A modified approach for multiaxial fatigue damage prediction

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A new high-cycle fatigue criterion based on the critical plane approach is proposed in this paper. Unlike most of the other multiaxial fatigue criteria based on the critical plane approach, the critical plane is obtained by 3D analysis of stress states. Also, the old critical plane criteria are modified in order to predict life assessment especially for three-dimensional stress fields. Furthermore, a variable critical plane concept is introduced and a new life assessment algorithm is presented. The stress analysis is carried out in the finite element method to achieve both computational efficiency and accuracy. The prediction life is obtained with MALTAB software based on new method. Finally, experimental fatigue lives are compared with prediction life by modified multiaxial fatigue criteria. Experimental results are prepared for both proportional and non-proportional cases. The results especially the results of orientation of critical planes in variable conditions are discussed in this paper.

1. Introduction

All fatigue damage models for crack initiation analysis can be classified into three groups: stress approach, strain approach and energy approach. The stress approach has been commonly used for high-cycle fatigue problems. The stress-based approaches can be divided into four groups based on empirical equivalent stress, stress invariants, average stress and critical plane stress.

Although there are many proposed models for multiaxial fatigue damage modeling, most of them are limited to specific materials or loading conditions. Some of them cannot predict the initial crack orientation, which is another distinct characteristic of multiaxial fatigue damage compared with the uniaxial fatigue problem. No existing multiaxial fatigue damage model is universally accepted.

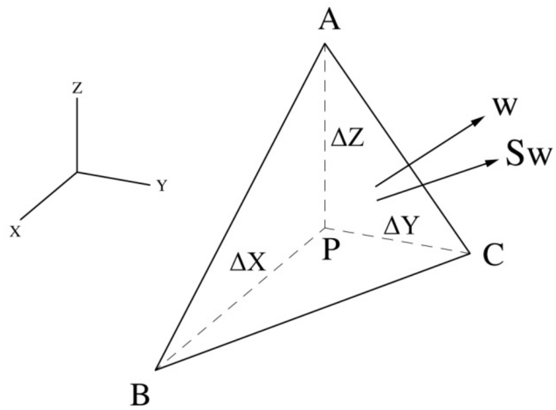
The fatigue process of mechanical components under service loading is stochastic in nature. In recent decades; numerous studies have attempted to develop multiaxial fatigue damage criteria. Despite the differences in different multiaxial models, the general idea is similar, which is to reduce the complex multiaxial stress state to an equivalent uniaxial stress state or an equivalent damage scalar. Thus the fatigue life is assessed based on the equivalent parameter.

In recent years, criteria based on the critical plane approach for multiaxial fatigue evaluation are becoming more popular because they generally give more accurate predictions of the fatigue damage.

The main purpose of this paper is to propose a new life assessment algorithm for the high cycle fatigue based on the critical plane approach. The presented theories are variants of Sines, McDiarmid, and Findley theories. In contrast to some previously presented algorithms, the plane on which the fatigue failure is more likely to occur and is considered as the critical plane is determined through succession of time. In the present research, a plane among all cross sectional planes on which variation of one stress component or a combination of several stress components lead to the maximum accumulated damage is determined and used in a fatigue life assessment procedure.

2. A modified approach for famous critical plane theories

Fig. (1) Demonstrates a Quadra-face element parted from an article which is under multi-axial loading. Point P is under three-dimensional stress and locates on element's corner.



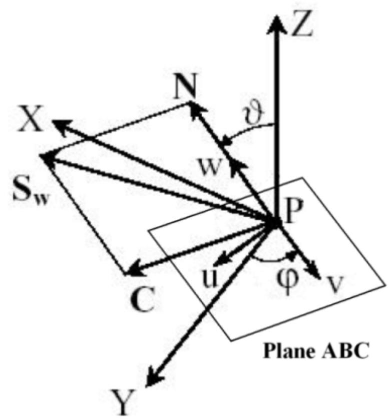
(Fig. 1) The position of Point P and resulted stress vector

In the assumed case in which the dimensions of element approximate to zero a plane like ABC will cross from P point. To adjust the position of the inclined plane besides from the angle of ABC plane unit vertical vector related to axis Z of XYZ system an additional definition is required in the same system. Therefore, a second clockwise system WUV is defined in relation to XYZ system according to Fig. (2) In the new system WUV, axe W is plane ABC's normal vector and has an angle θ with axe Z. Axe U is a vector is resulted from cross section of plane ABC and the plane which included normal vector W and axe Z. the vector V is normal to axes U and W concerning to plane WUV which is clockwise. Clockwise system UVW will find its form concerning to abovementioned expression. Based on above definition vector V locates in plane X-Y.

So, with refer to presented expression of coordination systems UVW and XYZ, conversion matrix of system PUVW into PXYZ will be as following:

$$\left.\begin{array}{c} e_i' = Q_{mi} e_m \\ Q_{mi} = e_m \,.\, Q e_i = e_m \,.\, e_i' = cos(e_m, e_i') \end{array}\right\} \qquad \text{(Eq. 1)}$$

$$\Rightarrow \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} cos\theta cos\varphi & -sin\varphi & sin\theta cos\varphi \\ cos\theta sin\varphi & cos\varphi & sin\theta sin\varphi \\ -sin\theta & 0 & cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$



(Fig. 2) Coordination systems XYZ, UVW

In other word, it could be expressed that the matrix Q in Eq. (1) is an interpretation of conductive cosines of coordinate system PUVW unit vectors in coordinates PXYZ as a function of φ and θ that varies in $0<\theta<180$, $0<\varphi<360$ domain. Normal unit vector of inclined plane ABC could be express based on matrix Q according to Eq. (2) if stress in point P defines by stress tensor Sp according to Eq. (3).

$$S_p(t) = \begin{bmatrix} \sigma_x & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_y & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_z \end{bmatrix} \qquad \text{(Eq. 2)}$$

$$w = \begin{bmatrix} sin\theta cos\varphi \\ sin\theta sin\varphi \\ cos\theta \end{bmatrix} \qquad \text{(Eq. 3)}$$

Then the quantity of traction $S_w(t)$ will be evaluated from Eq. (4). Heretofore,c_θ s_θ denote $sin\theta$, $cos\theta$.

$$S_w(t) = S_p \, . w = \begin{bmatrix} s_\theta\left(\sigma_x c_\varphi + \sigma_{xy} s_\varphi\right) + \sigma_{xz} c_\theta \\ s_\theta\left(\sigma_{xy} c_\varphi + \sigma_y s_\varphi\right) + \sigma_{yz} c_\theta \\ s_\theta\left(\sigma_{xz} c_\varphi + \sigma_{yz} s_\varphi\right) + \sigma_z c_\theta \end{bmatrix} \qquad \text{(Eq. 4)}$$

As it shows in Fig. (2) Stress vector could be fragmented into two vertical and shear components normal and tangent to plane ABC. Vertical stress vector oriented to axe W in coordination system UVW could be evaluated from Eq. (5).

$$N(t) = (w \, . S_w) w \qquad \text{(Eq. 5)}$$

$$= \begin{bmatrix} s_\theta c_\varphi \left[s_\theta^2\left(\sigma_x c_\varphi^2 + \sigma_y s_\varphi^2 + \sigma_{xy} s_{2\varphi}\right) + s_{2\theta}\left(\sigma_{xz} c_\varphi + \sigma_{yz} s_\varphi\right) + \sigma_z c_\theta^2 \right] \\ s_\theta s_\varphi \left[s_\theta^2\left(\sigma_x c_\varphi^2 + \sigma_y s_\varphi^2 + \sigma_{xy} s_{2\varphi}\right) + s_{2\theta}\left(\sigma_{xz} c_\varphi + \sigma_{yz} s_\varphi\right) + \sigma_z c_\theta^2 \right] \\ c_\theta \left[s_\theta^2\left(\sigma_x c_\varphi^2 + \sigma_y s_\varphi^2 + \sigma_{xy} s_{2\varphi}\right) + s_{2\theta}\left(\sigma_{xz} c_\varphi + \sigma_{yz} s_\varphi\right) + \sigma_z c_\theta^2 \right] \end{bmatrix}$$

The quantity of stress vector loaded on plane ABC could be evaluated from Eq. (6).

$$C(t) = S_w - N = S_p \, . w - (w \, . S_w)w \qquad \text{(Eq. 6)}$$

$$\begin{bmatrix} s_\theta\left[c_\varphi\left(\sigma_x c_\theta^2 + (\sigma_x - \sigma_y)s_\theta^2 s_\varphi^2\right) + \sigma_{xy} s_\varphi\left(1 - 2s_\theta^2 c_\varphi^2\right)\right] \ldots \\ s_\theta\left[s_\varphi\left(\sigma_y c_\theta^2 + (\sigma_y - \sigma_x)s_\theta^2 c_\varphi^2\right) + \sigma_{xy} c_\varphi\left(1 - 2s_\theta^2 s_\varphi^2\right)\right] \ldots \\ \sigma_z c_\theta s_\theta^2 - c_{2\theta} s_\theta\left(\sigma_{xz} c_\varphi + \sigma_{yz} s_\varphi\right) \ldots \\ \ldots + \sigma_{xz} c_\theta - s_\theta c_\varphi\left[s_{2\theta}\left(\sigma_{xz} c_\varphi + \sigma_{yz} s_\varphi\right) + \sigma_z c_\theta^2\right] \\ \ldots + \sigma_{yz} c_\theta - s_\theta s_\varphi\left[s_{2\theta}\left(\sigma_{xz} c_\varphi + \sigma_{yz} s_\varphi\right) + \sigma_z c_\theta^2\right] \\ \ldots - c_\theta s_\theta^2\left(\sigma_x c_\varphi^2 + \sigma_y s_\varphi^2 + \sigma_{xy} s_{2\varphi}\right) \end{bmatrix}$$

In view of the fact that the article is under multi-dimensional stress and this loading varies via time, stress tensor S_p is a function of time and vectors $S_w(t)$, $N(t)$ & $C(t)$ vary with time too. Therefore, position and angles of critical plane will be variable according to coordination. In previous studies, the position of critical plane is assumed fixed. For an ant phase sinusoidal loading on a plane, Carpinteri and Spagnoli presented an equation [1]. In form of a square combination of maximum vertical stress, they present the domain of shear stress functioned on critical plane as a fatigue criterion under cyclic loading.
The above assumption is based on a broad range of empirical job and fatigue experiments, in this study, the results of Gough [2]. study are used. He found that average shear stress has no effect on articles fatigue life time. Also average vertical stress tension decrease the fatigue resistance effectively and average contraction stress increase fatigue life time. Another factor in presenting above relation is arise from Papadopolous [3] and Lasser and Froustey [4] studies in which phase difference β between bending and twisting has no effect in fatigue limit of hard metals.
Carpinteri's Eq. is not proper for life estimation. Therefore, in present investigation Carpinteri's Eq. is multiplied by f_{-1} .The result is Eq. (7).

$$\sqrt{N_{max}^2 + \left(\frac{f_{-1}}{t_{-1}}\right)^2 C_a^2} \leq f_{-1} \qquad \text{(Eq. 7)}$$

In single axial condition the relation of rupture fatigue criterion has the simple form $f_N \leq f_{-1}$. So, multi-axial stress could be compared to single axial one by defining equivalent stress.
For each of the Findley [5], McDiarmid [6] and Matake [7] theories which have a criterion in common according to Eq. (8), an equivalent stress could be evaluated similar to Carpinteri-Spagnoli theory Eq. (9).

$$C_a(\varphi_c, \theta_c) + kN_{max}(\varphi_c, \theta_c) \leq \xi \qquad \text{(Eq. 8)}$$

$$\sigma_{eq} = \frac{f_{-1}}{\xi}\left(C_a(\varphi_c, \theta_c) + kN_{max}(\varphi_c, \theta_c)\right) \qquad \text{(Eq. 9)}$$

It is worth to note that each of the parameters of ξ and k should be evaluated based on theory which is used concurrently. Also in present study quantities C_a *and* $N_{\max}$ should be substituted based on three-dimensional stress, Eqs.(5, 6).

3. Components Fatigue Analysis Approach and Life Time Prediction

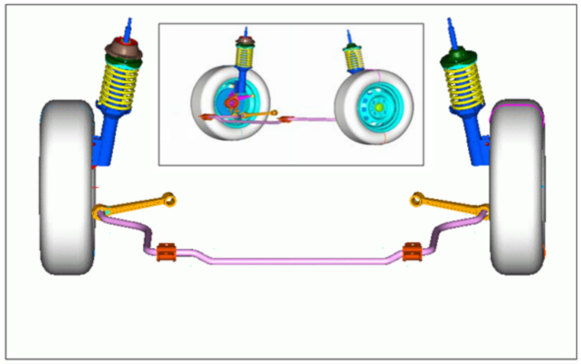
Various algorithms are presented for fatigue analysis of mechanical parts, especially automobile parts which are under random loadings. For having final result, it is required to acquire the history of stress arising from previous loadings using shear stress initially. This job could be accomplished by means of the history of variations in force components, torques or articles displacements using finite element software like ANSYS, intending exerted conditions and the material property of article. This stress record would be the input of software. It is used for an article life estimation which is under inadvertent three-dimensional stress. This software would be able to estimate the life of an article based on each of the theories of Carpinteri, Findley, Matake, McDiarmid, and VonMises by having the three-dimensional stress records and material properties as an input.

If an element of an article was being exerted under a stress record, based on the critical plane definition in each of the mentioned theories in each duration a critical plane would be existed that probably would not be critical in next duration anymore. Therefore, in an intended history of stress, the position of critical plane will be stationary and is a function of time. In previous studies, the critical plane is unique and is the plane which has the maximum quantity of rupture criterion. In all of them, the estimation of life is accomplished based on the variation of stress components on that plane.

In present study, it suggests that the variation of stress components evaluates on all of the planes and concerning to definition of equivalent stresses presented in previous section, the plane with maximum cumulative damage intends as a critical plane. Therefore, the provided software using the spherical coordination by change in angles φ, θ in $0<\varphi<360$, $0<\theta<180$ domain, calculates the vertical and shear components of stress and will provide the history of equivalents stress. Then, by using of numerating method of rain storm cycle the number of half-cycles together with the domain and average stress in each half-cycle will be calculated. Finally, the equivalent reverse stress using modified Goodman criterion will be calculated in corresponding cycles. To determine material's S-N curve and stress in corrected final fatigue limit the surface roughness, hardness and etc are required. So, the S-N curve could be corrected in attribution to case article. By having S-N curve and mirrored equivalent stress based on Miner's cumulative damage criterion that is used in automobile parts fatigue stress analysis, the life on each plan and the plane in which the life span in minimum will be calculated. The life span of this plan is the element's fatigue final duration under history intended loading. Among the various nodes, the node which has the minimum fatigue life span will determine the structures endurance.

4. The comparison of fatigue analysis results and life estimation of modified theories with empirical results of a stabilizer or antiroll bar

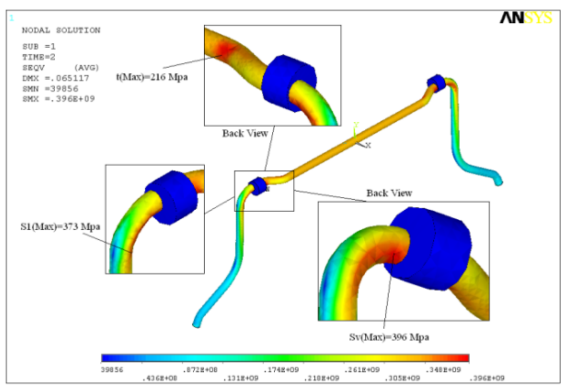
Schematic of an anti-roller bar which always is under oscillating bending and twisting loading is shown in Fig. (3). concerning to complicated shape of this article, a precise geometrical model is required to stress analysis. In making of this model the ANSYS software is used and element Solid95 is used in bar partitioning due to modeling ability of deformations and non-linear strains. After simulation it is determined that fundamental theories like Carpinteri, Findley, Matake and Mc-Diarmid are not suitable for life cycle calculation and modified theories are used in final result evaluation.



(Fig. 3) antiroll bar

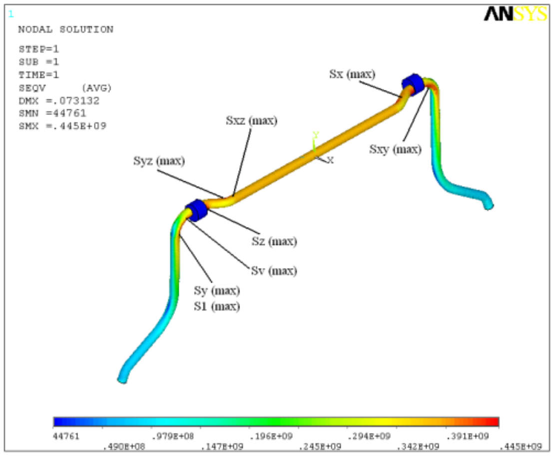
4.1 Life Prediction in fully reversal loading

The major part load exerted on anti-roller bar is normal on the plan which includes articles symmetry line. Also it is reported that after 10400 oscillating displacement cycle with 65 mm oscillation domain with sinusoidal nature, small ruptures will create on the surface. The abovementioned condition is used in software development. Fig. (4) Shows stress variation under this loading.



(Fig. 4) stress variation range in aphasic experiment under oscillating displacement with two-side 65mm domain

As it is shown the maximum stress is on bends of anti-roller bar and acquired stress in this point will be our reference in this study. Fig. (5) Shows fatigue analysis of nodes which have maximum stress components.



(Fig. 5) fatigue analysis of nodes which have maximum stress components

Life span of this nodes and plan with maximum fatigue damage which are calculated by software are shown in Table (1).

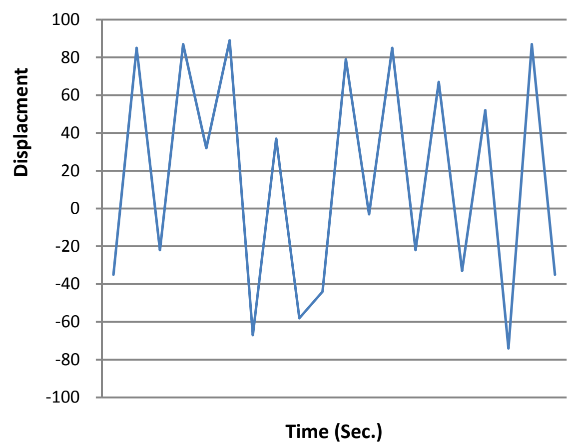
(Table 1) the results of anti-roller bar life prediction via cycle, using modified theories in mirrored loading

Maximum Stress	Element No	VonMises	Findley			Matake			McDiarmid			Carpinteri		
		life	life	θ	Φ	life	θ	Φ	life	θ	Φ	life	θ	Φ
Sx	1679	175,036	162,021	21	346	149,350	81	179	170,883	19	347	186,298	157	168
Sy,S1	13458	145,897	138,649	77	55	128,749	77	55	161,662	67	308	112,743	98	246
Sz	29	122,879	124,653	2	172	114,443	2	168	124,503	3	141	142,507	3	125
Sxy	145	Infinite	Infinite			Infinite			Infinite			Infinite		
Syz	242	112,157	157,290	15	131	143,466	15	132	143,867	14	135	131,317	10	163
SXZ	11	114,177	135,405	174	195	123,864	6	16	128,883	86	0	136,024	2	83
S2	4057	Infinite	Infinite			Infinite			Infinite			Infinite		
S3	13406	Infinite	Infinite			Infinite			Infinite			Infinite		
SV	6791	101,612	195,799	12	22	177,786	12	23	168,961	12	30	113,375	166	257

Referring to Table (1) it is acquired that critical node with minimum life span is node 29. Based on VonMises the node 6791 is critical node with maximum equivalent stress and based on Carpinteri theory node 13458 is critical node with least life prediction. Also there are obvious differences in equivalent stress in different theories.

4.2 Life estimation in Loading with Non-Zero Average Stress

For results evaluation concerning to random displacement the history of random displacement variation intended like Fig. (6).



(Fig. 6) displacement variation

This displacement loaded anti phase on two ends of anti-roller bar. Like fully reversal loading with zero average stress, stress variation calculates for all nodes and finally fatigue life span of those nodes that are under more severe critical condition will be calculated by software.

(Table 2) the results of fatigue life prediction of anti-roller bar via history of random loading by means of modified theories in loadings with non-zero average stress

Maximum Stress	Element No.	VonMises	Findley			Matake			McDiarmid			Carpinteri		
		life	life	θ	Φ	life	θ	Φ	life	θ	Φ	life	θ	Φ
Sx	1679	243486	141505	98	359	101013	22	347	183821	20	346	262931	23	348
Sy,S1	13458	162241	70557	69	305	60798	69	305	108671	102	234	83431	98	246
Sz	29	112692	84603	77	270	76161	77	270	87571	101	90	155946	3	125
Sxy	145	Infinite	Infinite			Infinite			Infinite			Infinite		
Syz	242	83616	178825	91	270	159657	15	129	161192	166	313	124938	10	163
Sxz	11	85057	123792	173	194	89506	7	13	118466	85	0	129532	2	83
S1	4057	Infinite	Infinite			Infinite			Infinite			Infinite		
S3	13406	Infinite	Infinite			Infinite			Infinite			Infinite		
Sv	6791	73067	284939	12	21	253390	12	22	237920	12	29	84000	14	77

As it shown in Table (2) based on theory of VonMises, the node 6791 is a critical plan with maximum equivalent stress. In despite of loading with non-zero average stress, in the same node other modified theories predict the critical plan's life span three times more.

Fig. (7) Shows components of exerted stress and Fig. (8) Shows equivalent stresses variation based on modified stress in a loading cycle for node 13458.

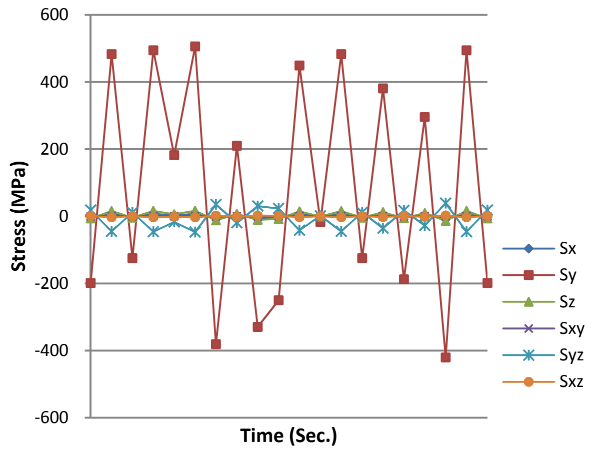


Fig. (7) Components of exerted stress for nod 13458

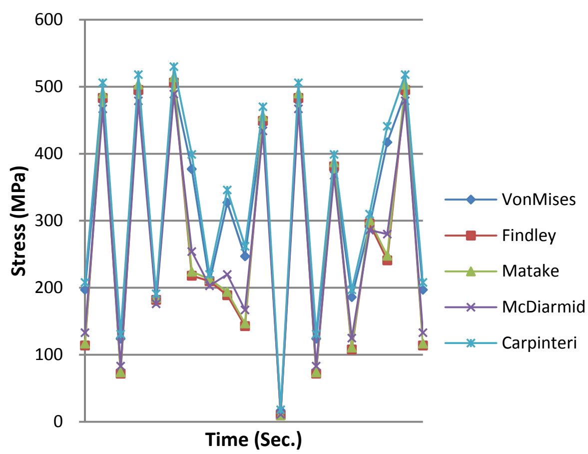


Fig. (8) Shows equivalent stresses variation based on modified stress in a loading cycle for node 13458

5. Conclusions

The main purpose of this paper is to propose a new life assessment algorithm for the high cycle fatigue based on the critical plane approach. In this regard, modified versions of the well-known theories are proposed and employed to enable fatigue life assessment of components with complicated geometries subjected to non-proportional random three dimensional stress fields. The presented theories are variants of Sines, McDiarmid, and Findley theories. In contrast to some previously presented algorithms, the plane on which the fatigue failure is more likely to occur and is considered as the critical plane is determined through succession of time. Many researchers have considered the orientation of the critical plane to be fixed. Other researchers have employed the stress intensity integral approach or the energy approach to avoid searching the critical plane. In the present research, a plane among all cross sectional planes on which variation of one stress component or a combination of several stress components lead to the maximum accumulated damage is determined and used in a fatigue life assessment procedure. An equivalent stress concept is used which considers the effects of the mean stress, phase shift, different frequencies of the stress components, etc. in conjunction with Miner-Palmgren linear accumulated damage rule to predict the fatigue lives. Surface finish, cross section size, and the surface hardness are considered to modify the fatigue strength values. The components are modeled in ANSYS FEM software and the stress components histograms are extracted. The rainflow cycle counting procedure is used to convert the effective stress history to an equivalent one with a zero mean stress. The obtained results confirm the efficiency and the accuracy of the theories and the relevant algorithm presented in the present paper.

6. References

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