A coupled two-scale computational approach for masonry out-of-plane failure

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1 Introduction

The formulation of macroscopic constitutive laws for the behaviour of masonry is a complex task, due to its strongly heterogeneous microstructure which considerably influences its overall mechanical behaviour. Due to the quasi-brittle nature of its constituents, this results in initial and damage-induced (evolving) anisotropy properties, accompanied with localisation of damage. In its structural use, such a material may be subjected to cracking, leading to localisation of damage at both the structural and fine scales. Closed-form laws have therefore been developed for equivalent anisotropic media for elastic and cracking behaviour [1], later applied for the modelling of plate failure [2]. The use of such models in the cracking regime is however impeded by their costly and cumbersome identification. As a complementary approach to closed-form constitutive relations, the multi-scale computational strategies aim at solving this issue by deducing a homogenised response at the structural scale from a representative volume element (RVE), based on constituents properties and averaging theorems.

2 Multi-scale modelling of thin masonry shells

The purpose of a computational homogenisation procedure is to obtain numerically the average macroscopic response of a heterogeneous material from its underlying mesostructure and the behaviour of its constituents. In a computational context, in each macroscopic or coarse scale point of the structural scale discretisation, a sample of the mesostructure is used to determine the material response. For this purpose the local macroscopic strain measure is applied in an average sense to the mesostructure and the resulting mesostructural stresses are determined numerically. The averaging of these mesostructural stresses and the condensation of the mesostructural tangent stiffness to the homogenised tangent stiffness then furnish the macroscopic material response associated with the macroscopic point. The definition of such a nested scheme essentially requires the definition of four ingredients: (i) a fine scale constitutive description for the constituents, (ii) the definition of a representative mesostructural sample, (iii) the choice of a coarse scale representation, and (iv) the set-up of scale transitions linking structural and fine scale quantities.

A scale transition for homogenisation towards an elastic Kirchhoff-Love shell behaviour was recently proposed in [3] for running bond masonry. This method is adapted to non-linear material response. Based on a periodicity assumption (see Figure 1 for the case of masonry), a strain-periodic displacement field may be imposed under the form

$$u_\alpha(\vec{x}) = E_{\alpha\beta}x_\beta + \chi_{\alpha\beta}x_\beta x_3 + u^p_\alpha(\vec{x}) \qquad u_3(\vec{x}) = -\frac{1}{2}\chi_{\alpha\beta}x_\alpha x_\beta + u^p_3(\vec{x}) \quad (1)$$

where $\vec{u}^p$ is a periodic fluctuation such that $\vec{u}^p(\vec{x}+\vec{V}^\alpha) = \vec{u}^p(\vec{x})$, see [3,4]. The strain measures associated with a shell kinematical description can then be expressed in terms of the controlling degrees of freedom depicted in Fig. 1. The constituents inside the unit cell may be modelled using any closed-form formulation. The simplest representation for mortar joints consists in cohesive zones equipped with a Mohr-Coulomb criterion. The bricks are assumed to exhibit a linear elastic behaviour. As a result, the response of a coarse scale point under any loading program may be computed.

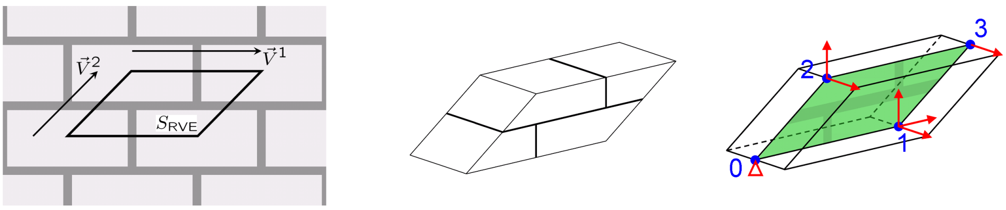

Figure 1: Periodic masonry structure (left) and controlling degrees of freedom (right), see [3].

3 Upscaling framework for failure in thin masonry shell

3.1 Localisation detection at the structural scale

Failure in masonry shells is accompanied by the localisation of deformation and of degradation. Any localisation enhancement by means of discrete failure zones at the structural scale to represent failure as performed in [5] for planar cases would require a criterion to detect localisation and to determine its orientation. In the same spirit, the definition of computational homogenisation-based failure should also take into account structural scale localisation detection. However, in a computational homogenisation procedure, the macroscopic material response is not postulated a priori but rather computed from the material laws introduced at the level of the mesostructural RVE. The detection and orientation of macroscopic localisation should then be based on the computationally homogenised quantities, the only available information related to the average material behaviour.

The detection of the structural scale localisation can be based on the acoustic tensor concept extended to the shell description, see [6]. This tensor has to be constructed based on the homogenised stiffness such that the localisation detection takes into account the coupling of flexural and membrane effects. It can be shown, see [7], that such a procedure allows to extract mesostructurally motivated average localisation orientations, based on the non positive definiteness of this tensor, for various coupled flexural-membrane loading paths. Note that a local maximum is found in the negative range of the related acoustic tensor determinant spectrum, which exactly matches the average orientation of the structural localisation, see Figure 2 for illustration.

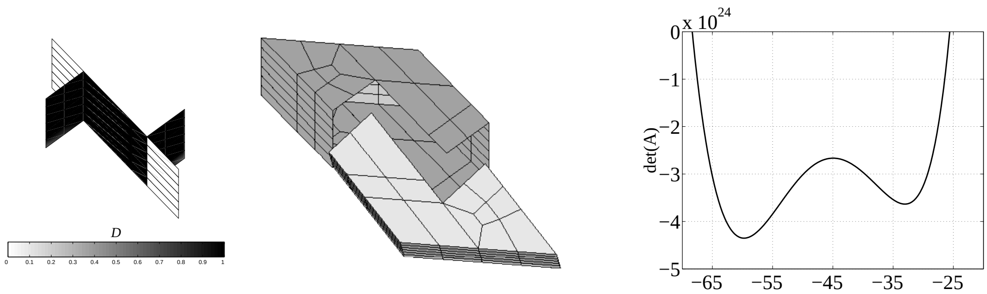

Figure 2: Out-of-plane stair-case bending failure at 45° (brick shape factor of 0.5): joint damage distribution (left), Deformed shape of the unit cell (centre) and related acoustic tensor determinant spectrum (right): non positive values indicate orientation for potential localisation, where the local maximum exactly matches the fine scale-motivated orientation.

3.2 Modelling localisation at the structural scale

At the coarse scale, a shell description is used since the appearance of tensile damage couples the in-plane and flexural effects. The structural scale problem is solved using the finite element method and using an embedded strong discontinuity model in which the behaviour of the discontinuity is obtained from fine scale computations. Once structural localisation is detected, the coarse scale displacement field is enriched by a strong discontinuity, as proposed by Armero and Ehrlich (2006). Displacement and rotation jumps $\vec{\xi}$ are introduced along a discontinuity line Γ_d, see Fig.3, the orientation of which is deduced from the acoustic tensor-based criterion. This jump is added to the regular continuous part of the displacement field according to

$$\vec{u}_e = \vec{u} + \Psi\vec{\xi} \tag{2}$$

where $\vec{u}$ represents the displacements and rotations degrees of freedom are collected, and Ψ represents a set of functions exhibiting a unit jump along a curve Γ_d. Based on the discretisation of the regular and discontinuous parts of the displacement field, the generalised strains (membrane deformations and curvatures) in the bulk of the material are obtained as

$$\mathbf{E}_e = (\vec{\nabla}\vec{u}_e)^{\text{sym}} = \mathbf{E}(\vec{u}) + \mathbf{G}(\vec{\xi}) + (\vec{\xi}\,\vec{n})^{\text{sym}}\,\delta_{\Gamma_d} \tag{3}$$

where $\mathbf{E}(\vec{u})$ is the strain tensor based on classical kinematics, $\mathbf{G}(\vec{\xi})$ is the regular part of the enhanced strain tensor $\mathbf{E}_e$ which depends on the displacement jump and δ_{Γ_d} is the Dirac function centered on the discontinuity line, see [8]. In order to determine the additional displacement jump fields, the weak form of equilibrium is solved together with a weak continuity condition on generalised stresses (bending moments, normal efforts and shear resultant forces) along the discontinuity

$$\int_{\Gamma_d} \left[\delta\vec{\xi}_u \cdot \left(\vec{N}_d - \mathbf{n} \cdot \vec{n}\right) + \delta\vec{\xi}_\theta \cdot \left(\vec{M}_d - \mathbf{m} \cdot \vec{n}\right)\right] \mathrm{d}\Gamma = 0 \tag{4}$$

where the stress resultants in the bulk are given by $\vec{N} = \mathbf{n} \cdot \vec{n}$ and $\vec{M} = \mathbf{m} \cdot \vec{n}$, and where $\vec{N}_d$ and $\vec{M}_d$ represent the generalised stresses in the discontinuity. A material response which links the discontinuity stresses to the displacement jumps is required to drive the discontinuity and reads

$$\delta\vec{\sigma}_d =^2 \mathbf{C}_d \cdot \delta\vec{\xi} \tag{5}$$

where ${}^2\mathbf{C}_d$ is the discontinuity tangent stiffness tensor and $\vec{\sigma}_d$ represents the generalised Kirchhoff-Love stresses. Once the embedded discontinuity is introduced, the bulk of the element is assumed to unload elastically from the state reached at that point.

Contrary to the approach proposed in [8] where constitutive laws are given by closed-form laws, both the bulk and discontinuity material behaviours are deduced from fine scale unit cell computations. A material secant stiffness is extracted from the unit cell in which the structural localisation has just been detected. The material behaviour of the discontinuity, described by Eq. (5) at the coarse scale, must be extracted from the fine scale description by means of an enhanced upscaling procedure. A further damaging unit cell is used for this purpose, which will be denoted in the sequel as localising volume element (LVE).

The extraction of the coarse scale discontinuity response requires the definition of an average strain to be applied on the LVE from the coarse scale displacement jump; as well as the evaluation of $\vec{\sigma}_d$ and ${}^2\mathbf{C}_d$ from the results of the LVE computation. An approximate energy consistency argument is used in order to build a relationship between the displacement jump vector $\vec{\xi}$ across a zero-thickness zone with an orientation $\vec{n}$ used at the coarse scale, and the average strain applied to a localising region with a finite volume detected at the fine scale. The localisation width defining the volume of the localising region therefore has to enter this relationship to take into account in the coarse scale description the finite fine scale volume on which damage localisation occurs. The overall procedure combining the localisation treatment at both the constituents and structural scales is depicted in Fig. 3.

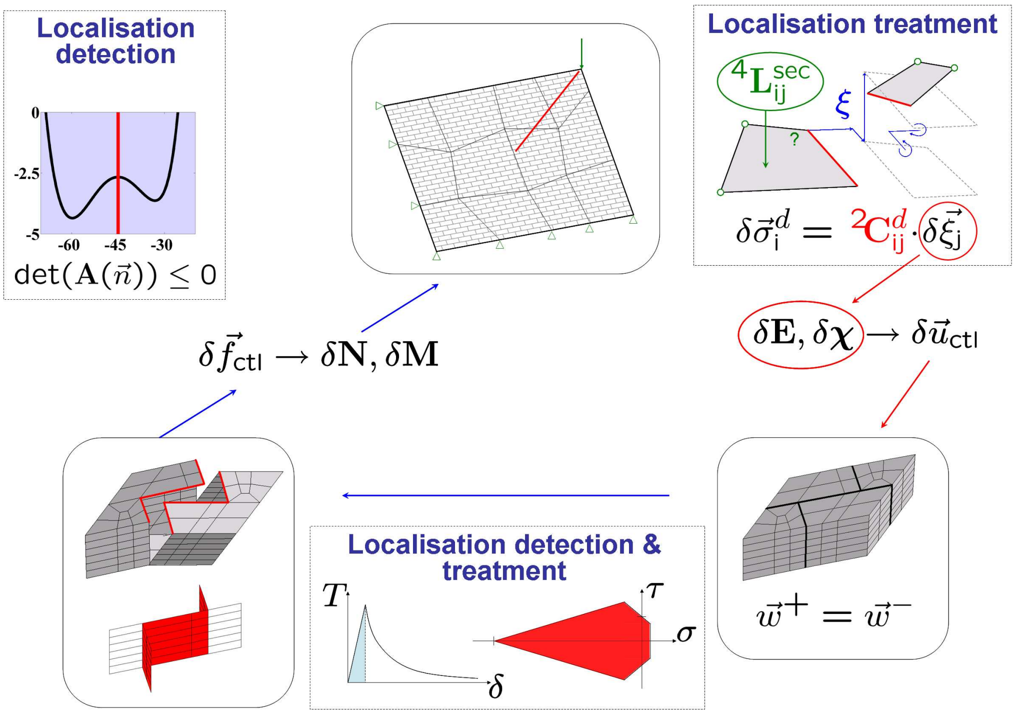

Figure 3: Outline of the complete multi-scale localisation-enhanced scheme

4 Application

The proposed multi-scale scheme was implemented within a parallel computational scheme. A planar case was illustrated in [9] where the multiscale results are compared to fine scale computations in order to analyse the effects of the periodicity and scale separation assumptions of the computational homogenisation procedure. Fig. 4 illustrates such a comparison at the peak load of a confined shear wall test. For the out-of-plane behaviour, the capacities of the proposed

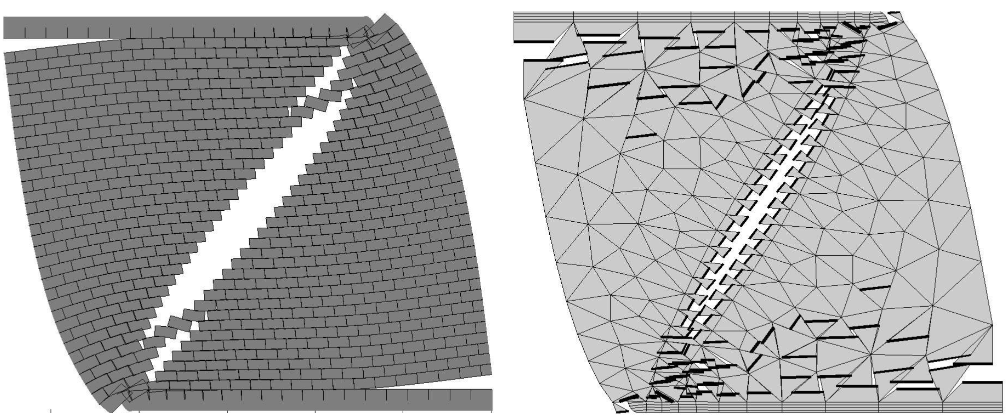

Figure 4: Confined shear wall test with comparison of (left) fine scale modelling results and (right) multiscale modelling results. The same fine scale material parameters were used in both computations.

approach will be shown by means of two structural computations. First, the case of bed joint out-of-plane failure mode propagation will be considered on thin masonry shell subjected to pure bending. A defect is introduced in the bed joint of one unit cell in order to initiate the crack propagation, see Fig. 5. The structural response of the masonry shell will be drawn for different values of the mortar joint fracture energy in order to show that the mesostructural material parameters are properly upscaled. This case will also allow to show the appearance of membrane-flexural couplings due to the different tensile and compressive strengths of the damaging joints are well incorporated in the homogenisation procedure and in the localisation analyses. Another structural computation will also be presented for the more complex stair-case out-of-plane failure mode propagation.

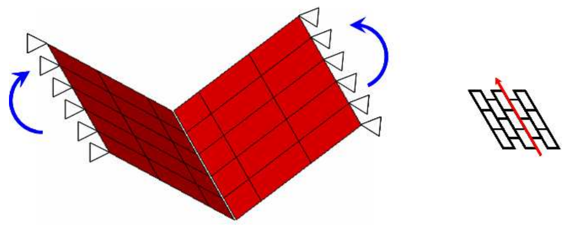

Figure 5: Application: bed joint out-of-plane failure mode (left) and bed joint orientation (right).

5 Conclusions

The multi-scale methodology proves to be a valuable tool for the investigation of masonry structures. In particular, it allows to account for the strong coupling between the structural response and the underlying mesostructural features of the material. Specific enhancements are however needed in order to account properly for the consequences of the quasi-brittle nature of the constituents. However, localisation therefore needs to be detected and treated at both the mesoscopic and macroscopic scales by means of an enhanced scale transition.

6 References

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