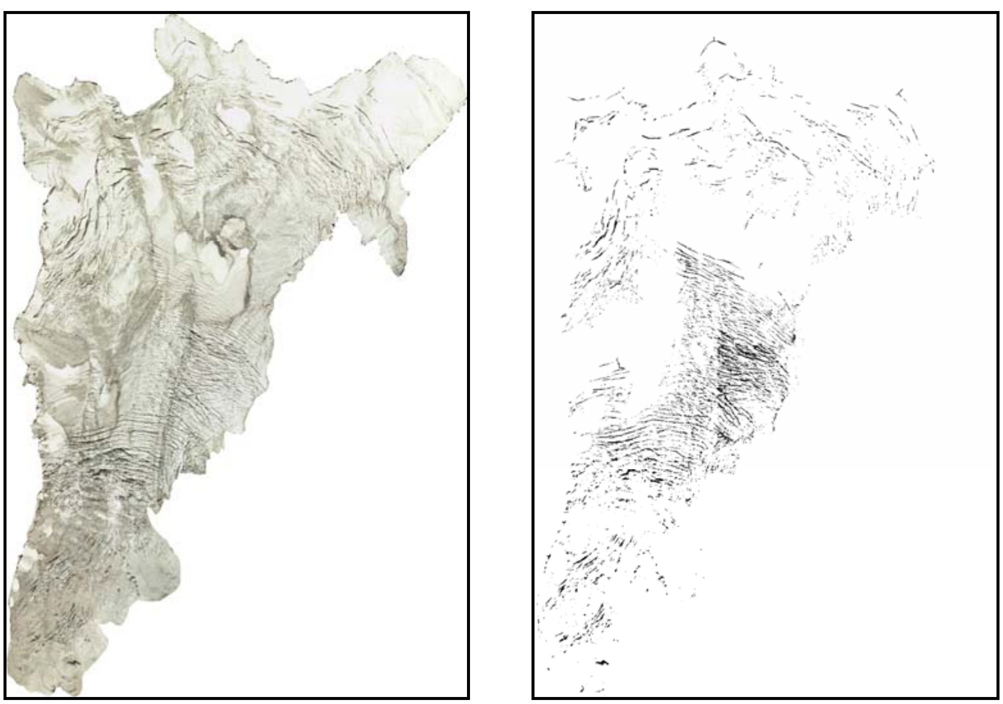
Among the various definitions of dimension of a fractal set, the Minkowski-Bouligand is a special case of the Hausdorff-Besicovitch dimension and represents the best definition for numerical implementation. To obtain the fractal dimension of a certain domain, it is necessary to cover it by means of regular Euclidean sets (usually square or rectangular grids) with falling linear size. The fractal dimension is obtained by computing the logarithmic density of the measure of these coverings [12]. This is called the box-counting method and, as a result, it gives the box-dimension Δ depending only on the metric characteristics of the set [13]. The mathematical definition of the box-dimension is given by:

$$\Delta = \lim_{b_i \to 0} \left(\frac{\log N(b_i)}{\log\left(\frac{1}{b_i}\right)} \right) \tag{1}$$

where b_i is the (progressively decreasing) linear size of the covering grid and N_i is the (progressively increasing) number of boxes that cover a part of the fracture network.

The box-dimension can be calculated in a straightforward manner by considering the slope α of a linear regression in the log$N$ vs. log$b$ plot. In this case, the box-dimension is equal to:

$$\Delta = -\alpha \tag{2}$$



(a) (b)

Figure 4. The Cherillon glacier: (a) Aerial image; (b) Crevasses network skeleton on glacier surface.

In the presence of self-affine scaling (as is the case of simple profile rouhness), this value depends on the shape of the covering grids. Generally, natural fractals do not exhibit an univocal value of the slope (mono-fractality), but show a geometric multi-fractality [14] with a continuous variation of the α parameter (some authors define this scaling behaviour as self-affinity). This behaviour is caused by the presence of two transition scales of phenomenon [15]. In particular, when the object is a digitalised image (i.e., a discrete set of pixels), it is necessary to define the lower limit of scaling to avoid to consider the external cut-off length of the object [15]. From this the lower limit of the scaling must be clearly defined.
In order to study the patterns of fracture networks on glacier, we have used a general-purpose version of the method originally developed for the fractal analysis of 2D lattices [16].
From the aero-photogrammetric view and the geo–referenced images of the Cherillon glacier (Figs. 3 and 4.a), the map of fracture networks on the ice surface can be outlined by image analysis, i.e. by skeletonising the areas corresponding to the crevasses (paying attention to eliminating shadows due to ice hills and valley) (Fig. 4.b). The box-counting method is then applied to this 2D pattern of fractures with dimension [1772 x 1024 pixels], by varying the linear size b of the square or rectangular grids (Figs. 5.a, 5.b, 5.c and 5.d). The ice fracture patterns on the Cherillon glacier possess a box-dimension Δ closed to 1.6.

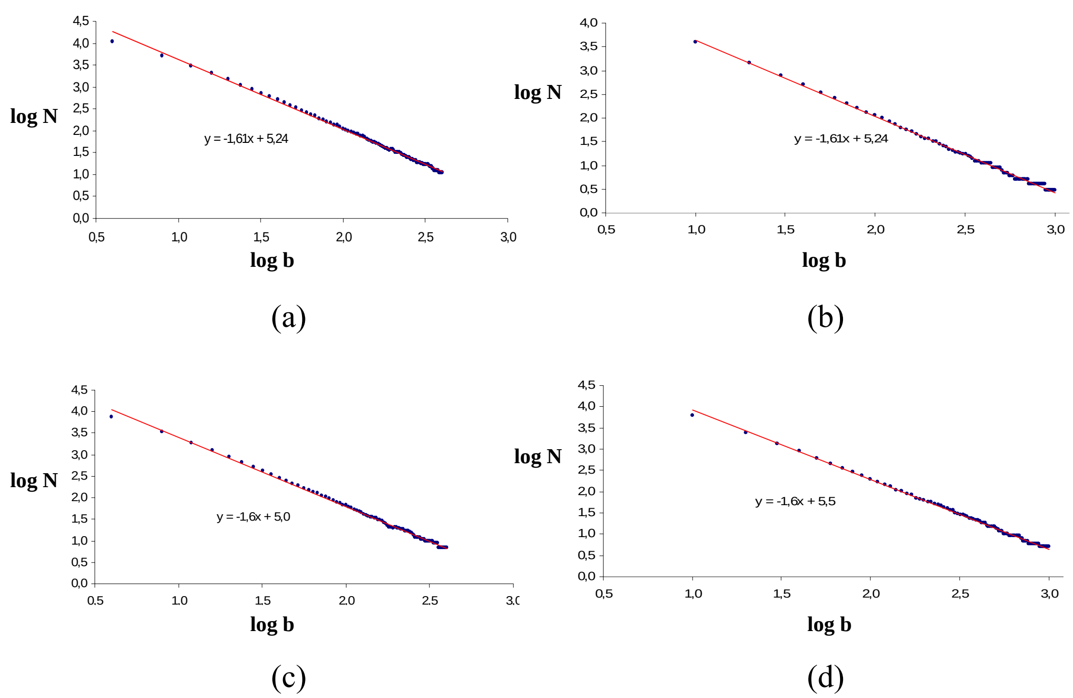


Figure 5. Application of the Box-Counting Method to the patterns of ice fractures: a) and b) square grids; c) and d) rectangular grids with different linear size b (the larger size of the rectangle is vertical and horizontal, respectively).

We note that varying the maximum dimension of square elements grids from 400 to 1000 pixels (respectively, Fig 5.a and Fig. 5.b), the box-dimension does not change. However the use of square grids (isotropic scaling implies the self-similarity of the network) could be in contrast with possible self-affine properties of fracture networks on ice.
To check this, rectangular grids have also been used to capture the possible direction-dependent scaling, adopting iterative rescaling as a function of the grid direction [16]. Fig. 5.c and 5.d show the box-counting dimension of the Cherillon glacier calculated by rectangular grids (vertical/horizontal ratio equal to 2 and 0,5, respectively). The box-dimension, calculated according to a self-affine grid, results again equal to $\Delta = 1.6$.
This means that the network of fractures on the glacier possesses self-similar fractal properties. From the mechanical point of view, this implies that the three prevailing stress rupture mechanisms are all active within the ice continuum, due to the particular geo-morphological shape of the valley.

4. CONCLUSIONS: MULTIFRACTAL SCALING OF STRENGTH AND TOUGHNESS

It is well know that some typical phenomena on the strength of materials are only the macroscopic results of the structural disorder. Also in ice mechanics, the investigation confirms that fracturing on glaciers surfaces shows a self-similar behaviour and induces a the invasive fractality ($\Delta > 1$) of the crack networks. This means that ice behaves as fragile material at small scales ($\Delta \approx 1$ with regular brittle fractures single paths) whereas becomes more ductile at large scales ($\Delta \cong 1.6$, with a dense fracture network on the ice surface) with a distributed damage and an increase of toughness.

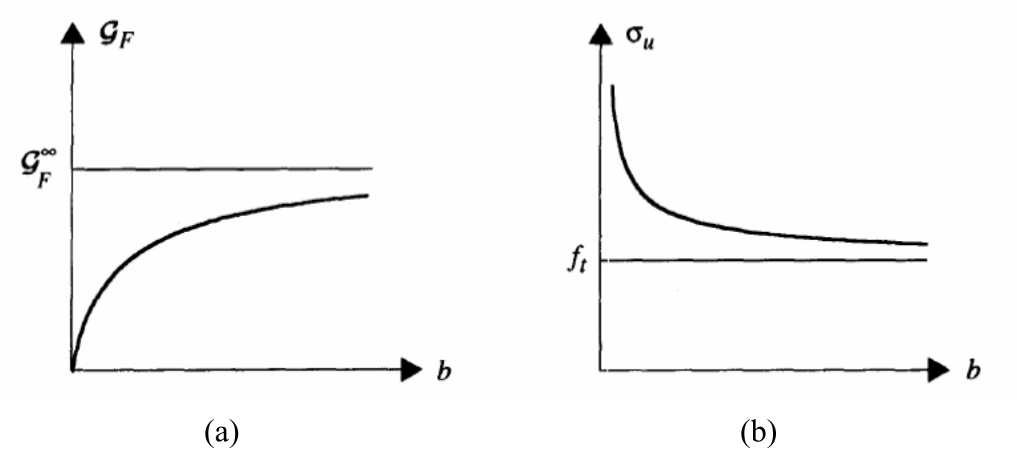
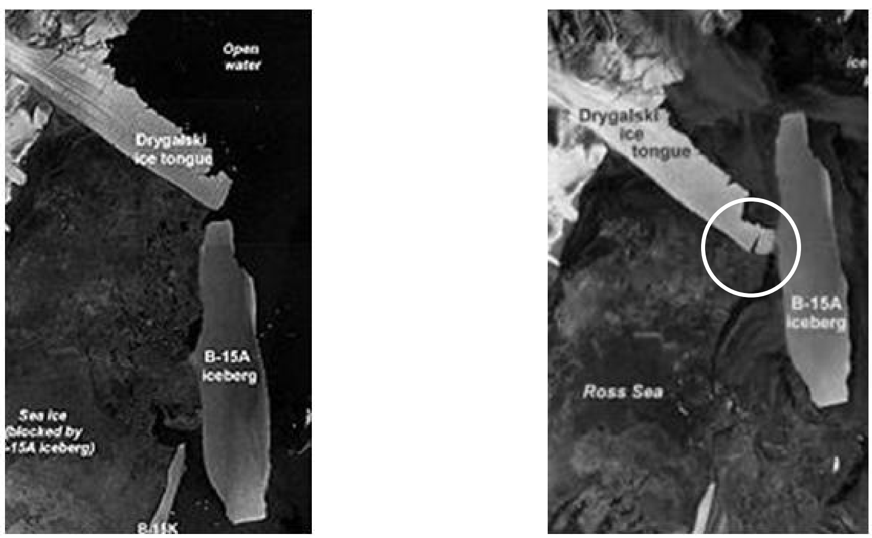


Figure 6. Multifractal scaling laws for the critical parameters $\mathcal{G}_F$ and σ_u [2].

As a consequence of the fractality of the domain of the fracture, the Multi Fractal Scaling Law - MFSL – [2] can be applied in ice mechanics to explain the decrease of ultimate tensile strength σ_u and the increase of toughness $\mathcal{G}_F$, as the scale of the specimen increases (Fig. 6 where $\mathcal{G}_F^{\infty}$ and f_t are the asymptotical values of the nominal quantities corresponding to the highest nominal fracture energy and to the lowest nominal tensile strength).
A further confirmation of the above mentioned scaling of the mechanical properties of ice can be obtained by comparing the data obtained at the small laboratory scale, by Dempsey et al. [17, 18, 19] on marine and freshwater ice, and the phenomenology of fracture evidences at the very large scale of Antarctic sea icebergs (see Fig. 7). In the Figure, two subsequent Envisat ASAR images taken on January and April 2005 show the steady progress of the massive B15-A iceberg (115 km long) towards the Drygalski ice tongue. The impact provoked a crack of 5 km length in the ice tongue.



(a) (b)

Figure 7. Two subsequent Envisat ASAR images of the B15-A iceberg and the Drygalski ice tongue: a) on January 2005; b) on 15 April 2005.

The analysis of the impact permits to highlight the dramatic fall of ice strength at the scale considered where the toughness, according to the MFSL depicted in Fig. 6.a, has already reached the asymptotic plateau.
In conclusion, the study of the scaling in ice mechanics can provide useful indications not only for the risk assessment of serac falls in mountain glaciers, but also in the case of iceberg impacts on offshore structures, for ice-breaking ships and in the Artic and Antarctic exploration and mining activities.

5. ACKNOWLEDGMENTS

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