Fracture Modeling of Interface Junctions on the Base of Multiscale Crack Bridging Concept

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1. Introduction

Reliability of interface junctions is the key problem in many industrial applications. To model adhesion, strength and fracture parameters of interface junctions the multiscale crack bridging concept is used. In the frames of this concept is supposed: there are bonds of the different scales (intermolecular forces, molecular bundles, fibers, nanofibers) between joined materials (the interface layer); any zone of weakened bond in this layer is considered as the interface crack with distributed nonlinear spring-like bonds between the crack surfaces (bridged zone). The bonds properties on the different scales define the stress state at the crack bridged zone and, hence, the reliability and the fracture toughness of the interface junction. In a general case, the size of bridged zone of the interface crack is comparable to the whole crack size. The conditions of a crack limit equilibrium and quasi-static growth for the case of the large scale bringing zone of the crack must be considered to model quantitatively the bridging effects. The quantitative analysis of the interface bridged crack growth consists of the following steps: 1) development the multiscale bond deformation law; 2) evaluation of stresses around the crack; 3) development and application the non-local crack growth criterion to analyze the fracture parameters of the interface junction.

Let's consider for simplicity the plane elasticity problem on a straight crack ($|x| \le \ell$) at the interface ($y = 0$) of two dissimilar joint half-planes. Assume that the uniform load σ_0 normal to the interface acts at infinity. The crack surface interaction exists in the end zones, $\ell - d(t) \le |x| \le \ell$. The size of the interaction zone $d(t)$ and the bond stresses depend quasi-statically on time t due to the possibility time changing of the bond properties. As a simple mathematical model of the crack surfaces interaction it is assumed that the spring-like bond, see details in [1], act through out the crack end zones at any level of bridging and the total traction $\sigma(x,t)$ arising in the bonds are

$$\sigma(x,t) = \sigma_{yy}(x,t) - i\sigma_{xy}(x,t)\,, \qquad i^2 = -1 \tag{1}$$

where $\sigma_{yy}(x,t)$ and $\sigma_{xy}(x,t)$ are the normal and shear components of the bond tractions, n - is the number of the different levels of bridging interactions.

The crack opening, $u(x,t)$, at $\ell - d(t) \le |x| \le \ell$ is determined by the prescribed bond deformation law

$$\sigma(x,t)=\sum_{s=1}^{s=n}\kappa_s\left(x,t\right)u(x,t) \qquad (2)$$

where n - is the number of the different bridging levels, $\kappa_s\left(x,t\right)$ - is the bond stiffness on the bridging level with number of s.
The bridged stresses can be defined using the approach proposed in [1-3] and based on the singular integral-differential equations (SIDE) method. The solution of the system of the SIDE can be obtained on an every bridging level and the total bridging stresses are the sum of the contribution of the every level and depend on the external loads and the position along of the crack surface.
The kinetic destruction of bonds due to elevated temperature and the aggressive agents is accounted on the every bridging level. The bonds kinetic model is based on the Zurkov's fluctuation model.

2. Fluctuation bond kinetic

According fluctuation theory of fracture [4] the lifetime of a body τ_b under the external tension loading σ is the exponential function

$$\tau_b=\tau_o e^{\frac{U(\sigma)}{kT}} \qquad (3)$$

where τ_0 is characteristic time (10^{-13}-10^{-12} s.), k is the Boltzmann constant, T is absolute temperature, $U\left(\sigma\right)$ is stress state dependent activation energy.

The function $U\left(\sigma\right)$ for wide range of external load and temperature is linear one

$$U(\sigma)=U_o-\gamma\sigma \qquad (4)$$

where U_o is the energy of the interatomic bond destruction, γ is the coefficient depends on the material microstructure.
The value $A=\gamma\sigma$ is the work of the bond deformation in supposing that the stress distribution over the bonds is uniform. Actually, the stress distribution over the bonds in the end zone of the crack is non uniform [1-3]. Seeing that we will use the relation (4) in the following form

$$U(\sigma)=U_o-A(x) \qquad (5)$$

where $A(x)$ is the work of the bond deformation at the point x of the crack end zone and suppose that the bond lifetime $\tau_b\left(x\right)$ is determined by the formula (3). In connection with, the lifetime of bonds in the bridged zone of the crack is the function of the bond position.

Let's assume that the time decrease of the bonds surface density $n(x,t)$ is governed by the equation

$$\frac{dn(x,t)}{dt} = -\frac{n(x,t)}{\tau_b(x)} \tag{6}$$

The solution of the equation (6) is

$$n(x,t) = N_0 e^{-\frac{t}{\tau_b(x)}} \tag{7}$$

where $\tau_b(x)$ is the lifetime of the bond.

The time decrease of the bonds surface density can be modeled by the changing of the bonds properties in the weakened zone. Denote by k_s the initial bond stiffness at the bridging level s. Then the effective stiffness of bonds per unit area in the crack end zone, $\kappa_s\left(x,t\right)$, is determined as follows

$$\kappa_s(x,t) = k_s n(x,t) = \chi_{s0} e^{-\frac{t}{\tau_b(x)}} \tag{8}$$

where $\chi_{s0} = k_s N_0$ is initial effective stiffness of bonds in the crack end zone.

The work of bond deformation (on the unit of the body thickness) over part of the bridged zone of the crack by size dx is equal to

$$dU(x,t) = \left[\int_o^{u_y(x)} \sigma_{yy}(u_y) du_y + \int_o^{u_x(x)} \sigma_{xy}(u_x) du_x \right] dx \tag{9}$$

Then, the work per one intermolecular bond is

$$A(x) = \frac{dU(x,t)}{dN_n} \tag{10}$$

where $dN_n = \varepsilon\, dn$, dn is number of the bonds over size dx, ε is the number of monomeric links between the crack surfaces.

Suppose that the bonds are the chains of polymer molecules and size of one monomeric link is a. If the bond elongation under loading is much less of the value a then the number of the monomeric links between the crack surfaces are

$$\varepsilon = \frac{(u_y^2(x,t) + u_x(x,t)^2)^{1/2}}{a} \tag{11}$$

The relations (8)-(11) enables us to model bond rupture kinetics in the crack end zone by means of the bonds stiffness variation in time.

3. Non-local criterion of bridged crack growth

The non-local fracture criterion [1, 2, 5] is used to evaluate the fracture toughness and the critical external loading in the frames of the bridged crack model. The state of the crack limit equilibrium corresponds to the following condition (Π is the total potential energy, $w(\varepsilon_{ij})$ is the density of the deformation energy, ε_{ij} are the components of the strain tensor; t_i, u_i are the tractions and displacements at the body boundary s_e; $\Phi(u)$ is the density of the strain energy of the bonds in the crack bridging zones, u is the crack opening at the bridging zones of area s_i, ℓ is a crack length)

$$-\frac{\partial \Pi}{\partial \ell} = \underbrace{-\frac{\partial}{\partial \ell}\left[\int\limits_{v} w(\varepsilon_{ij})dv - \int\limits_{s_e} t_i u_i ds\right]}_{G_{tip}(d,\ell)} \underbrace{-\frac{\partial}{\partial \ell}\int\limits_{s_i} \Phi(u)ds}_{G_{bond}(d,\ell)} = 0 \qquad (12)$$

The terms in this relation represent the strain energy release rate at creation of a new crack surface, $G_{tip}\left(d,\ell\right)$, and the rate of the energy absorption in the crack bridging zone, $G_{bond}\left(d,\ell\right)$, respectively. Note, that within the framework of the model the rate of the energy absorption depends on the bridging zone size and bond characteristics. The equilibrium bridging zone size is not assumed to be constant. It can be determined from condition (12) while the searching for the critical load needs additional conditions of the bond rupture. In the general case the strain energy release rate can be defined through the stress intensity factors.

The condition of the crack tip limit equilibrium (12) can be rewritten as follows

$$G_{tip}(d_{cr},\ell) = G_{bond}(d_{cr},\ell) \qquad (13)$$

Condition (13) is necessary but insufficient for searching for a limit equilibrium state of the crack tip and the bridging zone. This condition enables us to determine the bridging zone size, d_{cr}, such that the crack tip is in an equilibrium at the given level of the external loads. To search for the limit state of both the crack tip and the bridging zone within the framework of the model one should introduce an additional condition, e.g., the condition of bonds limit stretching at the trailing edge of the bridging zone $x_0 = \ell - d_{cr}$ (δ_{cr} is the bond rupture length)

$$u(x_0) = ([u_x(x_0)]^2 + [u_y(x_0)]^2)^{1/2} = \delta_{cr} \qquad (14)$$

Based on these two fracture conditions the regimes of the bridged zone and the crack tip equilibrium and growth are considered:

1) the crack tip propagates and the size of the bridged zone d increases without rupture of the bonds if

$$\begin{cases} G_{tip}(d,\ell) \geq G_{bond}(d,\ell) & (a) \\ u(\ell - d) < \delta_{cr} & (b) \end{cases} \qquad (15)$$

where $G_{tip}(d,\ell)$ is the energy release rate , $G_{bond}(d,\ell)$ is the rate of the energy dissipation by the bonds, ℓ is the half of the crack size, $u(\ell - d)$ is the crack opening at the trailing edge of the bridged zone, δ_{cr} is the bond limit stretching;

2) the size of the bridged zone decreases due to rupture of the bond without the crack tip propagation if

$$\begin{cases} G_{tip}(d,\ell) < G_{bond}(d,\ell) & (a) \\ u(\ell - d) \geq \delta_{cr} & (b) \end{cases} \qquad (16)$$

3) the crack tip propagates with simultaneous bond rupture at the trailing edge of the bridged zone if

$$\begin{cases} G_{tip}(d,\ell) \geq G_{bond}(d,\ell) & (a) \\ u(\ell - d) \geq \delta_{cr} & (b) \end{cases} \qquad (17)$$

The last case corresponds to quasi-static crack growth because the both fracture conditions are fulfilled.
The critical external loads, σ_{cr} the end zone size d_{cr} and the adhesion fracture resistance at the crack limit equilibrium state for the given crack length and bond characteristics can be determined from the solution of Eqs. (13) - (14).

4. Two-scale bridging zone

Let's consider the application of this criterion in the case of two-scale bridging. The adhesion junction of two semi-infinite plates is considered and it's assumed that there are two scale of bridging: 1) the bridging due to adhesion interaction between pristine materials of these plates; 2) the bridging due to the artificial interface layer of a polymer which is introduced to improve the adhesion. Supposing also that on the first bridging scale the fracture toughness is the constant value of G_c and bonds properties do not depend on time on the both scales. In this case the relation for $G_{bond}\left(d,\ell\right)$ can be written as follows [5]

$$G_{bond}(d,\ell)=\int_{\ell-d}^{\ell}\left(\frac{\partial u_y(x)}{\partial \ell}\sigma_{yy}(u)+\frac{\partial u_x(x)}{\partial \ell}\sigma_{xy}(u)\right)dx-G_b+G_c \tag{18}$$

where G_b the density of deformation energy allocated at break of the bond at the trailing edge of the crack end zone

$$G_b=\int_0^{u_{cr}}\sigma(u)du \tag{19}$$

The analytical consideration of the proposed criterion can be performed only for the case of the identical properties of the materials of planes and the bonds with the rectilinear law of the bond stress. In this very simple case the normal bridged stresses in the crack end zone are prescribed $(\sigma(x,t)=P_0)$, uniformly distributed along the end zone and independent on the crack opening and time. The normal displacements of an upper crack surface for this problem $u_0(x)$ are given in [6].

In the case small scale bridging condition we obtain from Eqs. (13) - (14) of the fracture criterion the critical end zone size which is *independent on the crack size in a small scale bridging limit* (see details in [5, 7])

$$d_{cr}=d_\infty=d_0\left(\sqrt{\eta+1}-\sqrt{\eta}\right)^2,\quad d_0=\frac{\pi E\delta_{cr}}{8P_0},\quad \eta=\frac{G_c}{G_b} \tag{20}$$

and the critical external stress (E is Young modulus of material)

$$\sigma_{cr}=\sqrt{\left(1+\eta\right)\frac{EP_0\delta_{cr}}{\pi\ell}}=\sqrt{\frac{E\left(G_b+G_c\right)}{\pi\ell}} \tag{21}$$

The size and the shape of the crack end zone do not change in the case of small scale bridging, therefore, the condition of autonomy of the end zone is satisfied and the energy absorbed to bonds in the end zone is equal to the energy released while breaking the bonds at the edge of the end zone. Thus, the total flow of the energy to the crack tip is spent on formation of a new surface of the crack. For this reason relationships (20-21) coincide with results which was obtained in [8] on the basis of the two-parametric fracture criterion with the first force condition of fracture $K_0-K_b=K_{Ic}$, where K_0 is the SIF due to an external loading, K_b is the SIF due to bonds and K_{Ic} is the matrix toughness. Noted that in the force fracture criterion [8] the work of bonds in the crack bridged zone is neglected and for the large scale bridging the noticeable difference is observed.

In the case of the uniform bridge stress it's possible to get the analytical solutions for the critical end zone size and the external stresses also for the large scale bridging case [5, 7]. The dependencies of the critical end zone size d_{cr}/d_0 vs the critical crack size $\lambda = \ell_{cr}/d_0$ is given in Fig. 1. It's interesting to note, that for materials with a weak interface $(\eta < 0.5)$ the critical end zone size is increasing during the crack growth (craze formation).

The dependencies of the apparent adhesion fracture energy (σ_{cr} is defined from the solution of the system of equations (13)-(14))

$$G^{ext} = (K_c^{ext})^2 / E, \quad K_c^{ext} = \sigma_{cr}\sqrt{\pi\ell} \qquad (22)$$

are shown in Fig. 2. As the crack length increases the apparent adhesion fracture energy tends to the constant value.

We shall further consider sub-critical growth of a crack with the bridging zone, assuming that an initial slit of the size $2\ell_0 \geq 0$ without bonds is initially made and the crack bridging zone of the size $2d$ is forming as the external loading monotonically increases. Then $t = \ell_0/(\ell_0 + d)$. Let the external load change in such a manner that at each current size of the crack bridging zone the condition (13) is satisfied, and the crack opening at the edge of the bridging zone does not exceed the critical value (condition (16b)).

As the external load increases a sub-critical growth of the crack occurs, and the size of the crack bridging zone reaches its critical value provided the condition (14) holds. To maintain a further quasi-static crack growth further, it is necessary to reduce the external loading; in this case the equations (13) and (14) are considered simultaneously. The curves for the sub-critical crack growth ($\eta = 1$) are presented in Fig. 3 where $\lambda = (\ell_0 + d)/d_0$ is a relative crack size, and $\lambda_i = \ell_i / d_0,\ i = 1...5,\ \ \ell_i$ is a size an initial slit without bonds. For example, (see Fig. 3) if $\lambda_2 = 0.463$ then under the increasing of the external load the initial slit will grow from $\lambda = \lambda_2$ till the intersection of the sub-critical crack curve with the curve of the critical loads.

5. Conclusion

The model of the interface crack with multiscale bridged zones enables one to study the sensitivity of the characteristics of the crack state and its limit equilibrium to the bond deformation curve without restrictions on the crack length scale. Hence, one can perform modeling of the behavior of nano-micro-meso and macrocracks taking into account the possible scale dependence of the bonding mechanisms and appropriate variations of the effective bond deformation law.

The application crack growth criterion leads to a classification of the possible regimes of the interface crack growth in dependence on the parameters of the bond deformation curve and crack scale.

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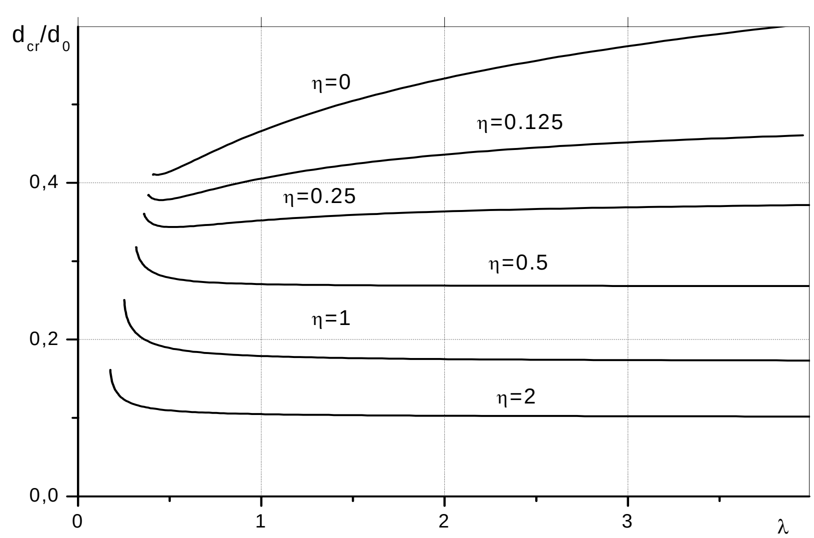

Figure 1 Critical bridging zone size vs. the critical crack size

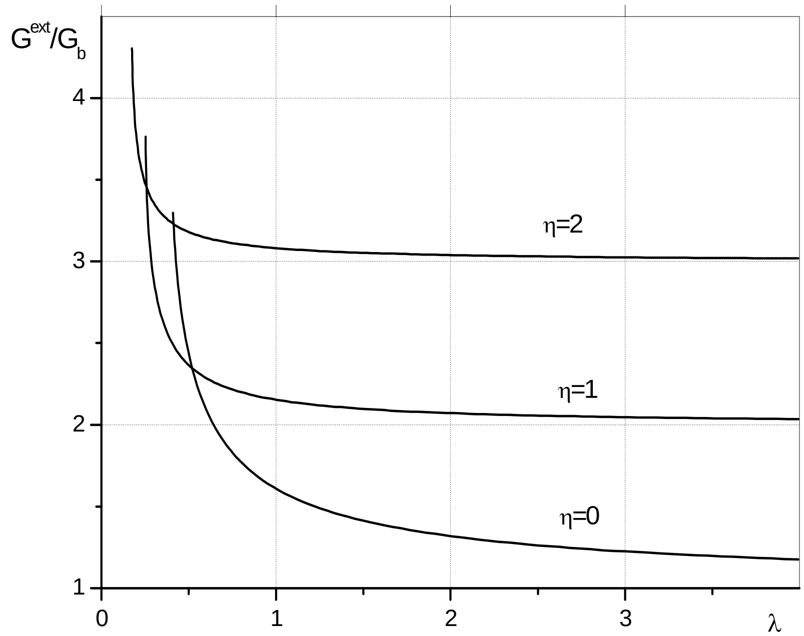

Figure 2 Apparent adhesion fracture energy vs. the crack length

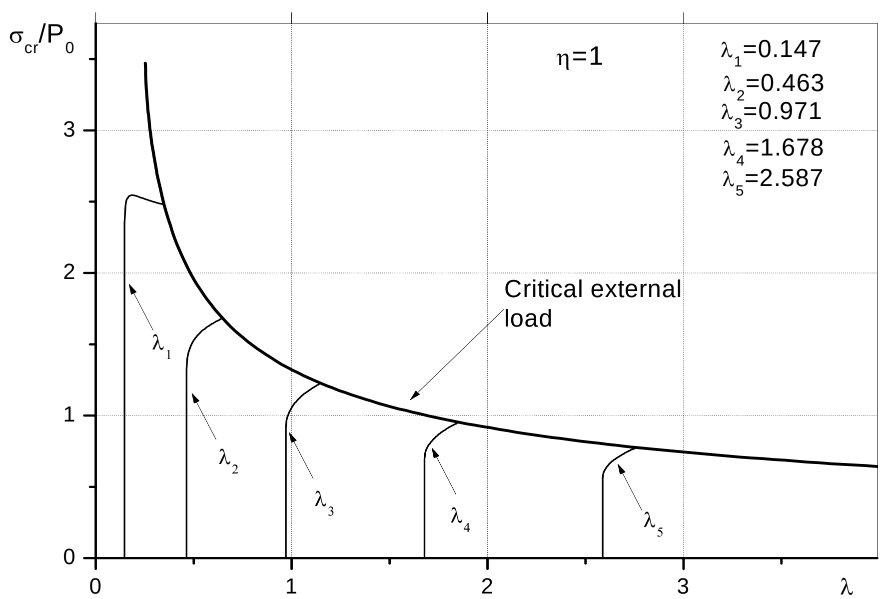

Figure 3 Curves of the sub-critical crack growth, $\eta = 1$