Effect of Crack-Crazing Patterns Interactions on Energy Release Rates

Seddiki, H [1] and <u>Chabaat, M</u> [2]

[1] Doctorate student, [2] Professor, Built Environment Research Laboratory, Civil Engineering Faculty, University of Sciences and Technology Houari Boumediene, B.P. 32 El Alia, Bab Ezzouar, Alger 16111, Algeria
mchabaat2002@yahoo.com & hseddiki2007@yahoo.fr

ABSTRACT

In this study, interactions between a main crack and a surrounding layer of crazing patterns are considered. Analysis of the stress field distribution as well as the energy induced during these interactions is based on the resolution of some differential equations along with appropriate boundary conditions and the use of a numerical approach. It is proven throughout this study that the crazes growth occurs along directions normal to the major principal stress directions and constitutes an important toughening mechanism. Thus, the mode I Stress Intensity Factor (SIF) is employed to quantify the effects of this damage on the main crack and the Energy Release Rate (ERR) due to the linear propagation of the main crack and also to the translational change in the growth of the damage. It is proven, herein, that crazes closer to the main crack dominate the resulting interaction effect and reflect an anti-shielding of the damage while a reduction constitutes a material toughness.

KEYWORDS: Displacement, major and minor principal stress, stress intensity factor, energy release rate, crack, crazing patterns.

1. INTRODUCTION

There is sufficient experimental evidence that in most cases, a propagating crack is surrounded by a damage zone which often precedes the crack itself. This zone usually consists of slip lines or shear bands in metals, microcracks in ceramics and polymers, and crazes in amorphous polymers. Thus, the existence of these defects affects progressively the propagation of cracks already present in some materials. Because this damage can constitute an important toughening mechanism, problems dealing with crack microcracks interactions have received considerable research attention since they were introduced to fracture mechanics. As a result, a wide body of literature, on this topic, exist [1-4]. Thus, analysis of the distribution of surface crazes in the vicinity of a stationary edge crack in a polystyrene (PS) sheet in tension has shown the craze growth occurs along directions parallel to the minor principal stress axis. This behaviour has been thoroughly documented and extensively discussed in a number of papers [5-7].

2. PROBLEM FORMULATION

Considering a two dimensional, linear elastic solid containing an edge crack of length L and a surrounding layer of crazing patterns as shown in Fig. 1. For an edge crack in a semi-infinite body under uniform traction, distant from and normal to the crack and considering Cartesian coordinates with the origin at the crack tip, the elastic plane stress field is given as follows [8].

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = K_I/(2\pi r)^{1/2} \begin{Bmatrix} \varphi_{xx}(\theta) \\ \varphi_{yy}(\theta) \\ \varphi_{xy}(\theta) \end{Bmatrix} + \sigma_\infty \qquad (1)$$

The first term of Eq. (1) expresses an asymptotic stress field near the singularity. The additional terms are homogeneous material stresses.

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = K_I / (2\pi r)^{1/2} \begin{Bmatrix} \cos\theta/2\,(1 - \sin\theta/2 \sin(3/2)\theta\,) \\ \cos\theta/2\,(1 + \sin\theta/2 \sin(3/2)\theta \\ \sin\theta/2 \cos\theta/2 \cos(3/2)\theta \end{Bmatrix} + \sigma_\infty \qquad (2)$$

where $K_I = 1{,}12(\pi L)^{1/2}\,\sigma^\infty$ is the effective stress intensity factor at the main crack tip for mode I.

Then, for the case of an edge crack specimen, the global stress field is given by the following expressions in Cartesian coordinates;

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = A.(L/r)^{1/2}.\sigma_\infty \begin{cases} \cos\theta/2\,(\,1 - \sin\theta/2 \sin(3/2)\,\theta\,) \\ \cos\theta/2\,(\,1 + \sin\theta/2 \sin(3/2)\,\theta + 1/A\,(r/L)^{1/2} \\ \sin\theta/2 \cos\theta/2 \cos(3/2)\,\theta \end{cases} \qquad (3)$$

where the constant A is equal to $1.12/\sqrt{2} \approx 0.8$

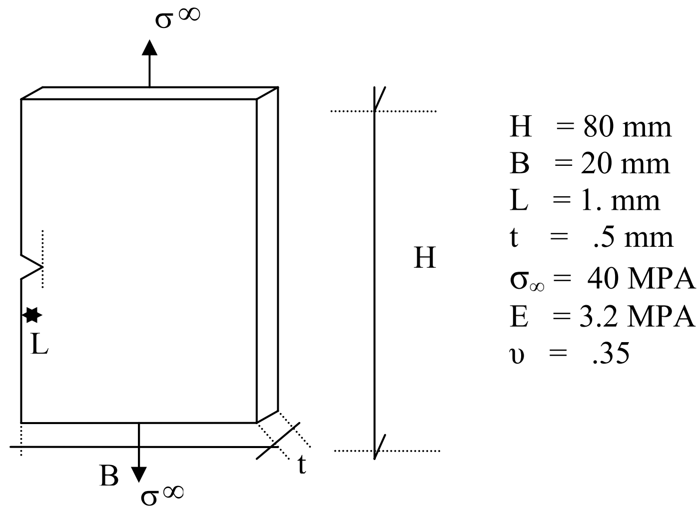

Figure 1: Geometry of the problem for general formulation.

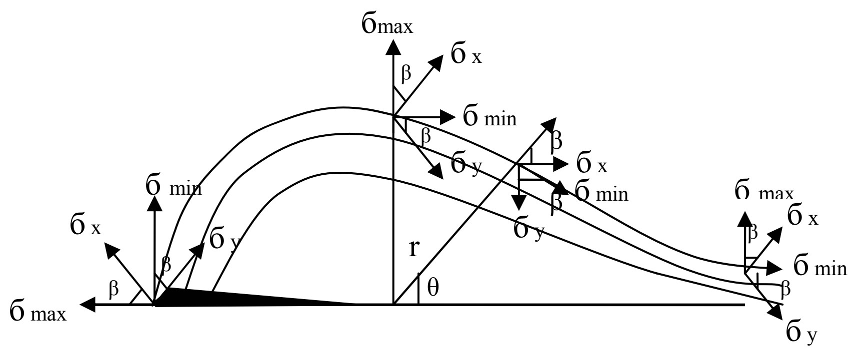

Figure 2: Schematic representation of principal stresses along the crazing patterns.

3. DIFFERENTIAL EQUATIONS FOR PRINCIPAL STRESS

According to Mohr's theory, principal stresses are orthogonal and act on a plane where shear stresses vanish. Then, the following geometrical transformations are taken in consideration (refer to Figure 2)

$$\tan 2\beta = 2 \tan \beta/(1-(\tan \beta)^2) \qquad (4)$$

Since $y' = \tan \beta$ stands for the slope of the minor principal stress, then; Eq. (4) takes the following form;

$$y'^2 + (2/ \tan 2\beta)\, y' - 1 = 0 \qquad (5)$$

and the principal stress directions are related as;

$$\tan 2\beta = 2\sigma_{r\theta}/(\sigma_{rr} - \sigma_{\theta\theta}) = ((-\sin \theta \cos (3/2\ \theta))/(\sin \theta \sin (3/2\ \theta) + (1/A)\sqrt{r}) \qquad (6)$$

Using $\theta = \arcsin (y/(x^2 + y^2)^{1/2})$ and $r = (x^2 + y^2)^{1/2}$, Eq. (6) can be written as;

$$\tan 2\beta = (-\sin (\arcsin(y/(x^2 + y^2)^{1/2})) \cos (3/2 \arcsin (y/(x^2 + y^2)^{1/2}))/ \\ (\sin (\arcsin (y/(x^2 + y^2)^{1/2}) \sin (3/2 \arcsin (y/(x^2 + y^2)^{1/2})) + \\ (1/A)(x^2 + y^2)^{1/4} \qquad (7)$$

Substitutions of Eq.(7) into Eq.(5), two differential equations related to the principal stress trajectories along with prescribed boundary conditions are set;

$$\Sigma_1 = y'_1 = -1/\tan 2\beta - 1/\sin 2\beta \qquad (8)$$

$$\Sigma_2 = y'_2 = -1/\tan 2\beta + 1/\sin 2\beta \qquad (9)$$

with; $x = x_0 =]-\infty, +\infty[$ and $y_1 = y_{01} =]-\infty, +\infty[$, $y_2 = y_{02} =]-\infty, +\infty[$

and; $$\beta = 1/2 \arctan ((-\sin (\arcsin (y/(x^2 + y^2)^{1/2})) \cos (3/2 \arcsin (y/(x^2 + y^2)^{1/2}))/ \\ (\sin (\arcsin (y/(x^2 + y^2)^{1/2}) \sin (3/2 \arcsin (y/(x^2 + y^2)^{1/2})) + (1/A) (x^2 + y^2)^{1/4}) \qquad (10)$$

One can notice that by changing β by β' such as $\tan 2\beta' = \tan (\pi - 2\beta) = -\tan 2\beta$, solutions for symmetries principal stresses trajectories are obtained.

Tresca's differential equations related to the or maximum shear stress function relative to the applied stress is given as follows

$$\Sigma_{12} = y'_5 = -2 / \sin 2\beta \qquad (11)$$

with, $x = x_0 =]-\infty,+\infty[$ et $y_5 = y_{05} =]-\infty,+\infty[$

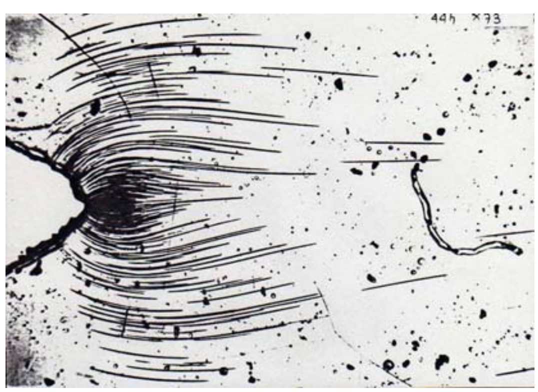

Figure 3: Microscopic observations of crazes in (PS) under tension [5].

4. STRESS INTENSITY FACTOR

Irwin [9] was the pioneer in the determination of a parameter as significant as the Stress Intensity Factor (SIF) using basically the theory of elasticity. The stress intensity factor for a fissured plane medium under mode I is defined as follows [10];

$$K_I = \lim_{r \to 0} [\sigma_{yy} (2\pi\, r)^{1/2}] \tag{12}$$

where,

$$\sigma_{yy} = A.(L/r)^{1/2}.\sigma_\infty (\cos \theta/2\ (1 + \sin \theta/2 \sin (3/2)\, \theta + 1/A\ (r/L)^{1/2}) \tag{13}$$

Substitution of Eq. (2) into Eq. (12), one can get;

$$K_I = 1.12\ .(\pi.L)^{1/2}.\ \sigma_\infty.\ \cos \theta/2\ (1 + \sin \theta/2 \sin (3/2)\, \theta) \tag{14}$$

Finally, the total SIF takes the following form;

$$K_I / K_0 = 1.12\ (\pi.L).\ \cos \theta/2\ (1 + \sin \theta/2 \sin (3/2)\, \theta) \tag{15}$$

Here, K_0 is the stress intensity factor in absence of the damage.

Figures (5) to (6) show that variations of the stress intensity factor K_I/K_0 are given according to the length of the principal crack and the position of the microscopic crazing patterns compared to the latter. It is noted that the fissured zone disturbs the propagation of the principal crack either by a progressive acceleration of the positive value of K_I/K_0 or by a rather drastic reduction thus delaying this propagation by the negative value of K_I/K_0

By fixing the position of the crazing patterns in the fractured zone and while varying the length of the crack gradually, one notices that the SIF K_I/K_0 presents two fields distinct from one to another. The increase in K_I/K_0 supports the crack to go forwards then propagating in a rectilinear path. The reduction in K_I/K_0 causes retardation in the propagation until the so-called “crack arrest”.

Fixing the length of the principal crack, one notes that K_I/K_0 decreases in a drastic way. In this case, the effect of reduction in the intensity of the stress delays the propagation of the main crack. On the other hand, by fixing the position of these crazing patterns and while varying the length of the crack, the amplifying effect takes over. Then, an increase in the intensity of stress field in the vicinity of the main crack is apparent and leads to an acceleration of the propagation of the crack.

5. TRANSLATIONAL ENERGY RELEASE RATE

Closed contour of integration including the crazing patterns is selected by the use of the previous differential equations corresponding to the minimal principal stresses trajectories. Energy Release Rate (ERR) is determined by computing the area limited by the contour itself and the X-axis coordinates in one side only because of the symmetry. This rate is given by the following relation;

$$J_I = \int_{-x}^{x} \int_{-y}^{y} (y_2(x, y))\, dy\, dx \tag{17}$$

where $y_2(x, y)$ are solutions to the differential equations.

The variation of the rate of energy due to the translation of the damage zone is plot with respect to the position of the crazing patterns using contour defined by the differential equations as shown in Figure (7). For reason of simplification, the translational ERR is given dimensionless as J_I / J_0 where $J_0 = G = K_0{}^2 / E$ corresponds to the ERR by the initial crack in the absence of the damage zone (Griffith's criteria).

One notices that the energy release rate J_I/J_0 corresponding to each length of the crack increases in an exponential way which leads to an acceleration in the propagation of the main crack. This process decreases the toughness of the material. On the other hand, for another contour, J_I/J_0 decreases in the same way proving that a deceleration in the propagation of the crack has taken place and consequently, increasing the material toughness.

6. CONCLUSION

Analysis of the distribution of surface crazes in the vicinity of a stationary edge crack in a polystyrene (PS) sheet in tension has shown that the craze growth occurs along directions parallel to the minor principal stress axis. It shown in this study that the stress field distribution in the vicinity of the main crack is obtained by the resolution of some differential equations. Reduced principal stresses trajectories are established according to Mohr's criteria. It is proven, herein, that the mode I Stress Intensity Factor and the Energy Release Rate are employed to quantify the effects on a crack of the damage consisting of crazing patterns.

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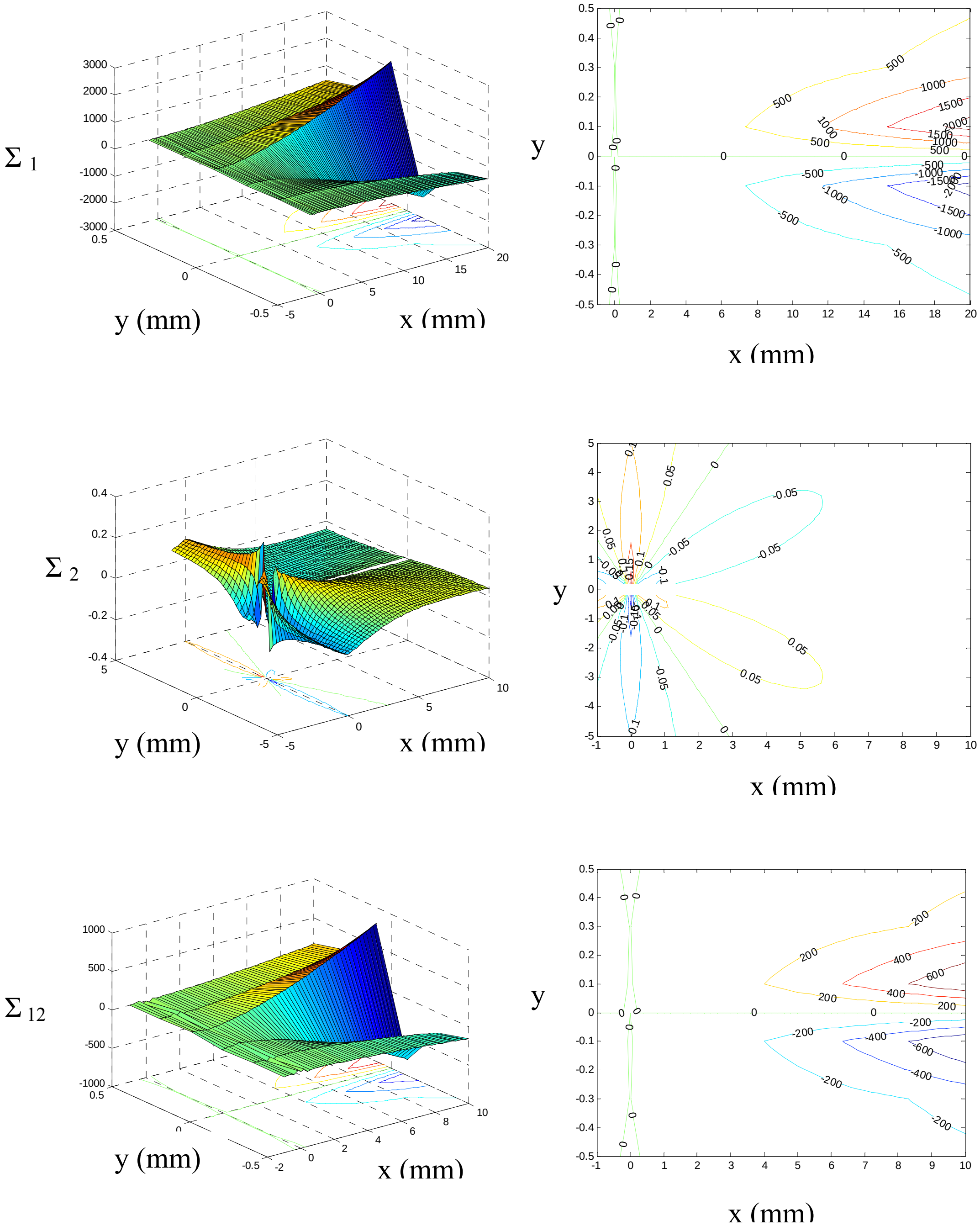

Figure 4: Variation of the reduced principal stress $\Sigma_{1,}$ Σ_2, Σ_{12} function of the orientation and the position of crazing patterns.

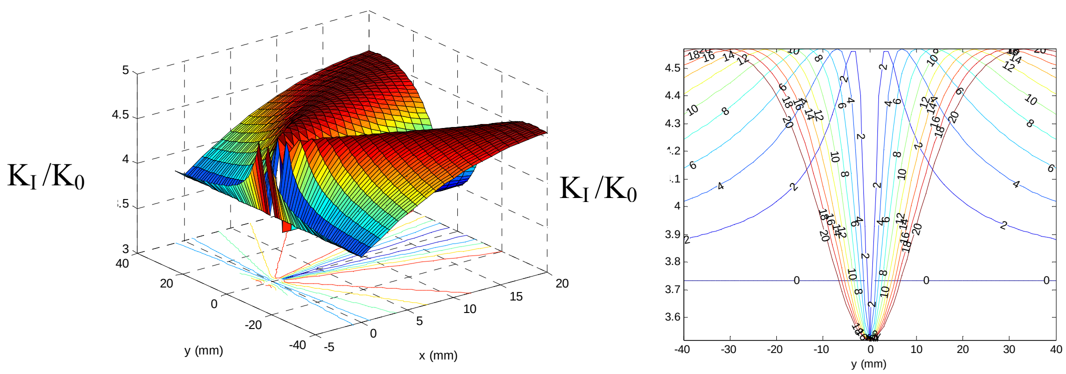

Figure 5: Variation of K_I / K_0 function of the length of the crack and the position of the crazing patterns.

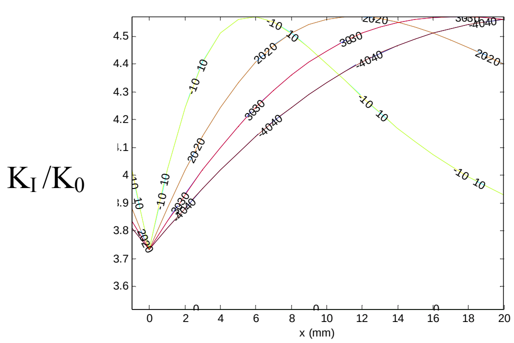

Figure 6: Variation de K_I / K_0 function of the position and the length of the crack.

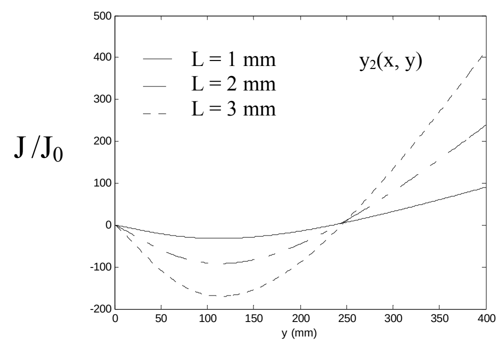

Figure 7: Variation of the energy release rate J/J_0 function of the position of the crazing