Diagnostics Cracks in Gears with Wavelet Analysis

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Abstract
A crack in the tooth root, which is the least desirable damage caused to gear units, often leads to failure of gear unit operation. A possible damage can be identified by monitoring vibrations. Time signals were obtained by experiments. Amplitudes of time signal vibration are, by frequency analysis, presented above all as a function of frequencies in spectrum using hybrid procedure for determining the level of non-stationarity of operating conditions primarily of rotational frequency. A non-stationary signal was analysed as well, using the family of Time Frequency Analysis tools, including Wavelets and Joint Time Frequency Analyses. Wavelet analysis is suitable primarily for non-stationary phenomena with local changes. The purpose was to obtain the location of the crack, i.e. to identify the tooth. Typical spectrogram and scalogram patterns result from reactions to faults or damages; they indicate the presence of faults or damages in a very reliable way.

1. Introduction

The aim of maintenance is to keep a technical system (gear-unit) in the most suitable working condition; its purpose is to discover, to diagnose, to foresee, to prevent and to eliminate damages. The purpose of diagnostics is to define the current condition of the system, and the location, shape and reason of the damage formation. Although a gear unit, with its elements enabling the transmission of rotating movement, is a complex dynamic model, its movement is usually periodical. Faults and damages represent a disturbing quantity or impulse, which is indicated by local and time changes in vibration signals [1,2]. As a result it is possible to expect time-frequency changes [3]. This idea is based on kinematics and operating characteristics.
Individual frequency components in signals often appear only occasionally. Classical frequency analysis of such signals does not suffice to determine when certain frequencies appear in the spectrum. Time-frequency analysis, however, makes it possible to describe in what way frequency components of non-stationary signals change with time and to define their intensity levels. Gabor, adaptive and wavelet transforms are representatives of various time-frequency algorithms [4]. The basic idea of all linear transforms is to carry out a comparison with elementary functions determined in advance [5]. It is possible to obtain different signal presentations on the basis of various elementary functions [6].

2. Adaptive Method Analysis

Qian [7] significantly improved the adaptive transform of a signal although many authors had been developing algorithms that would have no interferences reducing the usability of individual transforms as opposed to Cohen's class.
Adaptive transform of a signal *x(t)* is expressed in the following way:

$$x(t) = \sum_{p} B_p \cdot h_p(t) \tag{1}$$

where analysis coefficients are determined by means of the following equations

$$B_p = \langle x, h_p \rangle \tag{2}$$

whereby similarity between the measured signal *x(t)* and elementary functions $h_p(t)$ of transform is expressed.
A time-dependent adaptive spectrum can be defined as

$$P_{ADT}(t,\omega) = \sum_{p} \left| B_p \right|^2 \cdot P_{WV} h_p(t,\omega) \tag{3}$$

This is an adaptive spectrogram based on representations. No interferences and no cross terms are included, which makes it different from the Wigner-Ville distribution. Additionally, it also satisfies the conditions relating to energy conservation.

$$\left\| x(t) \right\|^2 = \frac{1}{2 \cdot \pi} \cdot \iint P_{ADT}(t,\omega) \cdot dt \cdot d\omega \tag{4}$$

The selection of elementary functions is the basic issue relating to linear presentations. In relation to a Gabor expansion, a set of elementary functions comprises a time-shifted and frequency modulated prototype window function *w(t)*. In concern to wavelets, elementary functions are acquired by scaling and shifting a mother wavelet $\psi(t)$. In these two examples, structures of elementary functions are determined in advance. Elementary functions in relation to adaptive representation are fairly demanding. In general terms, the adaptive transform is independent from the choice of elementary functions as it permits arbitrary elementary functions. In principle, elementary functions, used for adaptive representation of a signal with Eq. (1), are very general although this is not always the case in practice. Regarding time and frequency, elementary functions are preferentially localised in order to stress the time dependence of a signal. Also, it is necessary that they use the presented algorithm in a relatively simple way. A Gauss type signal has very favourable characteristics and is considered a basic choice when it comes to the adaptive representation.

3. Wavelet Analysis

The continuous wavelet transform of function $x(t) \in L^2(\Re)$ at the time and scale is expressed in the following way [8]:

$$W\ x(u,s) = \langle x, \psi_{u,s} \rangle = \int_{-\infty}^{+\infty} x(t) \cdot \frac{1}{\sqrt{s}} \cdot \psi^*\left(\frac{t-u}{s}\right) \cdot dt = x(t) \otimes \overline{\psi}_s(t) \tag{5}$$

$$\overline{\psi}_s(t) = \frac{1}{\sqrt{s}} \cdot \psi^*\left(\frac{-t}{s}\right) \tag{6}$$

$$\overline{\Psi}_s(\omega) = \sqrt{s} \cdot \Psi^*(s \cdot \omega) \tag{7}$$

where the transform is presented as the product of convolution; in the Eq. (6), the expression of an average wavelet function and the corresponding Fourier integral transform, Eq. (7), is expressed.

In concern to the continuous wavelet transform, the observed function *x(t)* is multiplied by a group of shifted and scaled wavelet functions. This brings about changes in time and frequency dissemination. With this, time and frequency dissemination of the continuous wavelet transform changes.

Wavelets, as locally limited functions, are used to analyse the observed function *x(t)*. The continuous wavelet transform is very sensitive to local non-stationarities. Gabor wavelet function is a representative of an approximately analytical wavelet function, acquired by means of a frequency modulation of the Gauss window function [8]:

$$\psi_{Gabor}(t,\sigma,\eta) = \frac{1}{\sqrt[4]{\sigma^2 \cdot \pi}} \cdot e^{-\frac{t^2}{2\cdot\sigma^2}} \cdot e^{i\cdot\eta\cdot t} \tag{8}$$

A family of wavelet functions, or shifted and scaled Gabor wavelet function is obtained as follows (8):

$$\psi_{Gabor}(t,\sigma,\eta) = \frac{1}{\sqrt{s}} \cdot \frac{1}{\sqrt[4]{\sigma^2 \cdot \pi}} \cdot e^{-\frac{1}{2\cdot\sigma^2}\cdot\left(\frac{t-u}{s}\right)} \cdot e^{i\cdot\eta\cdot\left(\frac{t-u}{s}\right)} \tag{9}$$

On the basis of the average moment of function:

$$\overline{\omega}_{u,s} = \overline{\omega}_{Gabor} = \eta \tag{10}$$

which represents the centre of the Fourier integral transform.

The relation between scale and frequency is defined as follows:

$$\omega_{skale} = \overline{\omega}_{u,s} = \frac{1}{s} \cdot \overline{\omega}_{Gabor} = \frac{1}{s} \cdot \eta \quad \Rightarrow \quad f = \frac{\omega}{2 \cdot \pi} = \frac{1}{2 \cdot \pi \cdot s} \cdot \eta \qquad (11)$$

4. Practical Analysis

All the measurements have been carried out in the test plant (Fig. 1) of the Computer Aided Design Laboratory of the Faculty of Mechanical Engineering, University of Maribor. A one-stage helical gear unit is located at the spot where vibration measurements have been performed.
A helical gear unit with straight teeth was integrated into the gear unit. Tests were carried out under constant loads and vibrations; measurements were performed directly, with accelometers fixed on the housings. Each gear unit had a carburised spur gear pair of module 4 mm, the pinion had 19 and the wheel 34 teeth. A nominal pinion torque was 20 Nm and nominal pinion speed was 1200 rpm (20 Hz). For this type of gear units, this is a very typical load condition frequent in industrial applications.

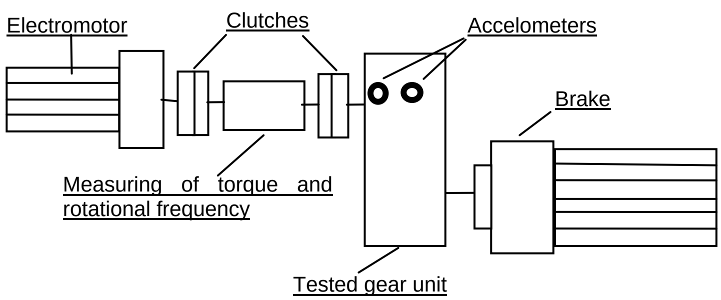

Figure 1: Test plant

A gear unit had a fatigue crack in the tooth root of a pinion; the operating conditions were typical of this type of a gear unit. A ground gear pair was a standard gear pair, with the teeth quality 6, but with the presence of a crack in the tooth root of a pinion. It is shown in Fig. 2. The length of the crack on one of the teeth in Fig. 2 is 4.8 mm. The whole measurement process and preparations for the analysis are presented in [9].
Elementary functions have restricted features. Therefore, adaptive spectrogram has a fine adaptive time-frequency resolution. Time-frequency resolution of the transform is adapted to signal characteristics. As an elementary function it is possible to use Gauss function (impulse) and linear chirp with Gauss window. If linear chirps that compose a signal are the result of a linear change in the rotational frequency of a gear unit, an adaptive spectrogram can be used to determine in what ways a possible frequency modulation is reflected in the time-frequency domain.

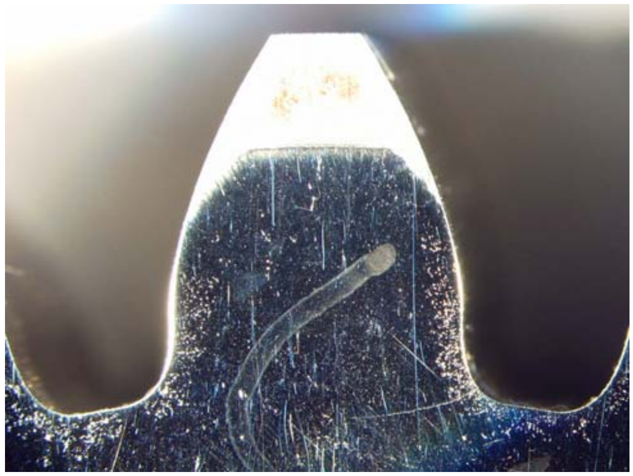

Figure 2: Pinion with a crack in the tooth root

The signal of measured values was 1 s long and composed of, on an average, 12500 measuring points. For comparison, spectrograms concerning Gabor transforms are given, the length of the window is 700 points. Spectrogram evaluation is based on an average spectrogram, which represents an amplitude spectrum of a measured signal, and on pulsating frequencies of individual frequency components.

In relation to the Gabor spectrogram, presented in Fig. 3, no rhythmic pulsation of harmonics is evident, with the exception of typical frequencies, determined on the basis of a power spectrum. It is possible to observe some pulsation sources but they are not very expressed in relation to the adaptive spectrogram (Fig. 4), which features a higher level of energy accumulation in the origins. It is very interesting to monitor the increase or decrease (complete disappearance) in appropriate frequency components with rotational frequency of 20 Hz. This is typical of the 3rd harmonic of mesh frequency; 1530 Hz is expressed only in relation to the presence of a crack. This phenomenon is much more expressed in the adaptive spectrogram (Fig. 6) than in the Gabor spectrogram (Fig. 5). In Fig. 6, pulsation (i.e. the area, marked with a continuous line) is expressed, whereby a single engagement of a gear pair with a crack within one rotation of a shaft is reflected. Similarly, sources denoting pulsating portions of individual components with the frequency of 20 Hz are indicated between the 6th and the 9th harmonics (i.e. the area marked with a dashed line).

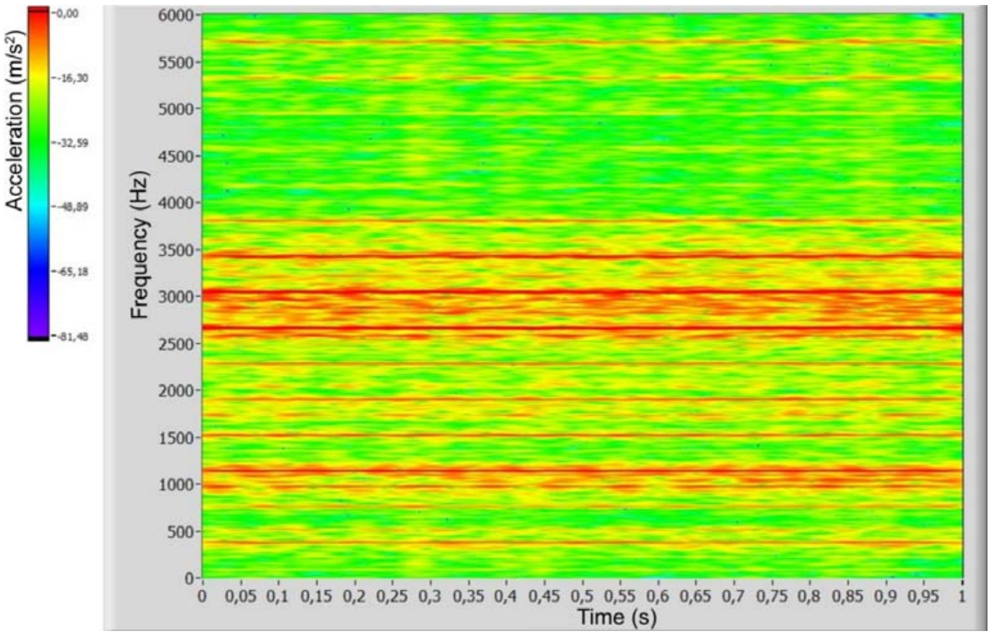

Figure 3: Gabor’s spectrogram of a faultless gear unit

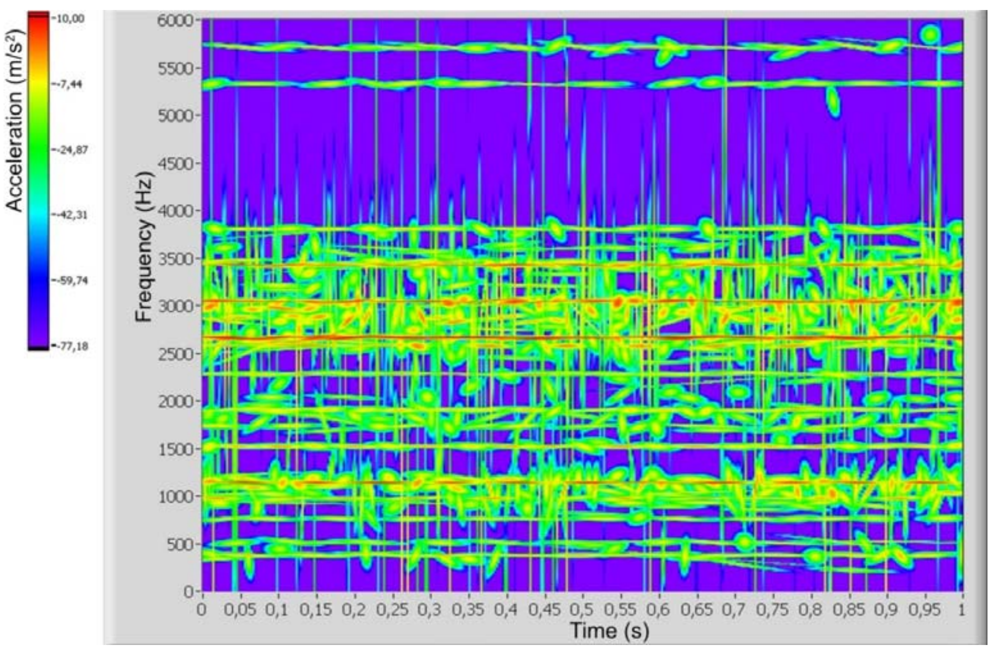

Figure 4: Adaptive spectrogram of a faultless gear unit

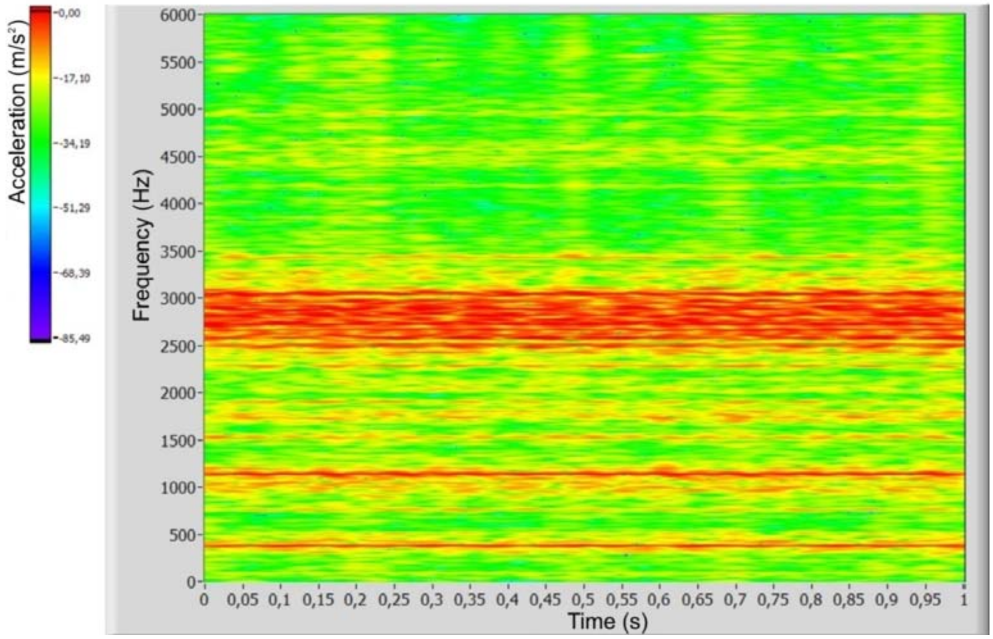

Figure 5: Gabor's spectrogram of a gear unit with a pinion with a crack

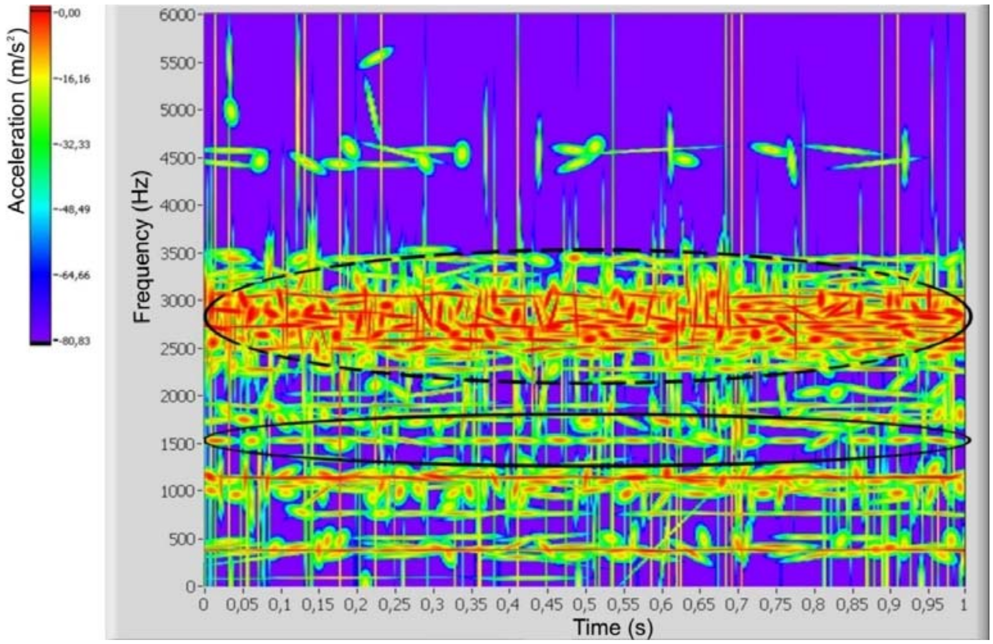

Figure 6: Adaptive spectrogram of a gear unit with a pinion with a crack

A scalogram of analytical wavelet transform with Gabor wavelet function represents normalised and square values of amplitudes of wavelet coefficients. The connection between the scale and frequency is established, and the representation is performed in a time-frequency domain. As it is much simpler to find adequate characteristics in time-frequency domain (frequency scalogram) than in time-scale domain (scalogram), this is very appropriate in technical diagnostics. Based on normalization, the transform matches the Parseval characteristic of energy preservation; this means that the energy of wavelet transform equals the energy of the original signal in time domain. Wavelet analysis is suitable primarily for non-stationary phenomena with local changes. Therefore, the analysis was carried out to establish the condition associated with the presence of a crack in a tooth root; the purpose was to identify the location of the crack, i.e. to define the tooth. For analysis, the analytical continuous wavelet transform, with parameters $\eta = 6$ and $\sigma = 1$, was used. The highest frequency in the signal (6250 Hz) was acquired on the basis of Nyquist frequency and the frequency of sampling the measured time signal. For analysis, a part of the signal, representing one whole rotation of the tooth, i.e. of a pinion with a crack, of 50 ms, was used. It is evident from the figures relating to the faultless gear, in the frequency scalogram, that there are no particularities in expressed components that would indicate local changes, which applies for a square representation (Fig. 7) of wavelet coefficients. The matter is different when analysing the signal produced by a gear with a crack; this signal shows a local change in wavelet coefficients, in time, at the value of 11 ms, in frequency scalograms (Fig. 8). Local change, i.e. the presence of transients, can be noted where there is the tooth with the crack in its root. If the wavelet length is 50 ms, which represents one rotation of the pinion, and there are 19 teeth along the circumference, the increased amplitude is located at 11 ms and belongs to the fourth tooth in the direction of rotation from the reference positional point of the gear unit.

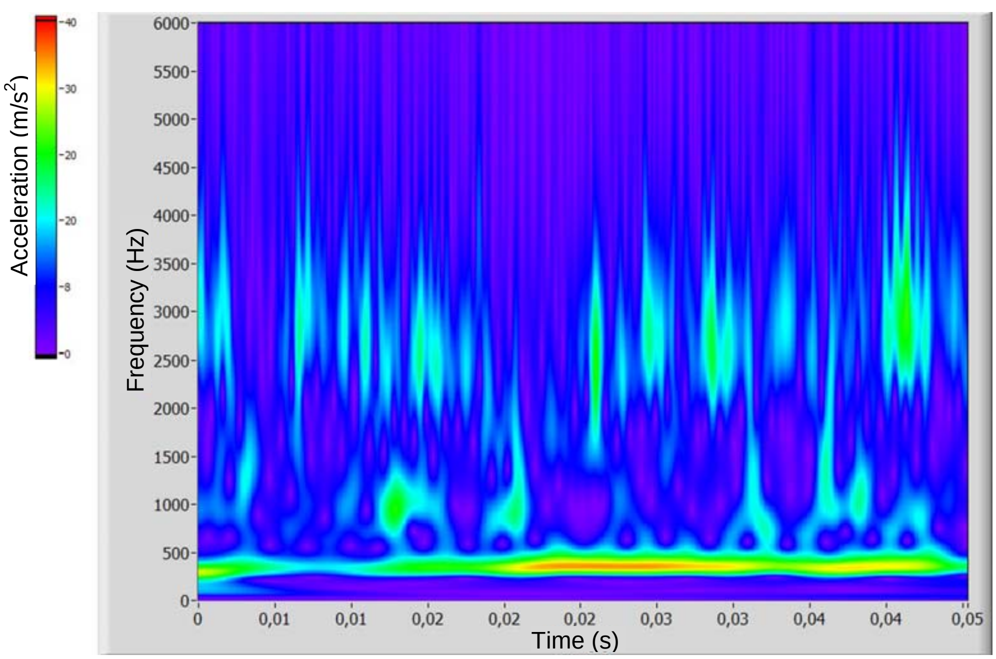

Figure 7: Average Gabor frequency scalogram of square wavelet coefficient of the reference gear unit

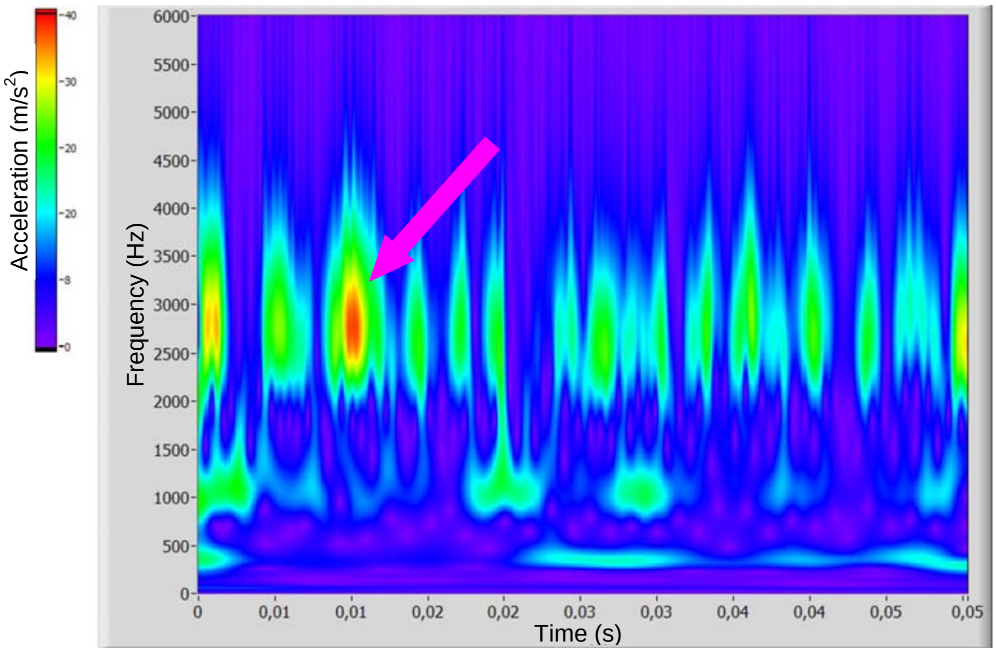

Figure 8: Average Gabor frequency scalogram of square wavelet coefficient of the gear unit with a gear with a crack in a tooth root

5. Conclusions

Vibration analysis for fault detection in industrial gear units is presented; the described methods can increase the safety of operation and, consequently, the reliability of monitoring operational capabilities.

The condition of a gear unit can be monitored in a more reliable way if appropriate spectrogram samples and a clear presentation of the pulsation of individual frequency components are used; they, along with the average spectrum, represent a criterion for evaluating the condition of a gear unit. Adaptive time-frequency representation primarily enables a reliable prediction. The representation is clearer, without increased dissemination of signal energy into the surroundings.

A wavelet transform can make it possible to identify changes in a very short time, and to determine the presence of a damage, at the level of an individual tooth. An appropriate method or criterion makes it possible to monitor the actual condition of a device and its vital component parts, which can have a considerable impact upon the operational capability. By detecting faults and damages in time, the reliability of operation control is significantly improved. A high level of reliability of detecting faults improves the prediction of the remaining life cycle of a gear unit.

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