A micromechanics based model for ductile damage in multiphase and composite metallic alloys combining extended Gurson and homogenization theories

C. Tekoğlu[1] *and T. Pardoen*[1]
[1] *Institute of mechanical, materials and civil engineering, Université catholique de Louvain, Place Sainte Barbe 2, B-1348 Louvain-la-Neuve, Belgium*
E-mail: thomas.pardoen@uclouvain.be, c.tekoglu@uclouvain.be

Abstract

A damage model based on Gologanu-Leblond-Devaux constitutive law, extended to account for strain hardening, is integrated with a homogenization scheme in order to investigate the ductile fracture of multiphase and composite metallic alloys. The homogenization scheme supplies the ductile fracture model with an accurate prediction of the load transfer between the matrix and the second phase particles, leading to a better estimate for the overall strength and strain hardening of the composite. Furthermore, the homogenization scheme also allows relating void nucleation -particle fracture and/or particle-matrix interface decohesion- directly to the stress in the particles. This paper focuses on the effects of the mechanical behavior of different phases on the fracture resistance of a composite: a parametric study on an elasto-plastic matrix containing brittle-elastic inclusions is performed by using the integrated homogenization-ductile fracture model, and the results are compared with the results of the damage model alone.

1. Introduction

Ductile fracture can be viewed as three consecutive and interacting stages, namely, nucleation, growth and coalescence of voids. Experiments performed on quite a large range of multiphase metals have shown that void nucleation occurs via two different mechanisms, i.e., particle fracture and particle-matrix interface decohesion (e.g. [1]); voids which nucleate in the matrix, away from the second phase particles, are rarely observed (e.g. [2]). If the particle is brittle and deforms elastically, a void nucleation criterion in terms of the effective stress in the particle can be found by equating the total energy released by formation of a crack in a particle, to the energy associated with the crack-surface formation (e.g. [3]). Void nucleation via particle-matrix interface separation, however, is much more complex due to the fact that both the separation energy and the interface strength play a role in the problem; therefore, neither critical stress nor critical strain based nucleation models are fully satisfactory. In either case, particle fracture or interface decohesion, the quality of a void nucleation model depends on a good assessment of the stress and strain condition at (and round) the particle. In the current literature, phenomenological models incorporating stress and/or strain controlled nucleation conditions are being used, but the critical stress/strain values are usually overall values and not the local values in the particle or along the interface (e.g. [4, 5]). Here, we use an incremental formulation of a Mori-Tanaka scheme for elasto-plastic composites to explicitly account for the stresses and strains in both the matrix and particle. In many engineering materials, such as dual phase (DP) steels, the volume percentage of the second phase particles reaches up to 30% and they have a considerable contribution to the overall strength and strain hardening of the composite. Therefore, the benefits of incorporating the Mori-Tanaka scheme into the fracture model are twofold: it supplies a much more physically sound void nucleation condition, and an explicit representation for each phase in the composite leading to realistic overall mechanical properties.

The following section describes the homogenization scheme (section 2.1) and the damage model (section 2.2). Section 3 briefly explains how the two models are combined. In section 4, the preliminary results of a parametric study are shown, and finally, section 5 concludes this paper.

2. Description of the model

2.1. Mori-Tanaka homogenization scheme

In this section, we briefly introduce the Mori-Tanaka (MT) homogenization scheme for elasto-plastic materials. The reader is referred to [6] and the references therein for an extensive account of the model.

The fundamental solution of Eshelby shows that if an elastic homogeneous ellipsoidal inclusion in an infinite linear elastic matrix is subjected to a uniform remote strain, $\boldsymbol{\varepsilon}^{\mathrm{r}}$, the strain inside the inclusion, $\boldsymbol{\varepsilon}^{\mathrm{i}}$, is uniform and given as

$$\boldsymbol{\varepsilon}^{\mathrm{i}} = \boldsymbol{A} : \boldsymbol{\varepsilon}^{\mathrm{r}} = \left[\boldsymbol{I} + \boldsymbol{\xi} : \left(\left(\boldsymbol{C}^{\mathrm{M}} \right)^{-1} : \boldsymbol{C}^{\mathrm{i}} - \boldsymbol{I} \right) \right]^{-1} : \boldsymbol{\varepsilon}^{\mathrm{r}}, \tag{1}$$

where $\boldsymbol{A}$ is the strain concentration tensor for the single inclusion, $\boldsymbol{I}$ is the fourth order identity tensor, $\boldsymbol{\xi}$ is the Eshelby tensor, $\boldsymbol{C}^{\mathrm{M}}$ and $\boldsymbol{C}^{\mathrm{i}}$ are the stiffness tensors (or the tangent operators, in case of incremental plasticity) of the matrix and the inclusion, respectively [7]. This expression is only valid for dilute composites. To account for interactions among inclusions, MT models relate the average strain in an inclusion, $\langle \boldsymbol{\varepsilon}^{\mathrm{i}} \rangle$, to the average strain in the matrix, $\langle \boldsymbol{\varepsilon}^{\mathrm{M}} \rangle$. In an incremental form, for a non-linear rate-independent mechanical response, Benveniste's version of the MT model reads

$$\left\langle \dot{\boldsymbol{\varepsilon}}^{\mathrm{i}} \right\rangle = \boldsymbol{A} : \left\langle \dot{\boldsymbol{\varepsilon}}^{\mathrm{M}} \right\rangle, \tag{2}$$

and the macroscopic response of the composite is modeled as

$$
\begin{aligned}
&\langle \dot{\boldsymbol{\sigma}} \rangle = \boldsymbol{C} : \langle \dot{\boldsymbol{\varepsilon}} \rangle , \\
&\boldsymbol{C} = \left[f^{\,\mathrm{i}} \boldsymbol{C}^{\mathrm{i}} : \boldsymbol{A} + \left(1 - f^{\,\mathrm{i}}\right) \boldsymbol{C}^{\mathrm{M}} \right] : \left[f^{\,\mathrm{i}} \boldsymbol{A} + \left(1 - f^{\,\mathrm{i}}\right) \boldsymbol{I} \right]^{-1} ,
\end{aligned}
\tag{3}
$$

where f^{i} is the volume fraction of the inclusions and $\boldsymbol{C}$ is the macroscopic tangent operator of the composite. The strain concentration tensor $\boldsymbol{A}$ in Eq. 3 is related to the Eshelby tensor $\boldsymbol{\xi}$ through Eq. 1, and $\boldsymbol{\xi}$ depends on the geometry of the inclusion and the tangent operator $\boldsymbol{C}^{\mathrm{M}}$ of the matrix. Note that even though the matrix material is isotropic, $\boldsymbol{C}^{\mathrm{M}}$ is an anisotropic tensor. Numerical simulations show that using only the isotropic part of $\boldsymbol{C}^{\mathrm{M}}$ (referred to as $\boldsymbol{C}^{\mathrm{M\_iso}}$ in the following) to calculate $\boldsymbol{\xi}$ gives much better results compared to the over-stiff predictions obtained by using the full (anisotropic) tangent operator $\boldsymbol{C}^{\mathrm{M}}$ [6]. In this study, we use a two-step recursive homogenization scheme [8]. The idea behind this scheme, as outlined at the top of Fig. 1, is to divide the composite into two subsystems, each composed of two subphases, one for the matrix (M) and one for the inclusion (i). First, each subsystem is homogenized separately by using the MT scheme, where the Eshelby tensor $\boldsymbol{\xi}$ is calculated with $\boldsymbol{C}^{\mathrm{M}}$ for subsystem-1 (M1-i1) and with $\boldsymbol{C}^{\mathrm{M\_iso}}$ for subsystem-2 (M2-i2). Then, the two subsystems are combined trough an iso-strain (Voigt) homogenization scheme. The volume fractions of both subsystems are arbitrarily taken to be 50%. Among many different homogenization strategies, the one described above was found to give the best comparison with direct unit cell finite element calculations in terms of the overall response of the composite, as well as per-phase behavior. Note that, here, the term "inclusion" is used in a generic sense to refer to a reinforcing phase which can be particles, fibers, etc.

2.2. Damage model

In order to account for the heterogeneity of the void nucleation process, we assume that it starts when the maximum principal stress in a particle reaches a critical value, $\sigma_{\mathrm{c}}^{\mathrm{min}}$, and continues within a range of critical stress values, $\Delta\sigma_{\mathrm{c}}$, corresponding to a distribution of particles with different size and shape, and therefore different fracture strength. The void nucleation rate in a functional form is taken as

$$
\begin{aligned}
&\dot{f}_{\mathrm{nuc}} = g\left(\sigma^{\mathrm{p}}_{\text{max-princ}}\right)\dot{\sigma}^{\mathrm{p}}_{\text{max-princ}}, \\
&\text{with} \\
&g\left(\sigma^{\mathrm{p}}_{\text{max-princ}}\right) = a_1\left(\sigma^{\mathrm{p}}_{\text{max-princ}}\right)^4 + a_2\left(\sigma^{\mathrm{p}}_{\text{max-princ}}\right)^2 + a_3,
\end{aligned}
\tag{4}
$$

where a_i are chosen to avoid discontinuities in the porosity evolution. The increase in porosity with void nucleation Δf_{nuc} is related to the particle volume fraction f_{p} and aspect ratio W_{p} (=1) via (see [1])

$$\Delta f_{\text{nuc}} = \frac{\Delta f_{\text{p}} W_0}{W_{\text{p}}} . \qquad (5)$$

Particles are assumed to lose all their load carrying capacity after void nucleation, giving birth to penny shaped voids with an initial aspect ratio of $W_0 = 0.01$.

As soon as they are nucleated, voids start to grow with the plastic deformation of the surrounding composite. Taking into account volume conservation of the composite, the void growth rate $\dot{f}$ reads

$$\dot{f} = \left(1 - f\right) \dot{\boldsymbol{\varepsilon}}_{ij}^{\text{p}} + \dot{f}_{\text{nuc}} , \qquad (6)$$

where $\dot{\boldsymbol{\varepsilon}}_{ij}^{\text{p}}$ is the plastic strain rate tensor. The constitutive law proposed by Gologanu et al. [9] accounts for spheroidal void shapes. For axisymmetric loading conditions analyzed in this paper, where the main void axis $\boldsymbol{e}_{\text{z}}$ does not rotate and remain parallel to the maximum principal stress, the evolution of the void aspect ratio reads

$$\dot{S} = \left(1 + h_S h_T h_f\right)\left(\dot{\varepsilon}_{zz} - \dot{\varepsilon}_{xx}\right) + h_{Sf} \dot{\boldsymbol{\varepsilon}}_{ii} , \quad \left(S = \ln W\right) , \qquad (7)$$

where the h parameters are functions of W (void aspect ratio), f (porosity) and a power-law strain hardening exponent n. The plastic strain rate is taken to be normal to the flow potential:

$$\dot{\boldsymbol{\varepsilon}}_{ij}^{\text{p}} = \dot{\gamma} \frac{\partial \phi}{\partial \boldsymbol{\sigma}_{ij}} , \qquad (8)$$

with

$$\phi \equiv \frac{C}{\sigma_{\text{y}}^2}\left(\sigma_{\text{zz}} - \sigma_{\text{xx}} + \eta \sigma_{\text{h}}\right)^2 + 2q\left(g+1\right)\left(g+f\right) cosh\left(\kappa \frac{\sigma_{\text{h}}}{\sigma_{\text{y}}}\right) - \left(g+1\right)^2 - q^2\left(g+f\right)^2 = 0. \qquad (9)$$

In Eq. 9, C, η, g and κ are parameters that are functions of W and f, and q is a heuristic adjusting parameter.

The relatively homogeneous plastic deformation (of the matrix) of the composite is interrupted by the localization of the plastic flow in the ligaments between neighboring voids, which corresponds to the onset of coalescence. The following criterion suggested by Thomason [10] is used:

$$\frac{\sigma_{zz}}{\sigma_y}\frac{1}{\left(1-\chi^2\right)}=\alpha\left(\frac{1-\chi}{\chi W}\right)^2+1.24\sqrt{\frac{1}{\chi}}, \tag{10}$$

where the parameter α is a function of the strain hardening exponent n. The Thomason criterion states that coalescence occurs when the stress normal to the localization plane reaches a critical value, which decreases as the voids open (W increases) and get closer (χ increases). For a full account of the damage model, the reader is referred to [1] and the references therein.

3. Integration of the damage model and the Mori-Tanaka scheme

The composite is homogenized using the MT scheme which is described in section 2.1. As shown in Fig. 1, at each plastic strain increment, the macroscopic elastic and plastic moduli of the composite that are calculated by the MT scheme are transmitted to the damage model. By this way, the damage model interacts with the composite as if it is a homogeneous matrix surrounding the voids. Note that the MT and the damage models are subjected to exactly the same axisymmetric triaxial loading conditions. When the maximum principal stress in a particle reaches the critical void nucleation value, σ_c^{min}, particle fracture starts,

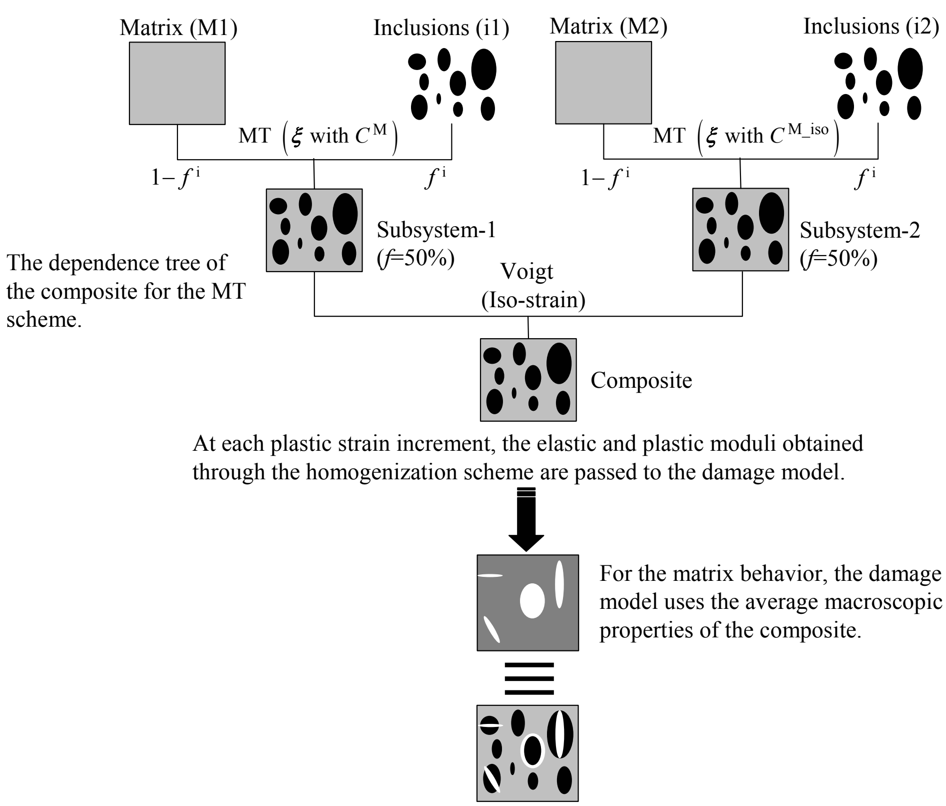

Fig. 1. A schematic diagram showing the dependence tree for the Mori-Tanaka (MT) model and the integration of the MT and the damage models.

and it continues within a range of critical stress values, $\Delta\sigma_c$. The particle volume fraction f_p is decreased in the MT scheme by an amount corresponding to the volume fraction of the particles fractured at that increment, while the increase in porosity due to particle fracture is updated in the damage model through Eq. 5. With increasing plastic strain in the composite, voids grow, change shape and get closer to each other. At each increment, the damage model checks the onset of void coalescence according to the Thomason criterion given in Eq. 10. The composite is assumed here to fracture at the onset of void coalescence.

4. Parametric study

The matrix of the composite is modeled as a J_2 elasto-plastic material with a simple power-law strain hardening defined as

$$\frac{\sigma_y}{\sigma_0} = \left(1 + \frac{E}{\sigma_0}\varepsilon^p\right)^n , \tag{11}$$

where E is the Young's modulus, σ_0 is the initial yield stress, n is the strain hardening exponent, and ε^p is the accumulated plastic strain. The material properties of the matrix are E/σ_0=125, ν=0.3, with n=0.1 and n=0.3, for two different cases analyzed, respectively. Particles are assumed to be brittle-elastic, with Young's modulus and Poisson ratio equal to those of the matrix. The aspect ratio of the newly nucleated voids is W_0=0.01. Axisymmetric loading is imposed under constant stress triaxiality T.

Figure 2 shows the normalized equivalent stress σ_{eq}/σ_0 versus the equivalent plastic strain ε_{eq}, for the MT homogenization scheme alone (no particle fracture), for the damage model alone (with all the particles assumed to have fractured right at the beginning of the plastic regime), and for the integrated MT-damage model (with relatively early and fast void nucleation, i.e., σ_c^{min}/σ_0=2 and $\Delta\sigma_c/\sigma_0$=0.5), for T=1/3. For the integrated MT-damage model, after the onset of void nucleation, there is a competition between the hardening due to the strain hardening of the matrix of the composite and softening due to the decrease in the load carrying capacity of the fractured particles. It is clear that the results of the integrated MT-damage model converges to the results of the damage model alone for $\sigma_c^{min}/\sigma_0 \to 0$ and $\Delta\sigma_c/\sigma_0 \to 0$, and to the results of MT scheme alone for $\sigma_c^{min}/\sigma_0 \to \infty$.

Figure 3 shows the evolution of the fracture strain ε_f plotted against the triaxiality ratio T, for the damage and integrated MT-damage models. The initial particle volume fraction is taken to be 10%, with void nucleation parameters (for the integrated MT-damage model) σ_c^{min}/σ_0=16 and $\Delta\sigma_c/\sigma_0$=12. We consider two cases, with n=0.1 and n=0.3, respectively. There is a considerable difference between the predictions of the two models, the fracture strains being much larger for the integrated MT-damage model. In the regime T<1, a high gradient in ε_f

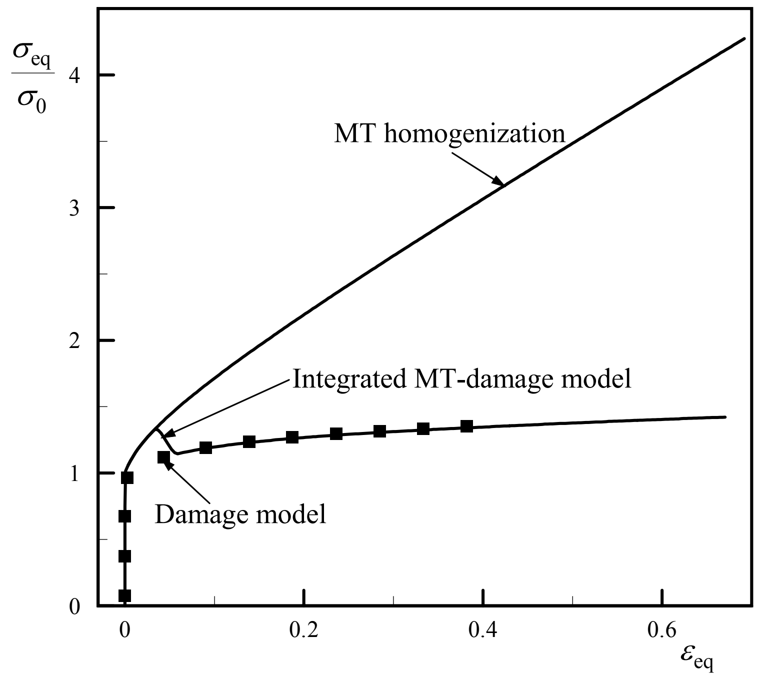

Fig. 2. Variation of the normalized equivalent stress σ_{eq}/σ_0 as a function of the equivalent plastic strain ε_{eq}, for the MT homogenization scheme alone (no particle fracture), for the damage model alone (with all the particles assumed to have fractured right at the beginning of the plastic regime), and for the integrated MT-damage model (with relatively early and fast void nucleation, i.e., σ_c^{min}/σ_0=2 and $\Delta\sigma_c/\sigma_0$=0.5), for T=1/3.

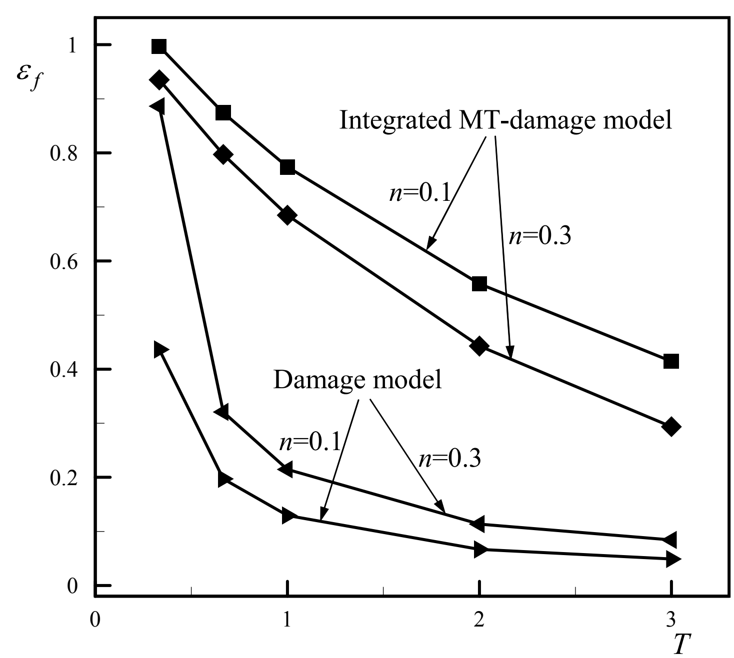

Fig. 3. Variation of the fracture strain ε_f as a function the triaxiality ratio T, for the damage and integrated MT-damage models, for two cases with a different strain hardening exponent, $n = 0.1$ and $n = 0.3$, respectively.

with increasing T is observed for the damage model, which is larger for $n=0.3$, whereas ε_f decreases almost uniformly with increasing T for the integrated MT-damage model, for both $n=0.1$ and $n=0.3$. Moreover, the damage model predicts an increase in ε_f with increasing n, while an opposite tendency, i.e. a decrease in ε_f with increasing n, is observed for the integrated MT-damage model. The reason behind this crucial difference is the fact that, with increasing n, void nucleation starts earlier and occurs faster in the MT-damage model: the softening introduced by particle fracture is more dominant than the hardening introduced by the strain hardening of the matrix of the composite. In the damage model, however, all the particles are assumed to have fractured right at the beginning of the plastic regime and therefore, the decisive parameter for the onset of coalescence is the strain hardening exponent n. For the integrated MT-damage model, coalescence occurs during void nucleation for all T values. In the case of $n=0.1$ ($n=0.3$), the percentage of the fractured particles at the composite fracture is 50% (72%) for T=1/3, while it decreases with increasing T and it is only 6% (28%) for T=3.

5. Discussions and conclusions

Advanced engineering metallic alloys, such as dual phase steels and multiphase or composite titanium alloys, contain a second phase volume fraction of up to 30% or more. The different phases in such composites not only have different mechanical properties, but also they usually obey different strain hardening behaviors. Therefore, for an accurate prediction of the overall macroscopic properties as well as the fracture strain of the composite, a fracture model should explicitly account for each phase in the composite. For this purpose, we integrated an incremental Mori-Tanaka homogenization scheme (see section 2.1) with an advanced micromechanics-based ductile fracture model (see section 2.2). The parametric study performed in section 4 showed that there is a dramatic difference between the fracture strain values predicted by the damage and the integrated MT-damage models, being much larger for the latter. Besides, the fracture strain ε_f increases with increasing n for the damage model, whereas it decreases with increasing n for the MT-damage model (see Fig.3). This difference originates from the difference in void nucleation mechanisms assumed in the two models, and it clearly confirms the importance of a good void nucleation model to have accurate predictions for the fracture behavior. We also compared the predictions of the integrated MT-damage model with experiments performed on dual phase steels, in terms of fracture strain, and the preliminary results are quite successful [11]. The next step is to implement a more realistic strain hardening behavior for each phase.

References

[1] D. Lassance, D. Fabrègue, F. Delannay, T. Pardoen, Micromechanics of room and high temperature fracture in 6xxx Al alloys, Prog Mater Sci 52 (2007) 62-129.

[2] A.W. Thompson and J.C. Williams, Nuclei for ductile fracture in titanium, in: D.M.R. Taplin (Ed.), Proceedings of the 4th International Conference on Fracture, Pergamon Press, Oxford, 1977, vol. 2, pp. 343-348.

[3] S. Jun, D. Zengjie, L. Zhonghua, T. Mingjing, Fracture strength of spheroidal carbide particle, Int J Fract 42 (1990) 39-42.

[4] C. Chu and A. Needleman, Void nucleation effects in biaxially stretched sheets, J Eng Mater Technol 102 (1980) 249-256.

[5] V. Tvergaard, Material failure by void growth to coalescence, Adv Appl Mech 27 (1990) 83-151.

[6] I. Doghri and A. Ouaar, Homogenization of two-phase elasto-plastic composite materials and structures – study of tangent operators, cyclic plasticity and numerical algorithms, Int J Solids Struct 40 (2003) 1681-1712.

[7] J.D. Eshelby, The determination of the elastic field of an ellipsoidal inclusion and related problems, Proc R Soc A 241 (1957) 376-396.

[8] T. Van Hoof, O. Piérard and F. Lani, A coupled mean-field/Gurson-Tvergaard micromechanical model for ductile fracture in multiphase materials with large volume fraction of voids. Part 1: Model description and validation, in preparation.

[9] M. Gologanu, J.-B. Leblond, G. Perrin, J. Devaux, Recent extensions of Gurson's model for porous ductile metals, in: P. Suquet (Ed.), Continuum Micromechanics, Springer-Verlag, New York, 1997, pp. 61-130.

[10] P. F. Thomason, Ductile Fracture of Metals, Pergamon Press, Oxford, 1990.

[11] A.P. Pierman, C. Tekoğlu, T. Pardoen, P.J. Jacques, Nucleation, growth and coalescence of voids in dual phase steels: from model microstructures to microstructure based modeling, submitted to: Proceedings of the 12th International Conference on Fracture, Ottawa, 2009.