Short cracks at notches and fatigue life prediction under mode I and mode III loadings

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Abstract

Accurate life predictions of notched components require the computation of the short crack propagation life spent after crack initiation at the microstructure scale. Several experimental studies have shown that the notch stress intensity factor (NSIF) provides a unifying parameter for crack propagation lives at different severe V-notches under axial loading. The analysis can be extended to blunt notches by adding the crack initiation life predicted by the notch stress concentration factor (NSCF). The present work makes a further step to test the NSIF approach under torsional loading.

Both axial and torsional fatigue tests were performed on plain cylindrical specimens made of 1045 steel with two different circumferential V-notches. S-N data were analyzed and compared to those previously obtained with smooth tubular specimens. The mode I and mode III NSIFs were computed for both notches by finite element analysis. In all cases investigated, the fatigue behavior is controlled by the material resistance to crack propagation. The short crack propagation lives at V-notches under under axial and torsional loadings are well correlated by the mode I NSIF and the mode III NSIF respectively. The fact that the fatigue notch factor is close to unity whatever the notch root radius is due to a very high resistance to mode III crack propagation.

1. Introduction

Fatigue life prediction of notched components is of great importance in engineering practice since fatigue cracks invariably start from notches or stress raisers. Many papers have been written on this topic since half a century. Local stress and local strain approaches have been proposed then extended to multiaxial fatigue [1]. Local stress approaches are based on the notch stress concentration factor (NSCF) and use empirical formulae (Peterson, Neuber, etc…) to compute the fatigue notch factor k_f , i.e. the strength reduction factor with respect to unnotched material, as a function of material and geometry-loading configuration. However, these formulae are restricted to high or very high cycle fatigue lives and they are overly conservative in torsional fatigue [1,2]. Local strain approaches take reversed yielding at notch tip into account but they require an accurate evaluation of the plastic strain amplitude at notch tip as well as low cycle fatigue data for the material under investigation [2]. Both local stress and local strain approaches are open to criticism since they use reference data from smooth specimens that can be not representative of mechanics and mechanisms prevailing at a notch tip.

Several authors have re-evaluated the Peterson-Neuber concept of critical distance or critical volume to interpret and predict notch effect in various conditions [3,4]. The material is not sensitive to the peak stress at notch tip but rather, for instance, to the local stress at a certain distance from notch tip. This so-called « critical distance » is determined as a function of the fatigue strength of the unnotched material and the fatigue crack propagation threshold [3]. However, despite good correlations have been made between many experimental data, the concept is somewhat artificial since the critical distance is not correlated to any specific microstructural features and it depends on fatigue life. Moreover it does not take directly into account the fatigue mechanisms at the notch tip. Contrary to smooth specimens, crack initiation at the microstructure scale can be very rapid in a notched specimen. The more severe the notch, the smaller is the crack initiation life and the larger is the crack propagation life [5].

Even if methodologies have been proposed in particular cases [6], the application of Fracture Mechanics for predicting the life spent in the short crack regime is not an easy task. There are several unknown or uncertainties when integrating a crack propagation law, such as initial crack dimensions, crack aspect ratio evolution and stress intensity factor solutions. Further LEFM is theoretically not applicable for a short crack propagating in the notch plastic zone. Indeed short cracks grow faster than long cracks due to closure effects [7]. However an easier approach can be used to make a global prediction of the short crack propagation life in the case of V-shaped geometries having a very small radius at notch tip [8]. It starts from the Williams analytical solution [9] that shows that a singular stress field exists in the region surrounding a V-shaped geometry. The stress field at notch tip can be expressed as,

$$\sigma_{ij}(r,\theta) = K^N r^{-\alpha} f_{ij}(\theta) \qquad (1)$$

where r and θ are the polar coordinates and K^N is the notch stress intensity factor (NSIF) in $MPa.m^{\alpha}$. It is proportional to the nominal loading and depends on the geometrical parameters. The order of stress singularity α is a function of the notch angle only. It increases with decreasing notch angle and reaches 0.5 in the limit case of a crack (0^0 V-notch). The practical consequence is that, for a given notch angle, different combinations of geometry and loading giving the same NSIF give the same stress field and thus should give the same events ahead the notch, provided they remain confined within the singularity. As a matter of fact remarkable correlations have been obtained in several experimental studies involving axial and bending loadings [8,10]. For instance, the size effect observed in S-N data for fillet welded joints disappears when fatigue lives are plotted against the NSIF at weld toe.

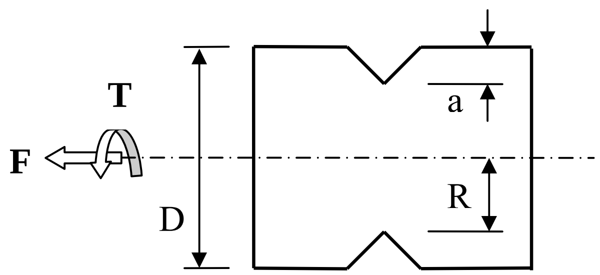
The previous analysis has been recently extended to V-notches with finite radius by adding the crack initiation life predicted from smooth specimen data and the NSCF [5]. It results in a new approach for predicting fatigue life of notched components under mode I loading (i.e. axial loading and bending), that rationalizes the effects of notch acuity and size, material and fatigue life on the fatigue notch factor. The present work shown in this paper makes a further step to test the NSIF approach under mode III loading (i.e. torsional loading). Even if the order of stress singularity α differs from mode I to mode III, Equation (1) is also valid in mode III. The question is the following. Can fatigue lives of short cracks propagating at V-notches be correlated by NSIF in mode III as well as in mode I ? The answer to this question is of practical interest for notched components working under torsional loads, such as stepped shafts or crankshafts [11].

2. Fatigue experiments

2.1. Material and specimens geometry

As-rolled 1045 steel was used as test material. The raw material was delivered in the form of a round bar with a diameter of 38.1mm. 1045 steel is a material used for automotive shafts where axial-torsional fatigue is often involved. The tensile yield and ultimate strengths are 387 MPa and 703 MPa respectively. The elongation at fracture is 31% using a 25 mm gage length. A constant hardness of 96 HRB was measured on several samples. The microstructure of this material consists of pearlite and ferrite. The average grain size of ferrite is 15µm and the average pearlite colony size is approximately 22µm. More details on material microstructure, axial-torsional fatigue properties measured on smooth tubular specimens and fatigue life correlations using critical plane parameters are given in reference [12].

Plain cylindrical specimens with circumferential 90^0 V-notches were machined from hot rolled bars to have a notch radius as small as possible (<0.075mm). Their central part is schematically drawn in Figure 1. Two notch geometries, N1 and N2 are used. Their respective dimensions are given in Table 1.



$$\sigma_{net} = \frac{F}{\pi R^2}$$

$$\tau_{net} = \frac{2T}{\pi R^3}$$

Figure 1. Schema of specimen central part and definition of nominal stresses over the net section (F = axial load; T = torque)

Table 1. Dimensions of the two V-notched specimens.

Specimen	D (mm)	2R (mm)	a (mm)	a/D
N1	19.05	13.335	2.86	0.15
N2	12.7	11.43	0.635	0.05

Before computing the 90^0 V-notch stress intensity factors (section 3), available solutions for the mode I and mode III stress intensity factors of cracks (0^0 V-notches) were used for the purpose of specimen design. The two specimens are designed so that the ratio between their stress intensity factors under the same nominal stresses is maximized. Their global dimensions are also determined by the grip size and the specimen design standard for axial fatigue [13].

2.2 Fatigue tests and S-N data

Fully reversed axial and torsional fatigue tests were performed on both types of specimens N1 and N2. All tests were load/torque controlled using a servo-hydraulic biaxial machine. Load and torque waveforms were sinusoidal with a frequency of 10Hz. Tests were conducted at room temperature in laboratory air.

Figure 2 shows S-N data obtained for axial loading. The total fatigue lives of N1 and N2 specimens as well as those of tubular smooth specimens are plotted against the amplitude of nominal axial stress. The higher the fatigue life, the lower the fatigue resistance of notched specimens as compared to smooth specimens. The fatigue notch factor of specimen N1 ($\approx$ 3.53 at 3.10^5 cycles) is higher than that of specimen N2 ($\approx$2.44 at 3.10^5 cycles). This is a well-known size effect in notch fatigue, that is also observed in structural components [8].

Figure 3 shows S-N data obtained for torsional loading. The fatigue lives of N1 and N2 specimens as well as those of smooth tubular specimens are plotted against the amplitude of nominal shear stress. For smooth specimens, the total life is used but for notched specimens, the fatigue life is the macroscopic crack initiation life defined by a 1% increase in rotation compliance, which corresponds to a crack depth of about 0.4-0.5 mm for both N1 and N2 specimens.

Note that the macroscopic initiation life in severe V-notched specimens is in fact a short crack propagation life since crack initiation at the microstructure scale can be very short in mode III as well as in mode I. It is necessary to measure this short crack life spent within the notch stress field in order to make good correlations between fatigue life and notch stress intensity factor (section 3). In axial fatigue, most of the life is spent before a crack reaches 0.5 mm in depth from the notch tip. Like for smooth specimens, the total fatigue life is representative of the macroscopic crack initiation life. However the corresponding life in torsional fatigue is only a small part of total life. Crack growth beyond 0.5 mm is very progressive and it does not increase rapidly as in axial fatigue.

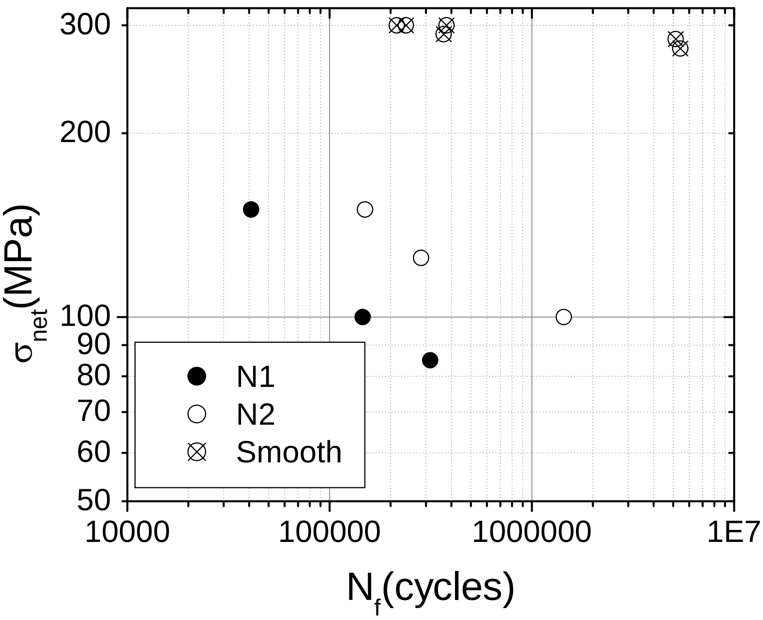


Figure 2 : Total fatigue lives plotted against nominal stress amplitude for notched specimens and smooth tubular specimens under axial loading.

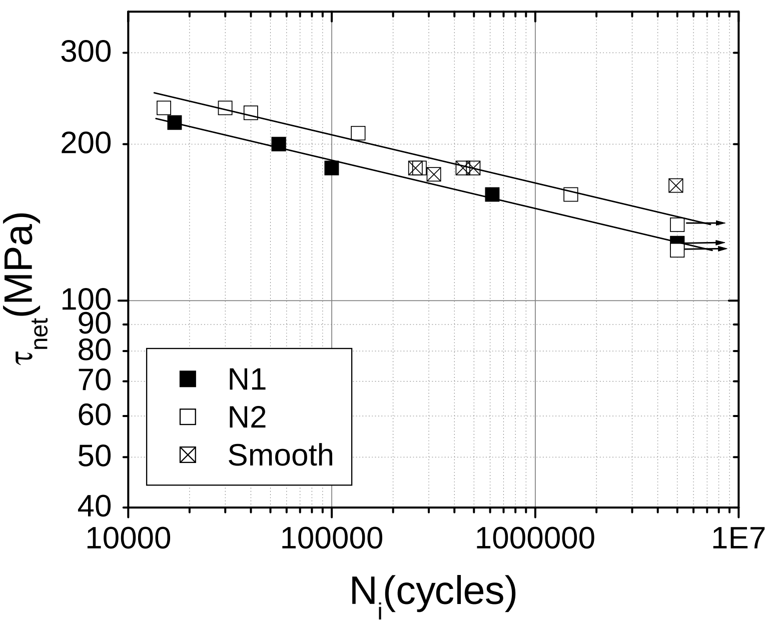


Figure 3 : Macroscopic crack initiation lives plotted against nominal stress amplitude for notched specimens and smooth tubular specimens under torsional loading.

The notch effect is very small in torsional fatigue in comparison to axial fatigue. Although it is much dependent on the definition of crack initiation life, the fatigue notch factor is hardly larger than unity. This result is well known in torsional fatigue [1,2]. Torsion loading is less damaging than axial loading in a circumferentially notched bar. However the size effect still exists even if there is a lower discrepancy between the fatigue performances of the two notched specimens. The slopes of the torsional S-N curves ($\tau \approx N^{-0.09}$) are also lower than those under axial loading ($\sigma \approx N^{-0.27}$).

3. Correlations between NSIFs and fatigue lives

Elastic finite element analyses for specimens N1 and N2 under both axial and torsional loadings were made using the Catia code. All the numerical parameters used in the computations are given in reference [13]. Only the general methodology and the final results are reported here. The mode I NSIFs were extracted from the stress distribution $\sigma_{zz}(r)$ at notch tip in the radial direction (θ=0). Except for the first elements at notch tip, this distribution is linear on a log-log plot until about 0.5 mm in depth for notch N2 and 1 mm for notch N1. Linear regression leads to the following expression,

$$\sigma_{zz} = K_I^N \, r^{-0.420} \tag{2}$$

The same methodology was used for extracting the mode III NSIFs. Linear regression leads to the following expression,

$$\tau_{z\theta} = K_{III}^N \, r^{-0.313} \tag{3}$$

The exponents of the stress singularities are close to theoretical values. The analytical solutions for 90^0 V-notches give α = 0.456 and α = 0.333 under mode I and mode III respectively [14]. The values of NSIFs computed from FEA as well as stress concentration factors at 0.1 mm from the notch tip are given in Table 2 for both notches N1 and N2. Axial and torsional fatigue lives of notched specimens can now be re-plotted against mode I and mode III NSIF respectively. Very good correlations are obtained as shown in Figures 4 and 5. The size effect observed in previous plots (Figs 2 and 3) has disappeared.

Table 2. Stress singularity parameters computed from FEA

	Mode I (α = 0.420)		**Mode III (α = 0.313)**	
Notch	K_I^N / σ_{net} ($mm^{0.42}$)	$\sigma_{zz} / \sigma_{net}$ at r = 0.1mm	K_{III}^N / τ_{net} ($mm^{0.313}$)	$\tau_{z\theta} / \tau_{net}$ at r = 0.1mm
N1	0.869	2.28	0.778	1.60
N2	0.634	1.67	0.703	1.445

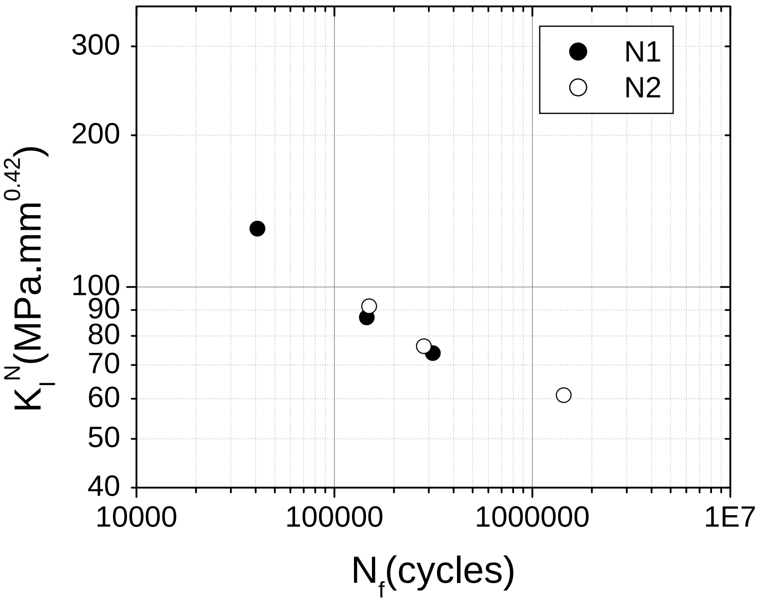


Figure 4 : Total fatigue lives of notched specimens N1 and N2 under axial loading against the amplitude of mode I notch stress intensity factor.

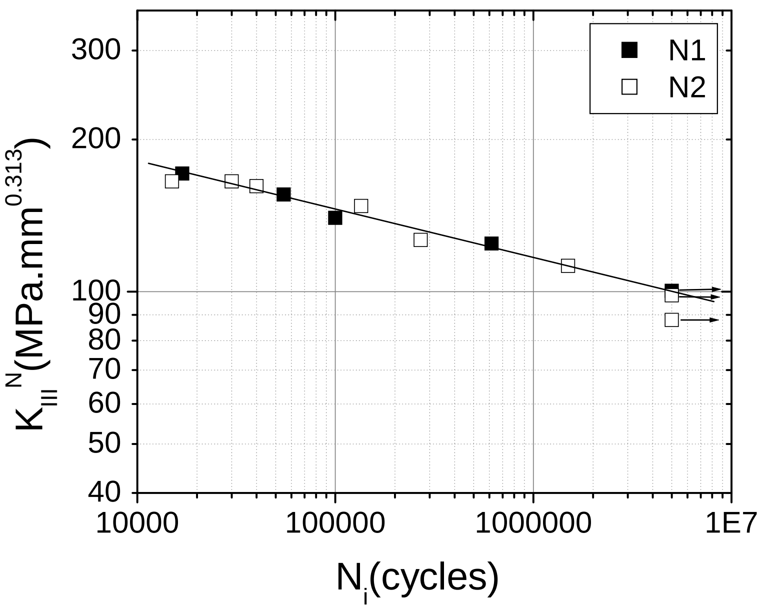


Figure 5 : Macroscopic crack initiation lives of notched specimens N1 and N2 under torsional loading against the amplitude of mode III notch stress intensity factor.

The shear yield strength of the material (τ_y = 223 MPa and 194 MPa according to Von Mises and Tresca criteria respectively) indicates that nominal plastic deformation can be present in torsion tests at lives shorter than 10^5 cycles (Fig.3). However a closer examination shows that cyclic plasticity is confined at notch tip. According to Tresca criterion, the plastic zone size is evaluated to 0.12 mm and 0.44 mm at K^N = 100 MPa.mm$^{0.313}$ and 150 MPa.mm$^{0.313}$ respectively.

4. Discussion

Fatigue strength of severely notched specimens under axial loading is controlled by crack propagation [5]. Most of life is spent within the first half millimeter but crack initiation life at the microstructural scale is very short even at low stress levels. In such conditions, the S-N curves are steep and there is an important size effect. The mode I NSIF takes this effect into account. Fatigue lives are well correlated by the NSIF, as already shown by previous studies [8,10].

The present study also validates the NSIF approach for V-notched components in mode III. The fatigue lives of both specimens N1 and N2 under torsional loading are well correlated by the mode III NSIF. As it can be expected, the same value of NSIF gives the same stress field at notch tip and thus gives the same rates of crack growth from the notch. The practical consequence is the same as in mode I [5]. Knowing the material K^N-N curve corresponding to a given crack depth (e.g. 0.5 mm), the macroscopic crack initiation life of any 90^0 V-notched component can be predicted after computation of the K^N value for this particular notch.

Another point of interest is the very small notch effect observed in torsional loading as compared to axial loading (Figs 2 and 3). Despite the notch is very severe, the fatigue notch factor is hardly greater than unity. Table 2 shows that the stress concentration factor in torsion is lower than that in axial loading. However S-N data of smooth and notched specimens are not really comparable in terms of stress concentration factor. First, a yielding effect is possible at lives shorter than 10^5 cycles as already discussed. Further, more importantly, fatigue strength of the V-notched specimen is controlled by crack propagation. Mode III crack propagation under torsional loading is a very difficult fracture mechanism.

Unlike axial loading where fracture surface is very flat, cyclic torsion of notched specimens N1 and N2 produces a « factory-roof » type fracture surface (Fig.6). This is typical of cylindrical specimens with a circumferential V-notch under low shear stress amplitudes [15]. All around the notch circumference, many cracks locally propagate by a mode I mechanism at ± 45 degrees to the plane of the notch and link up to give the serrated fracture surface. The macroscopic mode III crack is confined to the high stress region of the notch plane. The irregular crack pattern results in a significant degree of mechanical interlocking and sliding friction [1]. This is probably the major cause of the very high resistance of the V-notched specimens.

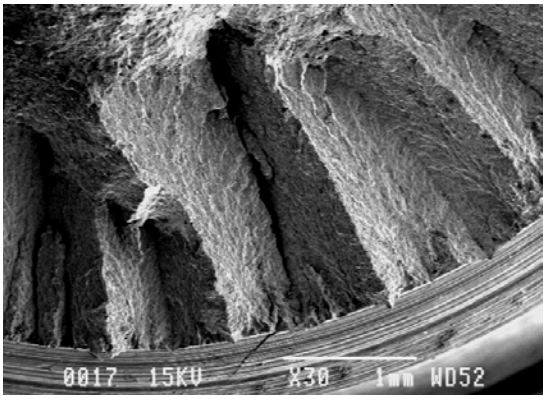


Figure 6 : Factory-roof fracture surface observed all around specimen circumference under cyclic torsion.

Under such conditions, unlike axial loading, torsional fatigue strength of V-notched specimens can be controlled by crack propagation even for blunt notches. This explains why the fatigue notch factor is close to unity whatever the notch root radius [1]. With reference to the new approach for life prediction [5] presented in Introduction, there is no need to add a crack initiation life to the crack propagation life predicted by the NSIF. However this conclusion cannot be generalized to stepped shafts (where a spiral, mode I crack can develop) and to biaxial fatigue [14]. A future challenge is to study the biaxial fatigue behavior of severe and blunt notches in order to make a fracture mechanism map that shows for a given material, the controlling fracture mechanism as a function of type of loading, stress level and notch geometry.

5. Conclusions

Axial and torsional fatigue tests were performed on plain cylindrical specimens made of 1045 steel with two different circumferential V-notches. The mode I and mode III NSIFs were computed for both notches by finite element analysis. The following conclusions can be drawn.

The fatigue behavior is controlled by the material resistance to crack propagation in all cases. The short crack propagation lives at V-notches under under axial and torsional loadings are well correlated by the mode I NSIF and the mode III NSIF respectively. The fact that the fatigue notch factor is close to unity is due to a very high resistance to mode III crack propagation.

The NSIF approach opens the door to a new way to make fatigue life prediction of notched components. The theoretical foundations are similar to those of Linear Elastic Fracture Mechanics. It is not required to take into account the notch tip plasticity as long it is confined within the singular stress zone.

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