Anisotropic and Unilateral Damage Application to Concrete

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Abstract

A theoretical model is presented suitable for the description of the damaging process in brittle materials. The model is based on damage mechanics: it features two second order symmetric damage tensors, representative of surface damage induced by tensile and compressive strains activates at different orientations to the principal strain directions, according to the sign of the principal strain which attains a damage threshold, and a scalar variable representative of volumetric damage. In this model, two separate damage variables, one for tension and the other for compression, allow to model the non-symmetric behaviour and the unilateral effect. Damage activation is governed by two different loading functions evolving with nonreciprocal hardening and softening. The capabilities of the model in describing the mechanical response of material elements subjected to non-proportional stresses are illustrated.

Keywords: concrete, non linear behaviour, damage, anisotropy, unilateral effect, numerical aspects.

1   Introduction

It is well known that the deformation of most engineering materials is often accompanied by irreversible changes in their internal structures. The nucleation and the growth of distributed microscopic cavities and cracks do not only induce the occurrence of macro-cracks, but also lead to the deterioration of material properties due to internal microstructural changes. Therefore, understanding and knowledge of the damaging process and its effects on the macroscopic behaviour of materials are very important prerequisites for accurate analysis of structural integrity of practical problems. An approach to these problems can be provided by the recently developed theory of continuum damage mechanics (CDM), which involves irreversible micro structural changes implicitly. Several CDM models have been proposed for brittle materials. For example, the Chow and Yang model 1991 [1], is based on the hypothesis of damage surface that is similar to the yield function in plasticity theory and uses one scalar valued internal variable to represent the damage state. The Chaboche model [2] assumes anisotropic damage basically and introduces tensorial damage variables. Therefore, it can be

considered that there exists a relationship between small cracks (damage) and strain direction. However, a detailed method to apply this model to anisotropic damage in any direction is still under development. Moreover CDM theories, which incorporate vectors [3], scalars [4], second-order tensors [5, 6], and fourth-order tensors [7] have been proposed. But these theories show either a discontinuous stress–strain response when the unilateral condition takes place or an unacceptable non-symmetric elastic behaviour for some loading conditions. In this paper, we attempt to describe the non-linear elastic constitutive equation using CDM and evaluate the structural integrity of brittle material components by the damage parameter introduced in this type of theory.

Concrete structures exhibits essentially elastic-brittle behaviour, the failure is accompanied by local loss in stiffness and strength, as a consequence of the concurrent coalescence and growth of micro cracks: a phenomenon which is usually defined as 'damage'. The complexity of microscopic mechanisms, which occur in concrete during the damage process, includes notably the formation of cracks in tension and the closure of cracks in compression, with possible friction between the crack lips.

In concrete materials the inelasticity is generally associated with irreversible thermodynamics process involving elastic degradation. Strength and stiffness degradation can be effectively modelled in the framework of CDM: isotropic damage models with a single damage variable [8] or a tension damage variable and a compression damage variable [3, 9], anisotropic damage models [6, 10, 11,], to cite a few.

The aim of the paper is to propose a thermodynamically consistent model, which accounts for some anisotropy induced by damage, and which includes unilateral effects related to crack closure. Attention is paid to limiting the complexity of the model, and especially the number of parameters, in order to make the model a viable candidate for practical engineering computations.

The present work is an extension of the model proposed to simulate the nonlinear mechanical behaviour of brittle materials, and successfully applied to structural analysis of concrete structures [12].

2   The damage model

2.1   Generality

Introduced in 1958 by Kachanov [13] for creep failure of materials under uniaxial loads, and proposed a creep damage evolution model. Later Rabotnov (1969) [14] improved Kachanov's creep model by introducing an extra coefficient to show that the damage rate is influenced more strongly by the degree of damage than by the global mechanical behavior. Continuum damage mechanics has been applied in the 1980 for simulating the non linear behaviour of concrete [8,15,16]. Thermodynamics of irreversible processes gave the framework to formulate the adapted constitutive laws (e.g. Lemaitre and Chaboche 1985) [17]. Considering the material as a system described by a set of variables and a thermodynamic potential, constitutive laws are systematically derived along with conditions on

the kinematics of damage. However, an adequate choice of the potential and of the damage variable (scalar, tensor, etc.) remains to be made. Several anisotropic damage models have already been proposed by Dragon and Mroz 1979 [18], Mazars and Pijaudier-Cabot 1989[19], Fichant and al. 1999[20]. Possible applications cover also dynamic problems [19, 21], porous materials [22], and chemical damage [27]. A recent literature review on damage mechanics can be found in Pijaudier-Cabot 2000 [30].

2.2 Elasticity modelling

The model is formulated within a strain space framework. In order to make the subsequent implementation in finite element code easier, Helmotz's strain based free energy has been chosen. The Cauchy strain tensor, $\underline{\varepsilon}$, will be split into a volumetric part, ε_v, and a deviator part $\underline{e}$, so that:

$$\underline{\varepsilon} = \varepsilon_v . \underline{1} + \underline{e} \tag{1}$$

The stress-strain relation:

$$\underline{\sigma} = 2G.\underline{e} + K.Tr[\underline{\varepsilon}]\underline{1} \tag{2}$$

Where K and G, are the bulk and shear modulus of the undamaged material, respectively.

The free energy is considered as sum of two contributions:
- a surface contribution : $\rho\psi_{surf} = G.Tr(\underline{e}.\underline{e})$
- a volumetric contribution : $\rho\psi_{vol} = \dfrac{1}{2}K.Tr^2[\underline{\varepsilon}]$

ρ is the material density and $\rho\psi_{el}$ is the Helmotz's free energy.

2.3 Elasticity coupled with anisotropic damage

To traduce the damage, two independent variables are chosen:
- a scalar variable δ representing the volumetric damage of material, i.e. the reduction, in effective volume of a damaged unit element volume compared with a virgin one. The material behaviour is due to the effect of a strong hydrostatic pressure applied to the micro porous structure of the concrete.
- a second order symmetric damage tensor $\underline{D}$. This tensor is meant to be representative of 'surface damage', i.e. the reduction in effective area of surface elements with different orientations from the undamaged to the damaged state.

The damage and the inelastic strains evolve simultaneously. These micro voids are the cause of the inelastic strains.

The thermodynamic forces associated to $\underline{e}$, $\underline{D}$ and δ have as expressions:

$$\underline{\sigma} = 2G.Tr\left[(\underline{1}-\underline{D})^{1/2}.\underline{e}.(\underline{1}-\underline{D})^{1/2}\right] + K.(1-\delta)Tr[\underline{\varepsilon}]\underline{1} + \frac{1}{2}\left(2G\beta(\underline{D}\cdot\underline{D})+\gamma K\delta^2\underline{1}\right) \qquad (3)$$

$$\underline{Y}_D = G\underline{e}.\underline{e} - 2\beta G[\underline{D}\cdot\underline{e}+\underline{e}\cdot\underline{D}] \qquad (4)$$

$$Y_\delta = \frac{1}{2}KTr^2[\underline{\varepsilon}] - \gamma K\delta Tr[\underline{\varepsilon}] \qquad (5)$$

Since $\underline{Y}_D$ and Y_δ are positive functions, the damage growth laws are such that
$$\underline{\dot{D}} \geq 0 \qquad \text{and} \qquad \dot{\delta} \geq 0$$

From Relation [3] one deduces the inelastic stress which is written:

$$\underline{\sigma}_{an} = \frac{1}{2}\left(2\beta G(\underline{D}\cdot\underline{D})+\gamma K\delta^2\underline{1}\right) \qquad (6)$$

β and γ are two new intrinsic parameters of material

We can observe that the total stress is divided into two parts
- an isotropic contribution, relating to the volumetric damage variable δ : $\gamma K\delta^2\underline{1}$
- an anisotropic contribution, relating to the surface damage variable
  $\underline{D}$: $2\beta G(\underline{D}\cdot\underline{D})$

2.4 Unilateral damage modelling

This unilateral damage effect usually leads to complex models when damage anisotropy is considered, one writes:

$$\underline{e} = \underline{e}^t + \underline{e}^c \qquad (7)$$

In the same way one postulates the existence of two independent internal variables, $\underline{D}^t$ and $\underline{D}^c$ for traction and compression respectively. Allowing for unilaterality, the stress-strain relationship of the material:

$$\underline{\sigma} = 2G.Tr\left[(\underline{1}-\underline{D}^t)^{1/2}\cdot\underline{e}^t\cdot(\underline{1}-\underline{D}^t)^{1/2}\right] + G.Tr\left[(\underline{1}-\underline{D}^c)^{1/2}\cdot\underline{e}^c\cdot(\underline{1}-\underline{D}^c)^{1/2}\right]$$
$$+ K.(1-\delta)Tr[\underline{\varepsilon}]\underline{1} \qquad (8)$$

Derivation of equation with respect to the damage variables yields the damage driving forces conjugate to and:

$$\underline{Y}_D^t = -\frac{\partial(\rho\psi)}{\partial\underline{D}^t} = G.\underline{e}^t.\underline{e}^t \ , \ \underline{Y}_D^c = -\frac{\partial(\rho\psi)}{\partial\underline{D}^c} = G.\underline{e}^c.\underline{e}^c \ , \ Y_\delta = -\frac{\partial(\rho\psi)}{\partial\delta} = \frac{1}{2}K.Tr^2[\underline{\varepsilon}]$$

3 Damage evolution laws

3.1 Evolution law of tensile damage

So that the law of evolution is written:

$$D_i^t = 1 - \left[\left(\frac{e_{D0}}{e_i^t}\right)\exp\left[-b(e_i^t - e_{D0})\right]\right] \tag{9}$$

b is a material parameter driving the slope of the softening branch.

3.2 Evolution law of Compressive damage

Compressive damage in a particular direction is considered only as a consequence of the tensile behaviour of the material and, is taken equal to a function of the state of tensile along the orthogonal directions [6, 20]; one uses a law power of the form:

$$D_i^c = \left(\frac{D_j^t + D_k^t}{2}\right)^\alpha \tag{10}$$

α is a material parameter

3.3 Evolution law of Volumic damage

One proposes an exponential form for the evolution of the volumetric damage:

$$\delta = 1 - \left[\left(\frac{\varepsilon_{D0}}{\langle Tr[\underline{\varepsilon}]\rangle}\right)\exp\left[-b(\langle Tr[\underline{\varepsilon}]\rangle - \varepsilon_{D0})\right]\right] \tag{11}$$

4 Numerical results

4.1 Uniaxial tension test

We present here, the response of concrete representative volume element (RVE) submitted to a uniaxial tension under controlled displacement. The calculation is carried out under the hypothesis of two dimensional plane strains.

The material parameters used for the analysis are: (The data of the tested specimen are the following)

E = 20000 MPa, $\quad\quad$ $\nu = 0.2$. $\quad$ $f_t = 2.5$ MPa $\quad\quad$ $f_c = 25$ MPa
$G_f = 0.05$ Nmm/mm^2 $\quad$ $\varepsilon_{Do} = 0.0001$
Hypothetical values of: $\beta = 8.10^{-4}$ $\quad$ $\gamma = -3 .10^{-4}$

The numerical results obtained from the simulation of the uniaxial tensile test are reported below
The response in tension is represented on the Fig.1. The model returns the principal characteristics of the material.
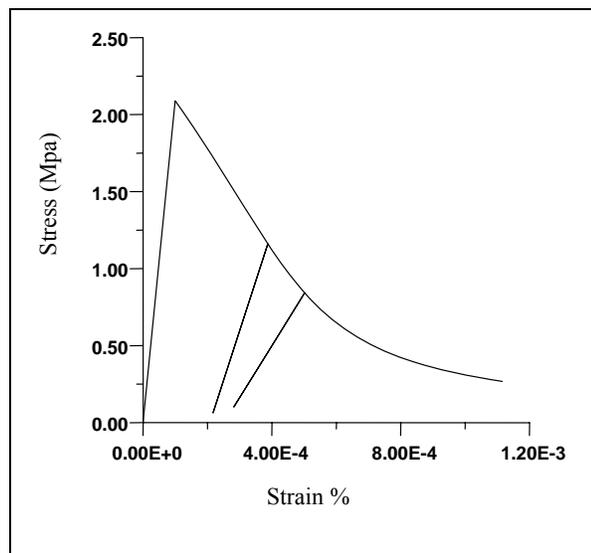
Figure 1: Tension test.

- a loss of stiffness beyond a certain stress point due to damage which is strongly anisotropic due the directionality of the defects (induced anisotropy);
- a permanent or irreversible strains with the loss of material stiffness (wich are due to growth of damage).

4.2 Response of the model under Compressive loadings

A unit volume of material is loaded in uniaxial compression under controlled displacement.

The analysis is performed at the material level.

The material parameters used for the analysis are:
E = 20000 MPa, $\quad\quad$ $\nu = 0.2$ $\quad\quad$ $f_t = 2.5$ MPa $\quad\quad$ $f_c = 25$ MPa
$G_f = 0.05$ Nmm/mm^2 $\quad$ $\varepsilon_{Do} = 0.0001$

In Fig.1 the stress-strain plot in uniaxial compression is shown. The asymmetric behaviour in compression typical of brittle materials is correctly described. The response of the model in terms of stress versus the longitudinal and transverse strain is represented on the Fig.2 and Fig.3. One can observe that the model is able to describe the volumetric response of the material satisfactorily.
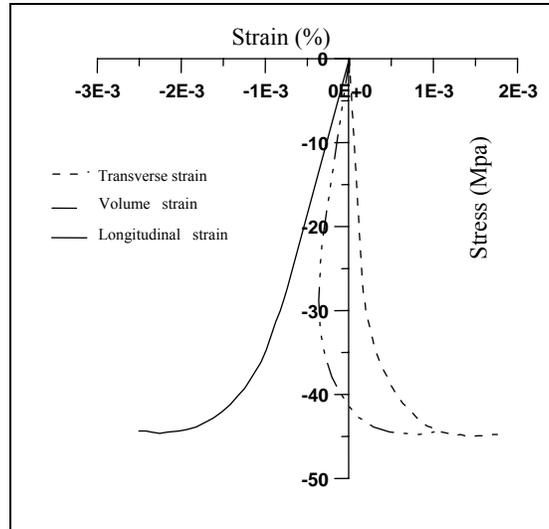

Figure 2: Compression test.

We represent below in Fig.3, the evolution of the surface damage and volume damage.
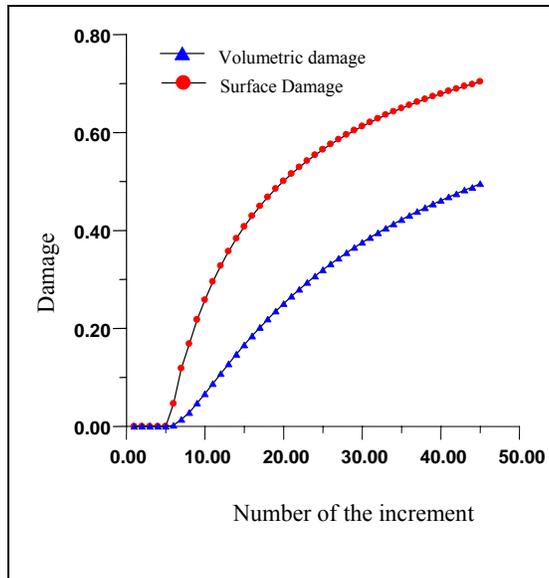

Figure 3: Surface and volumetric damage evolution.

4.3 Response of an RVE under cyclic loading

For instance, Fig. 4 show the numerical simulation of a uniaxial tension followed by compression on one element:
The dissymmetric behavior between traction and compression and a softening behavior during traction and compression is captured by the model. A Fig. 4 show that the unilateral condition applied to the anisotropic damage model preserve the continuity of the response and gives a stiffness recovery with the change from the damaging loading in traction to a compression loading.
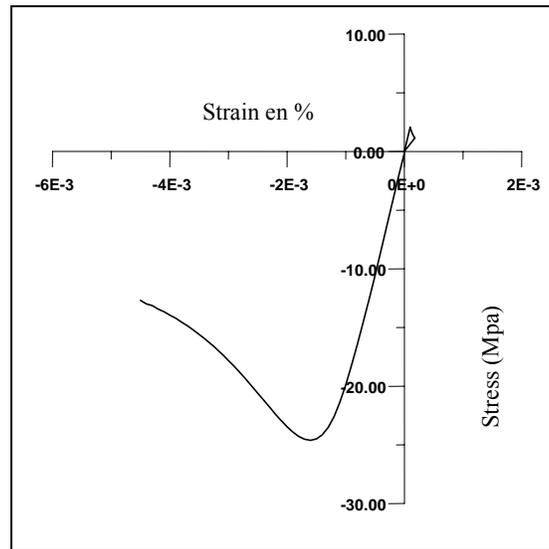
Figure 4: Unilateral effect.

5 Conclusion

A new anisotropic damage model is proposed for brittle materials such as concrete. The model is formulated in the framework of the continuum damage mechanics. A second-rank symmetric tensor is used to characterize material surface damage due to oriented microcracks; the volumic damage is characterized by a scalar damage variable.

This anisotropic model is combined with a unilateral condition in order to take into account the crack closure phenomenon and elastic moduli recovery.

The validity of the model is tested through numerical predictions in various loading conditions. The overall performance of the model is evaluated. The proposed damage model is capable of capturing several aspects of the nonlinear behaviour of brittle solids, namely, the damage-induced anisotropy, the unsymmetric behaviour in tension and compression, the 'unilateral' behaviour of the material.

Extensions are still required to incorporate in the model important phenomena, such as   the hysteresis loops at fixed level of damage during cyclic loadings, which were neglected in the present version.

Acknowledgement

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