Dissipation and mean stress effect in HCF and LCF

E. Charkaluk[1] and A. Constantinescu[2]

[1]*Laboratoire de Mécanique de Lille - CNRS UMR 8107, boulevard Paul Langevin, 59655 Villeneuve d'Ascq cedex, France*
[2]*Laboratoire de Mécanique des Solides - CNRS UMR 7649, Ecole Polytechnique, 91128 Palaiseau, France*

1. Introduction

Fatigue damage is generally separated in two domains : Low Cycle Fatigue (LCF) and High Cycle Fatigue (HCF) with a transition between limited and unlimited endurance. Even if fatigue failure is the result of complex microscopic phenomena which occur under cyclic loading, the common principal mechanism responsible of the crack initiation is the plastic strains and the damage developed in the grains due to irreversible dislocations motion. The essential difference between HCF and LCF regimes is that the scale of the plastic localization in a material volume is mesoscopic and respectively macroscopic. A general framework was already proposed by Charkaluk and Constantinescu [4], based on plastic dissipation, which allows to propose a unified vision of fatigue by considering plastic dissipated energy per cycle as a damage indicator. Results coming from temperature evolution measurements under cyclic loadings reinforced this proposition but its principal drawback is its independence on mean stress effect. In HCF as well as in LCF, this effect is then often postulated with linear relations between shear stress and respectively dissipated energy and maximal hydrostatic pressure [7, 1]. The objective of this communication is to show that the explicit introduction of damage in the modeling enables to take into account the mean stress effect in fatigue, in HCF as well as in LCF.

In the next section, a new micromechanical approach in HCF recently proposed by Monchiet et al. [12] and based on the introduction of damage at the grain scale, is presented. Damage is assumed to be the result of microvoids, representing microcracks nucleation and growth along slip bands. The damage growth is here taken into account with a Rice and Tracey type evolution law [15]. A local approach of fatigue crack nucleation at the slip band-matrix interface is then proposed. It corresponds to a critical value of the porosity; the fatigue criterion adopted is then a condition of no crack-nucleation. In a third section, elements are proposed for the extension of this approach to a dissipative framework in HCF. By using the thermodynamics of irreversible processes and more particularly the heat coupled equation, simulations of the temperature evolutions in HCF regime under cyclic loadings with different stress ratios are realized in order to evaluate the role of the mean stress on thermomechanical couplings. Then, in a fourth section, the extension to LCF is proposed. Here, cyclic plasticity can be considered macroscopic, i.e. at the specimen or structure scale. Therefore, the general assumptions of the modeling are recalled and the plastic dissipation is defined. This expression is compared to Park and Nelson [14] approach in low cycle fatigue and to Amiable et al. [1] proposition in thermomechanical fatigue.

2. Damage modeling in HCF

2.1 Modeling basis

Under low macroscopic loading as in the context of HCF, in the case of FCC structures, the plastic behavior is generally characterized by the activation of a predominant slip system and more precisely by the formation of strain localization band which are the potential sites for microcracks nucleation. This predominant slip system activated is characterized by a plane, defined by a unit normal vector $\underline{n}$ and a slip direction $\underline{m}$. Let us introduce $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ respectively the microscopic (i.e. at the grain level) stress and strain fields. As classically, an additive decomposition of the total strain $\boldsymbol{\varepsilon}$ into elastic strain, $\boldsymbol{\varepsilon}^e$, and plastic strain, $\boldsymbol{\varepsilon}^p$, at the microscopic scale is adopted : $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^e + \boldsymbol{\varepsilon}^p$. The microscopic plastic strain tensor reads : $\boldsymbol{\varepsilon}^p = \gamma^p \boldsymbol{\Delta}$ where γ^p is the slip plastic strain and $\boldsymbol{\Delta}$ being the second order symmetric tensor $\boldsymbol{\Delta} = \underline{n} \overset{s}{\otimes} \underline{m} = \frac{1}{2}(\underline{n} \otimes \underline{m} + \underline{m} \otimes \underline{n})$. We assume that the plastic strain is described, as classically, by Schmid's law :

$$f = |\tau - x| - \tau_0 - R(\gamma^p_{cum}) \quad (1)$$

where x is the kinematic hardening variable and R, the isotropic hardening variable with $\gamma^p_{cum} = \int_0^t |\dot{\gamma}^p| dt'$.
In the particular case of persistent slip bands (PSB), the strain localization is also accompanied by a dislocation annihilation mechanism which leads to the formation of vacancies along such bands which has been modeled by Essmann et al. [8] by defining a porosity associated to this mechanism, η_a, given by :

$$\eta_a(\gamma^p_{cum}) = A_0 \left\{ k_a \gamma^p_{cum} - 1 + \exp\left(-k_a \gamma^p_{cum}\right) \right\} \quad (2)$$

As the transition from vacancy production to the formation of microcracks is not yet well understood, it is assumed that it is the result of the agglomeration and the growth of vacancies formed by the previous process. Damage along slip bands is then the result of both two mechanisms: vacancies production and voids growth which is the result of the combined effect of the slip-like plastic activity and pressure. The total porosity at the grain scale, η, is then decomposed into two terms corresponding respectively to nucleation and the growth part as follows: $\eta = \eta_a + \eta_g$. However, in many cases, PSBs are not observed at the microstructure scale. Particularly, in a lot of engineering cases, it is well known that initial defects, as inclusions, precipitates, are the origin of the fatigue crack initiation. The proposed model is however still acceptable, but the definition of η_a has to be adapted, for example by taking into account an initial porosity due to the process.

As voids nucleations and growth induces volume change, the plastic strain at the grain scale can be decomposed in the following form, $\mathbf{1}$ being the second order identity tensor :

$$\boldsymbol{\varepsilon}^p = \gamma^p \boldsymbol{\Delta} + \varepsilon^p_h \, \mathbf{1} \quad (3)$$

where the volumetric plastic strain ε_h^p due to voids growth is related to η_g by using mass balance equation: $\eta_g = 1 - \exp(-3\varepsilon_h^p)$. A local fatigue criterion corresponding to no crack initiation is then defined by considering a critical value of the porosity, η_c, ideally corresponding to a critical crack size at the slip bands/matrix interface:

$$\eta_a + \eta_g < \eta_c \tag{4}$$

It appears that the determination of the total porosity η requires the calculation of ε_h^p.

2.2 Damage growth

Void growth in single crystal is the result of the activation of multiple slip activity around the circumference of the void. In order to derive a void growth model, single crystal is replaced by an equivalent von Mises material. A first step of the modeling consists in the consideration of a single void growth in an infinite perfectly plastic medium using the well known Rice and Tracey approach [15]. Plastic activity is assumed to be decomposed into an homogeneous plastic strain (the predominant slip system activated) and a heterogeneous symmetric plastic strain which accounts for multiple slip activity around the circumference of the void. The volumetric plastic strain ε_h^p, is the result of the combined action of pressure and the predominant slip plastic strain activity and is given by the following equation (see [12] for more details) :

$$\dot{\varepsilon}_h^p = \eta \frac{1}{2\sqrt{3}} \sinh\left\{\frac{\sqrt{3}}{2}\frac{\sigma_h}{\tau_0}\right\} \dot{\gamma}_{cum}^p \tag{5}$$

where σ_h is the hydrostatic part of the microscopic stress tensor and γ^p is described here by Schmid'law (1).

2.3 Determination of the macroscopic fatigue criterion

In the HCF framework of Dang Van [7], a plastic inclusion is supposed embedded in an elastic matrix and to describe the relation between macroscopic and mesoscopic fields, the Lin-Taylor scheme is used [7]. It supposes the equality between the macroscopic strain tensor $\boldsymbol{E}$ and $\boldsymbol{\varepsilon}$, the mesoscopic one. In the same context of very localized plasticity, the more generalized self-consistent scheme of Kröner can be used and includes the Lin-Taylor model [12]. By considering the same isotropic elastic behavior at both scales, the Kröner scheme gives the following relation between the mesoscopic and macroscopic stress fields :

$$\boldsymbol{\sigma} = \boldsymbol{\Sigma} - \mathbb{C} : (\mathbb{I} - \mathbb{P} : \mathbb{C}) : \boldsymbol{\varepsilon}^p \tag{6}$$

where $\mathbb{C}$ and $\mathbb{P}$ are respectively the fourth rank elastic moduli and Hill tensors and $\mathbb{I}$ is the fourth order identity tensor. Following Dang Van's reasoning, the elastic shakedown at the grain scale is a first *necessary* no crack initiation condition. The criterion (4) is a *sufficient* condition [12]. The shakedown condition (the cumulated plastic strain p is bounded) enables simplifications [12] and a final expression of the macroscopic fatigue criterion, deduced from (4) is the following:

$$A_0(k_a p - 1 + \exp(-k_a p)) + 3\varepsilon_h^p < \eta_c \tag{7}$$

In the general case, the different variables are computed after numerical integration. Some cases can however be determined analytically, in particular affine loadings characterized by: $\mathbf{\Sigma}(t) = \mathbf{\Sigma}_a \ \sin(\omega t) + \mathbf{\Sigma}_m$. By considering the Lin-Taylor scheme and a linear isotropic hardening model, the following expression of the criterion can be obtained for the particular case of an alternated torsion loading with a mean pressure:

$$\frac{T_a}{\tau_0} + \alpha\frac{\Sigma_{h,m}}{\tau_0} < \beta; \text{ with : } \alpha = \frac{R_0}{A_0 k_a k}; \ \ \beta = \frac{R_0}{A_0 k_a \tau_0}\left\{\eta_c + A_0\left\{1 + \frac{\tau_0 k_a}{R_0}\right\}\right\}$$

T_a is the macroscopic shear amplitude and $\beta\tau_0$ corresponds in this case to the fatigue limit under alternated torsion. This expression corresponds exactly to the criterion proposed by Dang Van [7], establishing a linear relation between shear stress and hydrostatic pressure. The criterion proposed by Monchiet et al. [12] is therefore a micromechanical based generalization, which exhibits explicitly the role of the mean and alternated part of the hydrostatic stress, as shown also in [13]. An illustration of this approach in an other example of macroscopic affine loading is presented on figure 1 where a repeated torsion with a mean tension is considered [16]. The results clearly shows the good capability of the model to recover explicitly the effect of the mean stress. In the next section, the extension of this approach to a dissipative framework is detailed.

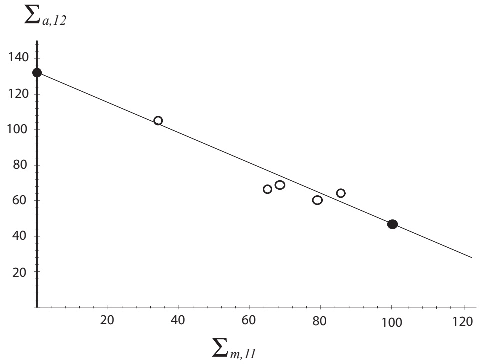

Figure 1: Repeated torsion with mean tension on En25T steel: comparison of the present criterion with the experimental data (o) of Ros [16]. The two experimental points used for the global identification of the model's parameters are in black.

3. Plastic dissipation in HCF

3.1 The plastic dissipation in the plasticity-damage framework

In the previous section 2.3, the considered volume of material is elastic and the plasticity is localized is some grains, represented as a plastic inclusion in an elastic matrix. This assumption was justified as the effect of very small macroscopic plastic strains is negligible in the interaction law (6) for the estimation of the macroscopic stresses. However, it is no more the case for the estimation of the macroscopic dissipation. Therefore, a macroscopically plastic volume of matrix-inclusion type is considered in this section; f_v is the volume fraction of inclusion. Let $\boldsymbol{E}^p$ be the macroscopic plastic strain tensor. For sake of simplicity, a "mean" grain is assumed for the plastic inclusion and the equation (3) becomes: $\boldsymbol{\varepsilon}^p = \boldsymbol{\varepsilon}^p_d + \varepsilon^p_h\, \boldsymbol{I}$, where $\boldsymbol{\varepsilon}^p_d$ is the deviatoric plastic strain associated to the plastic slip activity. Then, the interaction law (6) becomes:

$$\boldsymbol{\sigma} = \boldsymbol{\Sigma} - \mathbb{C} : (\mathbb{I} - \mathbb{P} : \mathbb{C}) : (\boldsymbol{\varepsilon}^p_d + \varepsilon^p_h\, \boldsymbol{I} - \boldsymbol{E}^p) \tag{8}$$

In the particular case of an idealized spherical inclusion, $\mathbb{P}$ reads:

$$\mathbb{P} = \frac{a}{3K}\mathbb{J} + \frac{b}{2\mu}\mathbb{K} \quad \text{with:} \quad a = \frac{3K}{3K+4\mu} \quad \text{and} \quad b = \frac{6}{5}\frac{K+2\mu}{3K+4\mu}$$

where $\mathbb{J} = \frac{1}{3}\boldsymbol{I}\otimes\boldsymbol{I}$ and $\mathbb{K} = \mathbb{I} - \mathbb{J}$. K is the bulk modulus, $K = 3\lambda + 2\mu$ and λ, μ are the Lamé's parameters. As a first approximation, one can also consider that $\boldsymbol{E}^p = \langle \boldsymbol{\varepsilon}^p \rangle = f_v \boldsymbol{\varepsilon}^p$; the interaction law (8) conducts to the following relation between $\boldsymbol{\sigma}$ and $\boldsymbol{\Sigma}$:

$$\boldsymbol{\sigma} = \boldsymbol{\Sigma} - (1 - f_v)(2\mu^* \boldsymbol{\varepsilon}^p_d + 3K^* \varepsilon^p_h\, \boldsymbol{I})$$

where $K^* = K(1-a)$ and $\mu^* = \mu(1-b)$. One can also remark that $tr(\dot{\boldsymbol{\sigma}}) = tr(\dot{\boldsymbol{\Sigma}}) - 9K^*(1-f_v)\dot{\varepsilon}^p_h$. Moreover, the mesoscopic plastic dissipation is defined as:

$$\boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^p = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^p_d + 3\sigma_h \dot{\varepsilon}^p_h \tag{9}$$

and the macroscopic one is deduced from the previous relations:

$$\boldsymbol{\Sigma} : \dot{\boldsymbol{E}}^p = f_v\,(\boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^p_d + 3\sigma_h \dot{\varepsilon}^p_h + 2\mu^* \boldsymbol{\varepsilon}^p_d : \dot{\boldsymbol{\varepsilon}}^p_d + 9K^* \varepsilon^p_h \dot{\varepsilon}^p_h) \tag{10}$$

3.2 Temperature evolutions and mean stress effect

The thermodynamics of irreversible processes (TIP) enables the determination of the heat coupled equation connecting the thermal field and the mechanical fields (see for example [10]). By defining a free energy Ψ depending on state variables α_j, $(j = 1, 2, \ldots, n)$, this heat equation can be written as:

$$\rho C_v \dot{T} = r + \mathrm{div}(\boldsymbol{k} \cdot \vec{\mathrm{grad}}(T)) + \left(\boldsymbol{\Sigma} : \dot{\boldsymbol{E}} + \rho T \frac{\partial^2 \Psi}{\partial T \partial \alpha_j}\dot{\alpha}_j - \rho \frac{\partial \Psi}{\partial \alpha_j}\dot{\alpha}_j\right) \tag{11}$$

where ρ is the density, C_v the specific heat, T the absolute temperature, r the distribution of external heat sources, $\boldsymbol{k}$ the second rank tensor of thermal conductivity. The intrinsic dissipation is $\Phi = \boldsymbol{\Sigma} : \dot{\boldsymbol{E}} - \rho \frac{\partial \Psi}{\partial \alpha_j}\dot{\alpha}_j$. One consider here that the inclusion admits an elastoplastic behavior and, for sake of simplicity, a linear kinematic hardening. As the plasticity is supposed confined in a few grains, the volume fraction of plastic inclusion, f_v, is considered low. The matrix is considered perfectly plastic and its volume fraction is $(1 - f_v)$. The state variables are then $\boldsymbol{E}^e$, $\boldsymbol{\varepsilon_e}$ and $\boldsymbol{\varepsilon_p}$ and their thermodynamical associated forces, $\boldsymbol{\Sigma}$ and $\boldsymbol{\sigma}$ the macro- and mesoscopic stress tensors and $\boldsymbol{x}$ the stress tensor associated to the kinematic hardening. Consequently, the Helmholtz's free energy is split into two parts: Ψ_{mat} associated to the matrix and Ψ_{in} associated to the inclusion. Therefore, following [5], the heat balance equation can then be simplified as:

$$\rho C_v \dot{T} - \mathrm{div}(\boldsymbol{k}\cdot \vec{\mathrm{grad}}(T)) = r - \alpha\, T\, \mathrm{tr}(\dot{\boldsymbol{\Sigma}}) - 9K\alpha^2 T\dot{T} + 9\, K^*(1-f_v)\, \alpha\, T\dot{\varepsilon}_h^p + \Phi \tag{12}$$

where α is the thermal expansion coefficient. The first term on the right side corresponds to an external source. The sum of the second and third terms is the thermoelastic coupling and the following term is a coupling between heat and damage. The last term is the intrinsic dissipation, Φ. With the previous assumptions, Φ is now equal to:

$$\Phi = \boldsymbol{\Sigma} : \dot{\boldsymbol{E}}^p + f_v \left[\boldsymbol{\Sigma} : \dot{\boldsymbol{E}}^e - \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^e - \frac{2}{3} h \boldsymbol{\varepsilon}^p : \dot{\boldsymbol{\varepsilon}}^p \right]$$

where h is the mesoscopic hardening modulus. The previous relations between mesoscopic and macroscopic mechanical fields conduct finally to:

$$\begin{aligned}\Phi = \; & f_v \left[\boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^p - \frac{2}{3} c \boldsymbol{\varepsilon}^p : \dot{\boldsymbol{\varepsilon}}^p \right] + f_v \left(2\mu^*(2-b) \boldsymbol{\varepsilon}_d^p : \dot{\boldsymbol{\varepsilon}}_d^p + 3K^*(2-a) \varepsilon_h^p : \dot{\varepsilon}_h^p \right) \\ & + f_v \left[(1-b) \left(\boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}_d^p + \dot{\boldsymbol{\sigma}} : \boldsymbol{\varepsilon}_d^p \right) + (1-a) \left(3\sigma_h : \dot{\varepsilon}_h^p + \dot{\sigma}_h : \varepsilon_h^p \right) \right]\end{aligned}$$

In this expression the f_v^2 and f_v^3 terms have been neglected. By taking into account this expression of the intrinsic dissipation, the equation (12) can be solved. The principle is detailed in [5] in the particular case of purely reversed tension-compression tests ($R_\sigma = -1$), without external heat sources ($r = 0$). These simulations are inspired by the experimental work of Boulanger [2] who realized cyclic tests with temperature measurements on a dual-phase steel at a 50 Hz frequency with different stress amplitude of 180, 250 and 300 MPa and different stress ratios. The specimens are flat and the conduction phenomenon is considered isotropic ($\boldsymbol{k} = k\boldsymbol{I}$). The estimation of the heat exchanges is similar as Boulanger [2] ones: $-\mathrm{div}(\boldsymbol{k}\mathrm{grad}T) \simeq \rho C \dfrac{T}{\tau_{eq}}$. The constant τ_{eq} is representative of the exchanges with the environment: convection with the ambient air and with the grip system. The heat coupled equation (12) can then be simplified and numerically solved with an explicit scheme in time for the temperature estimation. The plastic strain increments are computed beforehand with a radial return type implicit scheme. Details can be found in [5].

Here, tests at different stress ratios are simulated, in order to estimate the influence of the damage modeling on the temperature evolutions. The figure 2 shows the evolution of the mean temperature during cycling. One can observe that the stress ratio has an influence on this evolution, due to the spherical term in the relation (10). This influence is quantified for different stress ranges and different stress ratios on figure 3 and is qualitatively comparable to the experimental influence observed by Boulanger [2]. This shows that the proposed plasticity-damage framework enables a good representation of the mean stress effect in HCF. In the next section, this framework is now extended to LCF context, where plasticity is generalized at the scale of the material volume.

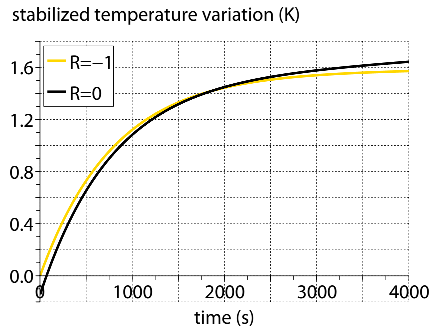

Figure 2: Evolution of the mean temperature during the cyclic loading for two cyclic tests at different stress ratios. The stress amplitude is $180 MPa$.

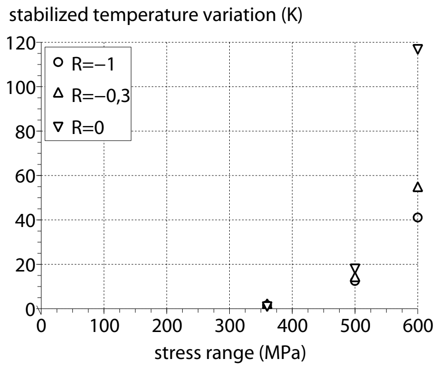

Figure 3: Evolution of the mean stabilized temperature as a function of the loading amplitude and the stress ratio.

4. Plastic dissipation in LCF

4.1 Extension of the damage model to generalized plasticity

In the LCF case, a generalized plasticity is considered at the scale of the material volume. Then, $\boldsymbol{\Sigma}$, $\boldsymbol{E}$ and $\boldsymbol{E}^p$ denote respectively the macroscopic stress, strain and plastic strain tensors. An additive decomposition of the total strain $\boldsymbol{E}$ into elastic strain, $\boldsymbol{E}^e$, and plastic strain, $\boldsymbol{E}^p$ is also adopted: $\boldsymbol{E} = \boldsymbol{E}^e + \boldsymbol{E}^p$. In LCF, it is well known that micro-cracks can quickly initiate in a specimen and the lifetime corresponds then to a slow crack growth until a macroscopic size. Therefore, similarly as for the previous proposed HCF model, lets denote by η_a the porosity associated to the initial small cracks initiation and η_g the porosity corresponding to the crack growth. The total porosity in the specimen is: $\eta = \eta_a + \eta_g$. Then the relation (3) becomes:

$$\boldsymbol{E}^p = \boldsymbol{E}^p_d + E^p_h \, \mathbf{1} \tag{13}$$

$\boldsymbol{E}_d^p$ corresponds to the macroscopic deviatoric plastic strain tensor and is deduced from the plastic criterion and from the normality law. The volumetric plastic strain E_h^p is due to crack growth and is related to η_g by using mass balance equation :

$$\eta_g = 1 - \exp(-3E_h^p) \tag{14}$$

The evolution of the hydrostatic plastic strain E_h^p is then required and can be similarly inspired by Rice and Tracey's work [15] dedicated to the growth rate of a spherical void embedded in an infinite perfect plastic matrix. A classical approximation of this evolution law available for high triaxiality ratios is the following:

$$\frac{\dot{a}}{a} = 0.283 \exp\left\{\frac{3}{2}\frac{\Sigma_h}{\Sigma_0}\right\}\dot{p} \tag{15}$$

where a is the void's radius, $\Sigma_h = \frac{1}{3}tr(\boldsymbol{\Sigma})$, Σ_0 is the macroscopic yield stress and $\dot{p} = \left[\frac{2}{3}\dot{\boldsymbol{E}}_d^p : \dot{\boldsymbol{E}}_d^p\right]^{1/2}$. In order to take into account arbitrary triaxiality ratios, Monchiet [11] obtained the following closer approximation, inspired by the Gurson's assumptions [9]:

$$\frac{\dot{a}}{a} = \frac{5}{9}\sinh\left\{\frac{3}{2}\frac{\Sigma_h}{\Sigma_0}\right\}\dot{p} \tag{16}$$

Then, the mass balance equation imposes that $\dot{E}_h^p = \eta\frac{\dot{a}}{a}$

4.2 Plastic work and mean stress effect

In LCF and in thermomechanical fatigue, crack initiation criterion based on plastic and/or elastic energy were recently proposed [3, 14, 1]. In the first case, Charkaluk et al. [3] proposed the dissipated plastic energy per cycle Δw_d^p as a damage indicator Φ_1 in thermomechanical fatigue:

$$\Phi_1 = \Delta w_d^p = \int_{cycle} \boldsymbol{\Sigma} : \dot{\boldsymbol{E}}_d^p dt$$

This proposition has the particular inconvenient, in a classical deviatoric plastic framework, to be independent of mean stress effects as Δw_d^p do not depend on hydrostatic pressure. Park et al [14] propose then to combine in Φ_2 the plastic dissipated energy Δw_d^p with the deviatoric elastic energy Δw_d^e and triaxiality factors TF_s and TF_m as follow:

$$\Phi_2 = 2^{k_1 TF_s}\Delta w_d^p + 2^{k_2 TF_m}\Delta w_d^e$$

with $TF_s = \frac{\Sigma_{h,a}}{J_2}$, $TF_m = \frac{\Sigma_{h,m}}{J_2}$ and k_1, k_2 are material parameters. The terms $\Sigma_{h,a}$ and $\Sigma_{h,m}$ are respectively the amplitude and the mean value of the hydrostatic pressure and J_2 is the second invariant of the deviatoric part of the stress tensor. To take into account mean stress effect, Amiable et al. [1] proposed recently a more simple indicator Φ_3 inspired by the linear relation proposed in HCF by Dang Van [7] between the shear stress amplitude and the maximal part of the hydrostatic pressure. Then, Φ_3 takes the following form:

$$\Phi_3 = \Delta w_d^p + \alpha \Sigma_{h,max}$$

Even if Φ_2 and Φ_3 exhibits a dependence on hydrostatic pressure, the forms of these damage indicators are however postulated. Therefore, it is interesting to define the macroscopic plastic dissipation d^p associated to the proposed damage based framework. Then, the plastic dissipation is precisely defined by $d^p = \mathbf{\Sigma} : \dot{\boldsymbol{E}}^p$, and, taking into account the equations (13) and (16), this conducts to:

$$d^p = \mathbf{\Sigma} : \dot{\boldsymbol{E}}_d^p + 3\Sigma_h : \dot{E}_h^p \text{ with } \dot{E}_h^p = \eta \frac{5}{9} \sinh \left\{ \frac{3}{2} \frac{\Sigma_h}{\Sigma_0} \right\} \dot{p}$$

It can then be underlined that the expression of d^p depends on the classical deviatoric plastic dissipation $\mathbf{\Sigma} : \dot{\boldsymbol{E}}_d^p$ which is present in the three previous damage indicators [3, 14, 1] but depends also explicitly on the hydrostatic pressure Σ_h. The next step of this work will then consist in a comparison of this theoretical expression of the plastic dissipation with experimental results coming from literature [14].

5. Conclusion

In this communication, the particular role of the mean stress in fatigue has been studied in HCF and LCF with the explicit introduction of damage in the modeling, inspired by the recent work of Monchiet [12]. In HCF, damage mechanisms are introduced in a multiscale approach in order to represent the initiation and growth of micro-cracks along the slip bands. The expression of the fatigue criterion, corresponding to a critical porosity, depends explicitly on the hydrostatic pressure. In the particular case of cyclic affine loadings and under particular assumptions, the Dang Van's criterion is recovered. This model is introduced in a thermodynamical framework which conducts to the heat coupled equation. It is shown that this model enables also a good representation of the mean stress influence on temperature evolutions of a specimen under cyclic loading with different stress ratio. An extension of this approach to LCF is proposed, based here on the macroscopic plastic dissipation. The introduction of damage enables to exhibit explicitly the role of hydrostatic pressure. Some work are in progress in order to compare the theoretical dissipative framework to experimental results. A coupling between damage and plasticity, inspired by the Gurson's work [9], is also possible [13]. This general framework seems to be a good candidate to propose a more unified vision of the fatigue phenomenon [6, 4].

[1] S. Amiable, S. Chapuliot, A. Constantinescu, and A. Fissolo. A comparison of lifetime prediction methods for a thermal fatigue experiment. *Int. J. Fat.*, 28(7):692–706, 2006.

[2] T. Boulanger. *Analyse par thermographie infrarouge des sources de chaleur induites par la fatigue des aciers.* Phd thesis, spécialité : mécanique, Université de Montpellier II, France, 2004.

[3] E. Charkaluk and A. Constantinescu. Energetic approach in thermomechanical fatigue for silicon molybdenum cast-iron. *Materials at High Temperatures*, 17(3):373–380, 2000.

[4] E. Charkaluk and A. Constantinescu. Dissipation and fatigue damage. *MP Materialprüfung*, 46(10):524–530, 2004.

[5] E. Charkaluk and A. Constantinescu. Estimation of thermoplastic dissipation in high cycle fatigue. *C.R. Mécanique*, 334(6):373–379, 2006.

[6] A. Constantinescu, K. Dang Van, and H. Maïtournam. A unified approach for low and high cycle fatigue based on shakedown concepts. *Fat. Frac. Engng. Mat. Struct.*, 26(6):561–568, 2003.

[7] K. Dang Van. Sur la résistance à la fatigue des métaux. *Sciences Technique Armement*, 47(3), 1973.

[8] U. Essmann and H. Mughrabi. Annihilation of dislocations during tensile and cyclic deformation and limits of dislocation densities. *Phil. Mag. A*, 40(6):731–756, 1979.

[9] A. L. Gurson. Continuum Theory of Ductile Rupture by Void Nucleation and Growth: Part I-Yield Criteria and Flow Rules for Porous Ductile Media. *J. Eng. Mater. Technol.*, 99:2–15, 1977.

[10] G. A. Maugin. *The thermomechanics of plasticity and fracture*. Cambridge University Press, Cambridge, 1992.

[11] V. Monchiet. *Contributions à la modélisation micromécanique de l'endommagement et de la fatigue des métaux ductiles*. Phd thesis, spécialité : Mécanique, Université de Lille I, 2006.

[12] V. Monchiet, E. Charkaluk, and D. Kondo. A plasticity-damage based micromechanical modelling in high cycle fatigue. *C.R. Mécanique*, 334(2):129–136, 2006.

[13] V. Monchiet, E. Charkaluk, and D. Kondo. A micromechanical explanation of the mean stress effect in High Cycle Fatigue (HCF). *Mech. Res. Com.*, 35(6):383–391, 2008.

[14] J. Park and D. Nelson. Evaluation of an energy-based approach and a critical plane approach for predicting constant amplitude multiaxial fatigue life. *Int. J. Fat.*, 22:23–39, 2000.

[15] J. R. Rice and D. M. Tracey. On a ductile enlargement of voids in triaxial stress fields. *J. Mech. Phys. Solids*, 17:201–217, 1969.

[16] M. Ros. Die bruchegefahr fester köper. Technical report, EMPA Bericht 173, Zurich, 1950.