A Consistent Anisotropic Brittle Damage Model Based on Kinking Elliptical Microcracks

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1. Introduction

It is known from experiments that all materials, and in more special case brittle materials, under general loading conditions develop anisotropic damage [1]. For a given stress state, materials damaged by microcracks in general accumulate additional damage through the growth of these microcracks. Considering this the concern of this paper is to provide a consistent, continuum damage model based on the micromechanical framework and the local anisotropy (orthotropy) induced by kinking and growing elliptical and/or circular microcracks. For clarity purposes and to explain the main issues of the proposed model in a more clear mathematical way, the complexity of the proposed damage model is reduced here by leaving out the thermal effects and other non-mechanical phenomena. Strains and rotations are assumed to be small; hence the framework of linear elastic fracture mechanics can be applied. Furthermore, viscous effects and permanent deformations are neglected and the material behavior is assumed to be linear elastic in its pristine state. The small strain assumption, and the lack of permanent deformations in this model makes it suitable to show the evolution of damage in structures with brittle and quasi-brittle fracture behavior experiencing high-cycle fatigue.

Generally, it is impossible to formulate a damage model covering the mixed-mode propagation of microcracks in a fully traceable way. Considering microcracks in the form of elliptical and circular cracks, one may calculate the kinking of the initial cracks analytically only for the first load increment. After the kinking of the initial cracks, however, the mathematical formulation of the next kinking steps is no longer possible. To overcome this difficulty, some researchers have introduced models based on simplifying assumption such as fixing the plane of the initiated microcracks, so that microcracks may only grow in a self-similar manner. This assumption may be acceptable for the case of monotonic loading or cyclic loading with a constant loading direction, but for loads with changing direction and amplitude, as is the case for non-proportional loads or even sequential loads, this assumption leads to underestimating the damage, since it does not allow for crack kinking. The assumption of self-similar growth of mixed-mode cracks, in general, results in a smaller damage accumulation than what the real mixed-mode kinking results in.

To cover the mentioned shortcomings of other models, and to overcome the difficulties in the formulation of a damage model, which accounts for the kinking and growth of microcracks in a mathematical traceable manner, a micromechanical based continuum damage model is proposed here, which is based on the reduction of stiffness due to kinking elliptical microcracks. To be able to formulate the model in a fully mathematical traceable way, the concept of an equivalent crack is introduced in the sense that a kinked crack is replaced with an equivalent elliptical crack, resulting in an equivalent dissipation of energy. Basically, eight degrees of freedom can be considered for each equivalent elliptical microcrack replacing the kinking one. These are the major and minor axes of the ellipse, orientation of the microcrack given by three Eulerian rotation angles, and the position of the crack in space. Considering the concept of unit cell and assuming that the microcrack is located in the center of the cell, the position of the microcrack can be fixed and may be left out of the formulation. This is because in the case of non-interacting cracks, the position of the crack does not have an impact on the elastic properties of the material. To calculate the other five unknown characteristics of the equivalent crack, different postulates may be proposed. Here, to determine the geometry and orientation of the equivalent elliptical crack, the postulates of equivalent dissipation and equivalent damage induced anisotropy are considered.

Such a formulation of the dissipative damage process due to kinking equivalent elliptical microcracks, taking into account the damage induced orthotropy of an elliptical crack in a local sense, results in a consistent damage model capturing the load history through the local orthotropic degradation of the mechanical material properties.

2. Effective continuum elastic properties of damaged media

2.1. Presence of a single internal elliptical crack

Consider a single internal elliptical crack in an infinite, homogeneous, isotropic and elastic continuum subjected to mechanical loads applied at infinity. This problem can be decomposed into two sub problems: that of the continuum without a crack subjected to the remote traction field, and that of the same continuum where only the crack faces are subjected to the traction field. The traction field of the second sub-problem is determined from the condition requiring that the total tractions over the mating faces of the crack vanish.

Within the framework of linear elastic fracture mechanics, the compliance tensor of the damaged unit cell can be decomposed to that of the matrix and the change of compliance due to the crack (Eq.1):

$$\tilde{\mathbf{S}} = \mathbf{S}^{\mathbf{Matrix}} + \Delta\mathbf{S}^{\mathbf{Crack}} \quad \text{(Eq.1)}$$

Compliance tensor of an isotropic homogenous matrix is known and can be calculated in terms of Young's modulus and Poisson's ratio. The change of compliance tensor due to the presence of crack can be derived from the contribution to the complementary strain energy corresponding to this sub problem (Eq.2 and Eq.3)

$$\psi^* = \frac{1}{V}\int_0^{r}\left(\oint_c \vec{J}_1\,dl\right)\cdot d\vec{r} = \frac{1}{V}\int_0^{r}\left(\oint_c J_1\,\mathbf{e}_n\cdot\mathbf{e}_r\,dl\right)dr \qquad \text{(Eq.2)}$$

$$\Delta S_{ijmn}^{Crack} = \frac{\partial^2\psi^*}{\partial\sigma_{ij}\,\partial\sigma_{mn}} \qquad \text{(Eq.3)}$$

Here, J_1 is the first component of the J-integral vector, and c is the path encircling the ellipse boarder.

Substituting the expressions for the stress intensity factors and performing the requisite integration, the strain energy released (complementary energy) by a single elliptical crack in an infinite, homogeneous, isotropic solid in its pristine state is resulting [2], from which the components of the fourth-rank compliance tensor attributed to the presence of a single elliptical crack are resulting.

2.2. Kinking of an internal elliptical crack

In a similar manner to previous section, the compliance tensor of a material containing a kinked crack can be decomposed into the compliance tensor of the matrix material in its pristine state, change of compliance due to the presence of the microcrack, and change of compliance due to the kinking and growth of the microcrack (Eq.4, see also [3])

$$\tilde{\mathbf{S}} = \mathbf{S}^{Matrix} + \Delta\mathbf{S}^{Crack} + \Delta\mathbf{S}^{Kink}, \qquad \text{(Eq.4)}$$

where the analytical evaluation of the first two tensors were described in previous section.

The rate of the change of the compliance tensor for a volume element of elastic material, attributable to an extension rate, through which a point on the perimeter of a single crack kinks to a new position, is given by Eq.5 [3]

$$\dot{S}_{ijmn}^{Kink} = \frac{\partial^2\dot{\psi}^{**}}{\partial\sigma_{ij}\,\partial\sigma_{mn}} \qquad \text{(Eq.5)}$$

where rate of complementary energy associated with the kinking of the crack is given by Eq.6:

$$\dot{\psi}^{**} = \frac{1}{V}\oint_c \left(G(s)\,\dot{s}\right)dl, \qquad \text{(Eq.6)}$$

Here, G(s) is the energy released during the kinking of crack with a local extension s, and is given in terms of stress intensity factors at the propagated crack front.

Considering the expansion of the stress intensity factors [4] in terms of the extension length and the crack tip parameters prior to kinking, i.e. SIF's and T-stresses [5], the expression for the fourth-rank tensor of the rate of the change of compliance due to the kinking of an elliptical crack reduces to Eq.7:

$$\dot{S}_{ijmn}^{Kink} = \frac{1}{V}\int_0^{2\pi} M_{\alpha\beta}\sqrt{\alpha^2\sin^2\varphi+\beta^2\cos^2\varphi}\,\Bigg\{F_{\alpha\lambda}F_{\beta\mu}\frac{\partial K_\lambda}{\partial\sigma_{ij}}\frac{\partial K_\mu}{\partial\sigma_{mn}} + \left(F_{\alpha\lambda}G_{\beta\mu}\frac{\partial K_\lambda}{\partial\sigma_{ij}}\frac{\partial T_\mu}{\partial\sigma_{mn}} + G_{\alpha\lambda}F_{\beta\mu}\frac{\partial T_\lambda}{\partial\sigma_{ij}}\frac{\partial K_\mu}{\partial\sigma_{mn}}\right)\sqrt{s} + G_{\alpha\lambda}G_{\beta\mu}\frac{\partial T_\lambda}{\partial\sigma_{ij}}\frac{\partial T_\mu}{\partial\sigma_{mn}}s\Bigg\}\,\dot{s}\,d\varphi\,, \quad \text{(Eq.7)}$$

It should be noted that there is no closed form solution for the compliance tensor attributable to the kinking of a crack, since depending on the crack problem and the resulting mode-mixity, the kinking angle φ and the propagation rate would change accordingly. Hence, for each crack problem above integral should be solved individually.

3. A fracture based anisotropic continuum damage model

Based on the hypothesis of statistical homogeneity and weak interaction of micro defects, which is reasonable for a modest distribution of heterogeneities [6], the first step in the formulation of the proposed damage model requires the formulation of the change of continuum elastic properties due to the presence, kinking and growth of elliptical and/or circular microcracks. In previous sections, based on the assumption of non-interacting cracks, the effect of a single mixed-mode elliptical crack and its kinking on the effective elastic properties of materials was formulated in an analytical manner. After the kinking of the initial cracks, however, mathematical formulation of the next kinking steps is no longer possible. To overcome this difficulty in the formulation of a damage model, which accounts for the kinking and growth of microcracks in a mathematical traceable way, the concept of an equivalent elliptical crack may be considered. In this regard, a kinked crack is replaced with an equivalent elliptical crack in a thermodynamically consistent manner (Fig.1), resulting equivalent dissipation of energy and equivalent type of damage induced anisotropy.

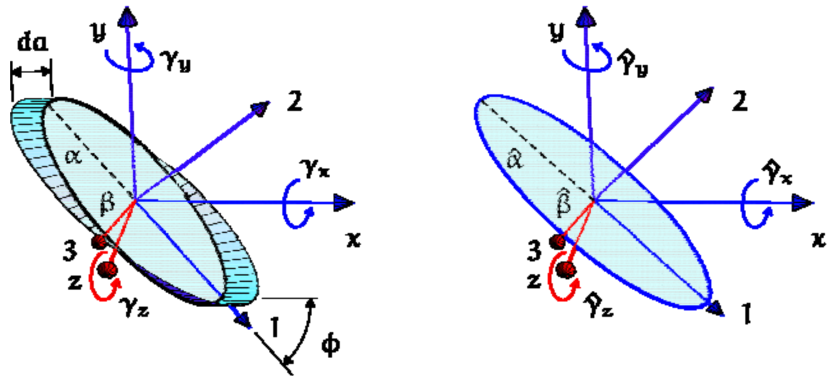


Fig.1: kinked crack and its equivalent elliptical crack

Basically, eight degrees of freedom can be considered for each equivalent elliptical microcrack replacing the kinking one. These are the major and minor axes of the ellipse (2 unknowns), orientation of the microcrack given by three Eulerian rotation angles (3 unknowns), and the position of the crack in the space (3 unknowns). Considering the concept of the unit cell and assuming that the microcrack is located in the center of the cell, the position of the microcrack can be considered a priori known (fixed) and be left out of the formulation. To calculate the other five unknown characteristics of the equivalent crack, i.e. the geometry and orientation of the equivalent crack, different postulates may be considered. In this work, the equivalent replacement crack model is based on the postulates of equivalent dissipations and equivalent damage induced anisotropy. The geometry and the orientation of the equivalent elliptical crack replacing the kinked one then result considering two optimization problems. The first problem takes into account the fact that the local damage associated with a single planar elliptical crack results in an orthotropic material symmetry, thus it can be argued that changing the type of material symmetry from isotropy to orthotropy may imply the existence of local damage due to an elliptical crack [7]. Thus it is deduced that the orientation of the equivalent crack replacing the kinked one is such that the resulting orthotropy axes are aligned with the ones due to the damage associated with the kinked crack. The second optimization problem results in the geometry of the equivalent elliptical crack in the sense that the components of the change of compliance due to the kinked crack and the equivalent crack are approximately identical.

Such a formulation of the dissipative damage process due to kinking elliptical microcracks, taking into account the damage induced orthotropy of an elliptical crack in a local sense, results in a consistent damage model capturing the load history through the local orthotropic degradation of the mechanical properties. The proposed continuum damage model based on the reduction of stiffness due to the kinking of equivalent elliptical microcracks results the effective elastic properties of a damaged material volume element in a completely analytical and consistent way. Based on the incremental analysis of the effective elasticity tensor for the given current values of the local stress and strain tensors, and taking the load history into account by introducing the concept of an equivalent elliptical crack, the propagation of microcrack is calculated by considering a crack evolution law. In this study, the propagation of microcracks is governed by the modified Paris' law given in [7] coupled with the fracture criterion of maximum driving force [8].

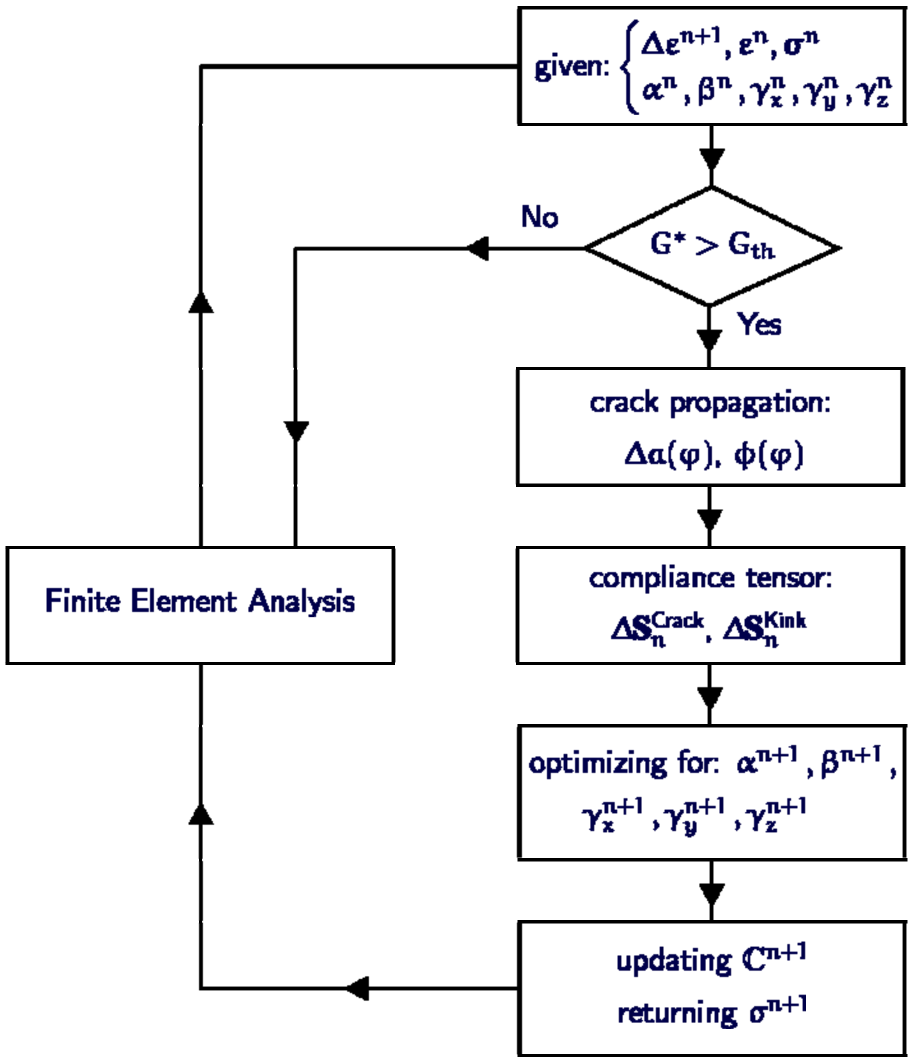


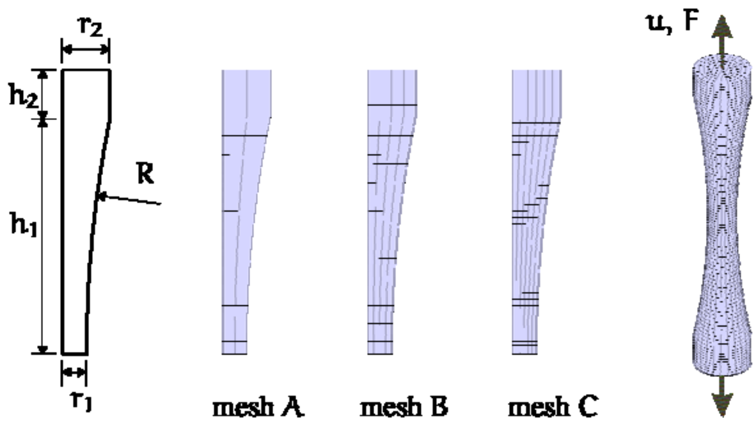
Table 1: algorithm for a consistent fracture based continuum damage model

The proposed continuum damage model based on the reduction of stiffness due to kinking elliptical microcracks can be easily implemented in a finite element code. The algorithm of the proposed damage model based is summarized in table 1 in an incremental manner.

The proposed damage model can also capture the unilateral effect observed in tension-compression tests, observed for a certain class of materials including ceramics and concrete, provided that for a passive crack the components of the corresponding compliance components are recovered, and they return to their degraded state upon the activation of the crack.

4. A numerical example; mesh sensitivity

The objective of this example is to show the degree of the mesh dependency for the proposed damage model. For this, the specimen of the form given in Fig.2 is considered.



$$r_1 = 10\,\text{mm}, \quad r_2 = 20\,\text{mm}$$

$$h_1 = 100\,\text{mm}, \quad h_2 = 20\,\text{mm}$$

$$R = 505\,\text{mm}$$

Fig. 2: specimen geometry and different mesh patterns

Displacement controlled analyses have been performed for different discretization levels for 2,000,000 cycles. All models are meshed with the help of hexahedral elements with quadratic displacement behavior, resulting 48, 384 and 1728 elements. Material is AISI-4130 Steel with mechanical properties given in Eq.8.

$$E = 203\,\text{GPa}, \quad G = 77\,\text{GPa}$$

$$\nu = 0.32, \quad K_{Ic} = 162\,\text{MPa}\sqrt{m}$$

$$\sigma_u = 1,232\,\text{MPa}, \quad \sigma_y = 1,036\,\text{MPa} \qquad \text{(Eq.8)}$$

The resulting force-cycle curves for these experiments are given in Fig.3, where F_N and F_o are the resultant forces at the end cross section of the specimen of radius r_2, which correspond respectively to the current load-cycle and the initial load-cycle. The good agreement between the results corresponding to different discretization levels demonstrates the very small mesh dependency of the model. This is due to the fact that in the considered fatigue microcrack evolution law, the rate of the driving force is not present.

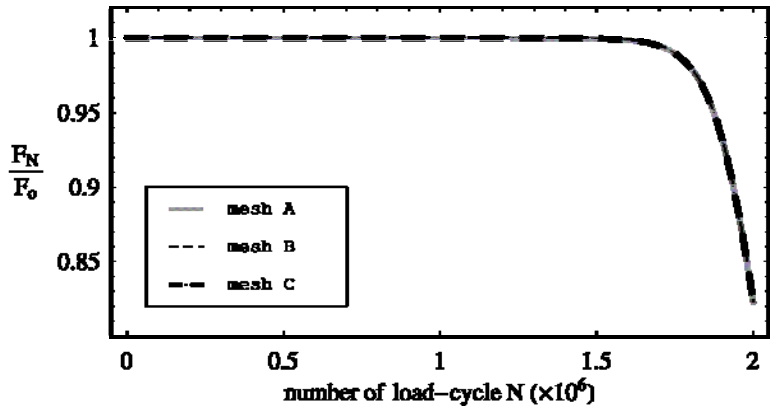


Fig.3: load-cycle curves for different mesh patterns

5. Summary and conclusions

A micromechanical based continuum damage model based on the reduction of stiffness due to kinking elliptical microcracks is proposed to show the anisotropic irreversible process of damage accumulation due to microcrack kinking and growth in brittle and quasi-brittle materials. The model is formulated consistently in a fully analytical way and degradation of the elastic properties is associated with the irreversible process of crack kinking and growth. In order to make the formulation of the model mathematically traceable, the concept of an equivalent elliptical crack is proposed. The geometry and the orientation of the equivalent crack are resulting from the postulates of equivalent dissipation and equivalent damage induced anisotropy. The evolution of the cracks is governed by the criterion of maximum driving force coupled with a fatigue crack evolution law. The proposed formulation yields a consistent damage model which considers the kinking and growth of microcracks, and accounts for the type of damage induced anisotropy in a local sense. The proposed model is appropriate to show the evolution of damage in brittle and quasi-brittle materials under cyclic loading.

6. Acknowledgements

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7. References

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