Fatigue Life of Components with Two Types of Defects

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In design against fatigue with models that take defects into account, most attention is paid to the largest defects. Medium-sized defects, however, are seldom considered as potential sites of failure. In this paper, a model material is studied, containing a substantial proportion of medium-sized defects. The influence of their distributions has been investigated, using FCG computations. These computations were performed as post-processing of FE-results, using a modified Paris law. It is shown that the medium-sized defects can cause fatigue failure, given that there is a relatively large region of intermediate stresses. Further, it is shown that the statistical distributions used for describing the defects have a clear influence on the fatigue life of a component. The effects of removing the largest defects, or reducing the density of the defects, are studied as well.

1 INTRODUCTION

Fatigue is a common failure mode in engineering. Traditionally, design against fatigue is based on extraction of stress cycles from a complex loading history which are then compared to the fatigue limit of the material. Fatigue failure is usually due to defects (inclusions, voids *etc*.) present in the material. This means that methods that take the defects into consideration should be used, in order to refine the description of the fatigue failure. Murakami [1] has studied the influence from defects. Since there is a vast amount of defects with different sizes present in a structure, statistical distributions have to be used for characterization of the defect size.

In many papers, [2] for example, all attention is paid to the largest defect present in a structure. Here, instead, all defects are considered important. Two methods of quality control can be envisioned when dealing with defects. The first one is to reduce the number of defects in the material, which is herein done by reducing the number of defects per unit volume (*i.e.* the defect density). The second method is only to remove the largest defects present, which can be done using a quality control. If the defects are completely removed above a particular size, the defect size distribution will be truncated. The goal of this paper is to investigate the effect of; *i*) reducing the density of defects with the same size distribution, *ii*) truncate the size distribution, with the same density of defects.

2 CHARACTERIZATION OF DEFECTS

Several methods are used for characterizing defects, as described in [3]. The inclusion size data used here are based on unpublished data by Barsoum. The observed cumulative distribution function (CDF) is shown in Figure 1. Also, a small plot shows the observed probability density function (PDF). It can be noted that the defect size distribution is a consequence of two types of defects which lead to two maxima that are partially overlapping. There is a local peak at about 2.2 µm and another one at about 6 µm. The data are taken from a low-alloy steel and will be used in this study of a model material. The details of the observed defect size distribution can not be carried over to the fatigue crack growth simulations. Instead standardized statistical distributions are used for describing the defect size data and the following distributions are used here: Generalized Extreme Value (GEV) distribution, Generalized Pareto distribution (GP), exponential distribution, normal distribution and lognormal distribution.

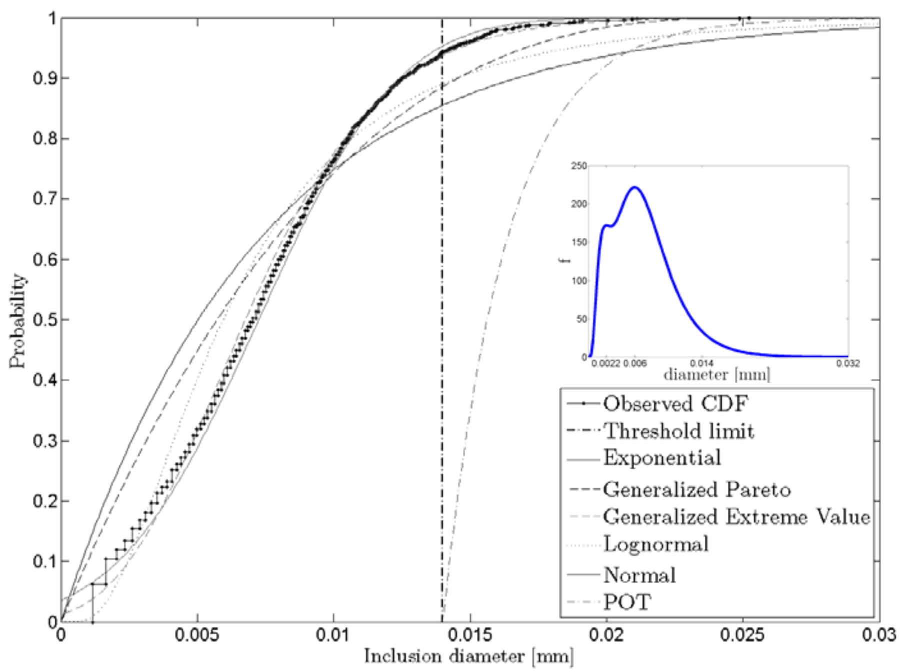

Figure 1. Different distributions fitted to the defect size data from Barsoum.

In Figure 1, the distributions mentioned above have been fitted to the defect size data. The fitting was performed using MATLAB's in-built distribution fitting functions. The dash-dotted curve starting at 14 µm indicates the Peak Over Threshold (POT) method, [4], which emphasizes on the largest defects present in the structure. The vertical dash-dotted line indicates the threshold limit between medium-sized and large defects, where large defects have values larger than the threshold and medium-sized smaller. The value 14 µm is chosen so that 95 % of the observed data is below this value. It should be noted that the exponential, lognormal and GP distributions describe this type of defect size distribution poorly. The GEV and normal distributions describe the observed CDF best. Obviously the POT method describes the defect size distribution very poorly for the medium-sized defects, which is logical since this method emphasizes the largest defects present.

The defect size distributions have been truncated as well, both using upper-limit and lower-limit truncation. When used, the upper truncation limit was set to 14 μm, the threshold between medium-sized and large defects. For some distributions a lower limit truncation is set to zero. The lower limit truncation had to be used for the GEV and normal distribution; otherwise these distributions can generate negative defect radii. The defect size distribution is a consequence of the manufacturing process and the chemical composition of the material. If a subsequent quality check is performed, large defects can be eliminated *i.e.* the distribution is truncated. Of course if the density function is integrated between its lower and upper limit, the integration should be equal to one. A complete truncation like this might not be very realistic, but the analysis will show if it decreases the scatter in the fatigue life and increases life.

Another defect characteristic is the defect density, the number of defects per unit volume, which governs the total number of defects present in a structure. Here two defect densities are used; the very low density of 2 defects/mm^3 and the low value of 20 defects/mm^3.

3 SIMULATIONS

The specimen, Figure 2, was first analyzed in ABAQUS and then the results were post processed in the defect based fatigue post processor P•FAT. For the theoretical background of P•FAT see [5] and [6]. The mesh was checked for convergence and the elements used were eight noded elements with reduced integration (C3D8R), see [7].

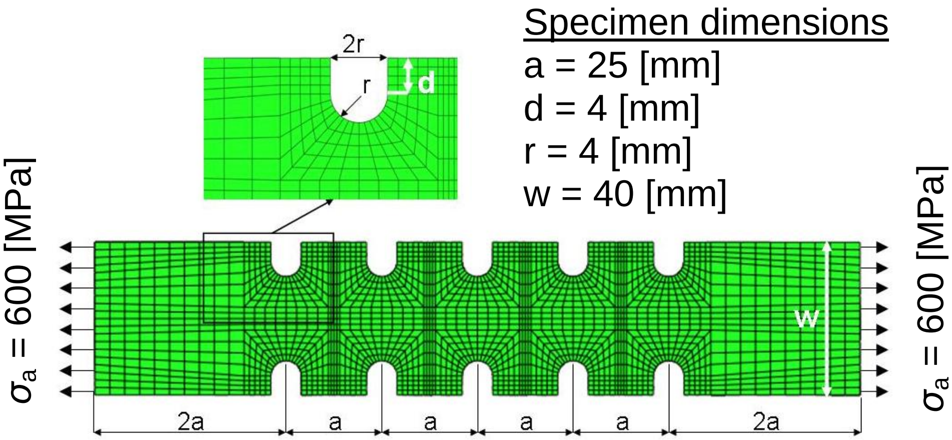

Figure 2. FE-model of the specimen used in the simulations, the thickness of the specimen (which is equal to 5 mm) is not shown.

The stress amplitude was approximately twice the fatigue limit after converting the fatigue limit to the stress ratio used in the simulations, which was R = -1. The conversion of the fatigue limit was done using a Walker correction. This high stress amplitude obviously gives short fatigue lives. In the FE-simulations a linear elastic material model was used. After the FE-simulations, the results were post processed in P•FAT. The option 'random defect approach' was used, which uses some aspects of probability theory. In P•FAT, all defects are initially treated as circular embedded cracks perpendicular to the maximum principal stress direction. The first step is to determine the number of defects per finite element, which is done by using a Poisson process (with the intensity equal to the defect density). This is followed by the determination of the defect location, which is done by letting the location be described by a uniform distribution. Here, three uniform distributions are used, since the specimen is in 3D (one distribution per direction). After determining the number of defects in each finite element and the location of the defects, the size is to be determined. This is done by letting the defect size be described by one of the distributions presented previously; hence each defect will have different size. When the characteristics of the defects have been determined, the Kitagawa-Takashi diagram, [8], is used for determining if the defect (crack) is potentially life-controlling or not, *i.e.* if $\mathrm{d}a/\mathrm{d}N > 0$ in Eq. (1) below. The next step is to determine the initial crack growth rate of the defect, since only the defect with the highest initial crack growth rate will be used in the fatigue life computation. The procedure explained above is repeated n times, thus n life controlling defects are generated. When these defects have been generated, the fatigue life can be calculated and n defects yield the fatigue life distribution at a constant stress level. The reason for choosing the defect with the highest initial crack growth rate is that this defect generally turns out to be controlling the life of the component, [5] and [6]. The fatigue crack growth rate is computed from

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C\Delta K_{\mathrm{th}}^{m}\left[\left(\left(\frac{\Delta\sigma}{\Delta\sigma_{\mathrm{A}}}\right)^{2} + \left(\frac{\Delta K}{\Delta K_{\mathrm{th}}}\right)^{2}\right)^{m/2} - 1\right]. \tag{1}$$

In Eq. (1), C is the crack growth constant, ΔK_{th} is a threshold stress intensity factor range, m is the crack growth exponent, $\Delta\sigma$ is the stress range, $\Delta\sigma_{\mathrm{A}}$ is the fatigue limit and ΔK is the stress intensity factor range. The values of these parameters are taken from [9] and presented in Table 1.

Table 1 Material properties used in the P•FAT simulations.

Fatigue limit ($\Delta\sigma_{\mathrm{A}}$)	Crack growth coefficient (C)	Crack growth exponent (m)	SIF threshold (ΔK_{th})
1100 [MPa]	$2.08*10^{-14}$ [MPa,m]	4.8 [-]	4.4 [$\mathrm{MPa}\sqrt{\mathrm{m}}$]

The crack growth constant, C, the threshold stress intensity factor range, ΔK_{th}, and the fatigue limit, $\Delta\sigma_{\mathrm{A}}$ are determined using R = 0. After this, these material properties are corrected to match the load ratio used in the simulation, which is R = -1, using the Walker correction γ = 0.78. It is assumed that the crack growth exponent, m, is constant and thus does not change for different load ratios.

The material data required for the present investigation is currently not available. Therefore, a model material is studied with fatigue properties taken from a bearing steel and the defect data taken from a low-alloy steel.

4 RESULTS

4.1 Different statistical distributions

Several distributions, presented in Section 2, have been used to describe the defect size. Here, the choice of statistical distribution is investigated with regards to fatigue life and selection of the critical defect. In Table 2 the outcome of 100 simulations are shown. Depending on which distribution is used, different defects will be critical; either medium-sized or large. It can be seen that the normal distribution has the most medium-sized defects that lead to failure, followed by the GEV distribution. For the exponential distribution and the GP distribution using the POT method no medium-sized defects led to failure.

Table 2 Effect of defect density and different size distribution type on critical crack size.

20 defects/mm^3 (Low density)		Defect density	2 defects/mm^3 (Very low density)	
Medium-sized	**Large**	**Distribution**	**Medium-sized**	**Large**
0	100	*GEV*	22	78
0	100	*GP*	3	97
0	100	*GP using POT method*	0	100
0	100	*Exponential*	0	100
0	100	*Normal*	32	68
0	100	*Lognormal*	1	99

In Figure 1 it can be seen that some size distributions are more prone to yield a fatigue failure from large defects. These distributions are the ones having larger portion of probability for large defects (*i.e.* lognormal, GP and exponential). This is analogous with the results presented in Table 2. When using the POT method, the threshold is set to the threshold limit, and thus it should not be possible to get fatigue failure from medium-sized defects.

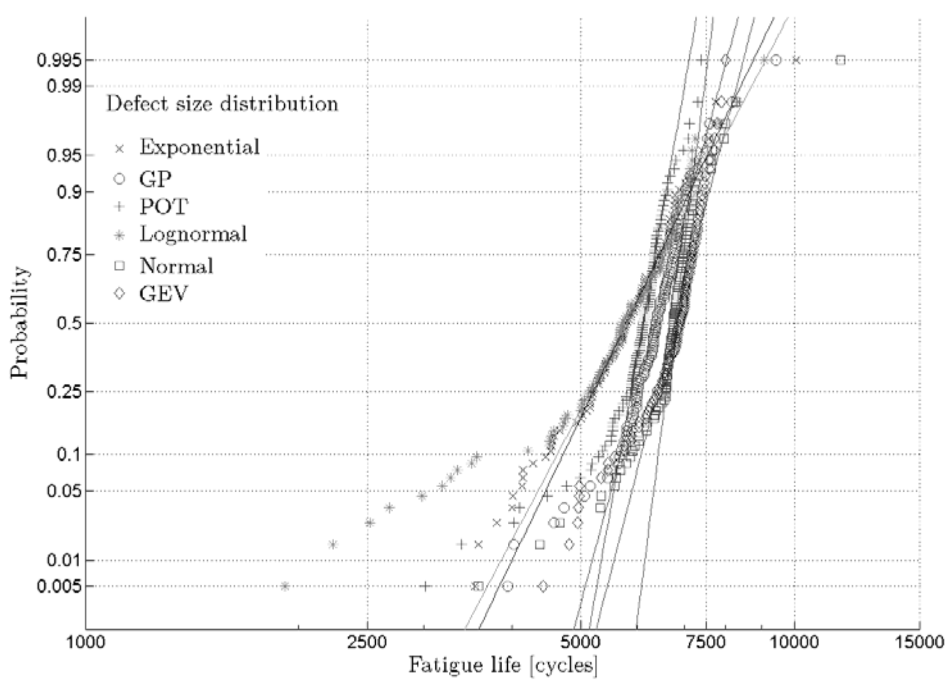

Figure 3. Fatigue life distribution for the different defect size distributions. A lognormal scale is used.

In Figure 3 the effect on the fatigue life distribution of using different defect size distributions is presented. The straight lines indicate data fitted to a lognormal distribution, since the fatigue life at a constant stress level usually is assumed to be described by a lognormal distribution. One hundred simulations are performed for each different defect size distribution. It can be seen that fatigue lives follow the distribution well on an interval from around 0.20 to 0.95. In the tail regions *i.e.* low and high probabilities, there is large discrepancy between the fitted lognormal distribution and the defect size distributions, especially in the lower tail region. It is remarkable that there is such a large difference between the fatigue lives for the different distributions, especially in the lower tail region.

4.2 Effect of truncation of the statistical distribution

As mentioned previously, truncation can be used as quality measure. This means that large defects are removed; hence the defect size distribution will be truncated. The upper truncation limit here is 14 µm. If the defect size distributions are truncated at the upper truncation and plotted using a lognormal scale, the results shown in Figure 4 are obtained.

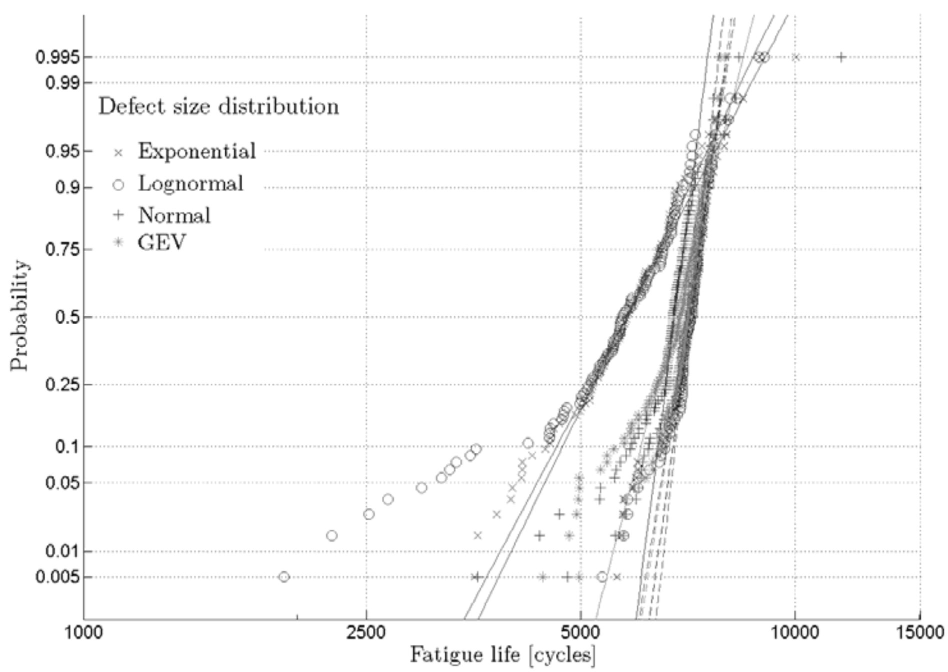

Figure 4. Comparison of fatigue life distributions for original and truncated data. A lognormal scale is used. Filled lines indicate non-truncated distributions and dashed lines indicate truncated size distributions.

In Figure 4 a comparison between truncated and original data is presented. One hundred simulations are performed for each different defect size distribution (both original and truncated). It can be seen that the slopes for the truncated distributions are higher. This slope is the reciprocal of the standard deviation. Hence, the standard deviation of the fatigue life for the truncated distributions is smaller. Also, the total range of the different fatigue lives is about twice as large for the original distributions compared to the truncated distributions. This means that fatigue life is improved and the scatter is decreased, when using truncation. This can also be seen by looking at the slopes for the different distributions; the truncated distributions have a higher slope compared to the original distributions.

5 SUMMARY AND CONCLUSIONS

In this paper it has been shown that not only the largest defects lead to fatigue failure of a component. Also, medium-sized defects can lead to fatigue failure given that there is a relatively large region of intermediate stress present. This may be found between stress-raisers (notches for example). In order to get fatigue failure from medium-sized defects, a multiple-notch specimen, Figure 2, had to be used. The notches in this specimen were so close that they interacted. For the FCG computations a modified Paris law was used. The FCG computations were performed using the post processor P•FAT.

The effect of describing the real defect size with different statistical distributions is presented in Figure 3. From Figure 1, it is seen that the GEV and the Normal distribution agree best with the observed CDF. Hence, in Figure 3, these distributions are expected to describe also the fatigue life of the component best. As can be seen in Figure 3, the GEV and the Normal distributions agree well with each other over a wide range of probabilities. In the tail region of low probability, there is a large difference between the different distributions. This region is of particular interest in fatigue design that aims for high reliability. It is therefore of utmost importance to use an appropriate statistical description of the defect size in simulations based on defect tolerance. A general result is that the lognormal distribution describes the fatigue life distribution poorly. When using a lognormal defect size distribution the discrepancy with the fatigue life distribution in the tail regions is large. This indicates that the fatigue life distribution does not only depend on the defect size distribution but also the stress distribution (*i.e.* the specimen geometry). It can also be observed that the exponential and lognormal distributions display a larger scatter than the other distributions (smaller slope than the other distributions). This might be explained by the fact that these distributions had zero medium-sized defects whereas the normal distribution, which has the largest slope, had the most medium-sized defects that lead to fatigue failure.

In Figure 4 the effect of truncating the defect size distributions was presented, where the fatigue lives where fitted to a lognormal distribution. It can be noted that the range of fatigue lives decreased with a factor of 2 when truncation was used. Further, slopes of the different fitted distributions, *i.e.* the standard deviation, and the mean values were almost the same. This is logical since the distribution function has to be 1 at the distinction limit and 0 for crack size equal to zero. Hence, the characteristics of the different distributions can only be seen on a certain interval and no tail behavior is permitted. If the Coefficient Of Variation (COV), the ratio between the standard deviation and the mean value of a normal distribution, was determined for the different distributions it could be noted that the relative difference between the different fatigue life distributions was less than 10 %.

In this paper the effect of medium-sized defects has been investigated. It has been shown that medium-sized defects, defined here as defect with a radius smaller than 14 µm, can also lead to fatigue failure given that there is a sufficiently large region with intermediate stresses present. Further, the distribution used for describing the defect size plays a decisive role for the fatigue life. However, it was seen that the fatigue life does not only depend on the defect size distribution. When the number of defects per unit volume was increased with a factor of 10, there were no medium-sized defects that led to fatigue failure. This is shown in Table 2. When truncating the defect size distribution, the scatter decreased drastically and the difference between the different life distributions was almost negligible. Hence, if truncation is used as a quality measure it will decrease the scatter in fatigue life as well as increase the average fatigue life.

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