Pre-fracture zone for the antiplane shear crack in structured material

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For description of pre-fracture zone, the modified Leonov-Panasyuk-Dugdale model is proposed. The simple relations have been derived for the following critical parameters: shear stresses, pre-fracture zone length, and III mode SIF. Passage to the limit from the sufficient fracture criterion to the necessary one takes place when the pre-fracture zone length vanishes. The critical stresses obtained under the necessary and sufficient criteria differ substantially. The critical SIF obtained under the sufficient criterion are calculated within the framework of the model by the grain diameter, and conventional parameters of the diagram of shear stresses versus shear strain.

1. Introduction. Complex setting up the problem of the distribution of stresses and displacements for the antiplane shear crack in a pre-fracture zone for elastic-plastic materials relate to nonlinear fracture mechanics. This complex nonlinear problem will be significantly simplified in the paper. To accomplish this, we make use of classical representations of linear fracture mechanics when antiplane shear cracks are modeled by bilateral cuts. In order to derive closed relations, we apply the modification of Leonov-Panasyuk-Dugdale model [1, 2]. Consideration of pre-fracture zones at the tips of an antiplane shear crack turns to be very successful [3] for description of these zone lengths and crack opening (see also pp.244-245 in [4]).

Let shear stresses τ_∞ be given at infinity. An inner rectilinear crack of length $2l_0$ is most often modeled by some imaginary crack-cut of length $2l = 2l_0 + 2\Delta$ (Δ is the pre-fracture zone lengths each of which is situated on the inner crack continuation). The simplest description of the pre-fracture zone can be probably obtained using modification of the Leonov-Panasyuk-Dugdale model [5, 6]. The modification of this model, as a matter of fact, differs from the classical model in that the pre-fracture zone is characterized by a width. Using one more parameter allows fracture of a fiber in the pre-fracture zone nearest to the middle of a real crack to be estimated applying data about parameters of the standard $\sigma - \varepsilon$ diagram of material [6]. As structured materials are under consideration, we make use of the Neuber-Novozhilov approach [7, 8].

Let the origin of Cartesian coordinate system Oxy and that of polar coordinate system $Or\theta$ be coincident with the right-hand tip of an imaginary crack, and the Ox-axis is directed along the crack. The stress field for antiplane shear cracks can be viewed as a sum of two summands [9]

$$\tau_{xz}(r,\theta) \cong -\frac{K_{III}}{\sqrt{2\pi r}}\sin\frac{\theta}{2} + O(1),\ \tau_{yz}(r,\theta) \cong \frac{K_{III}}{\sqrt{2\pi r}}\cos\frac{\theta}{2} + O(1),$$

$$K_{III} = K_{III\infty} + K_{III\Delta}, \quad K_{III\infty} > 0, K_{III\Delta} < 0, \quad K_{III\infty} = \tau_\infty \sqrt{\pi l} \tag{1}$$

where $K_{III} = K_{III}(l, \Delta)$ is the total stress intensity factor (SIF); $K_{III\infty}$ is the SIF generated by stresses τ_∞; $K_{III\Delta}$ is the SIF generated by constant stresses $\tau_{m\Delta} = const$ acting in compliance with the classical Leonov-Panasyuk-Dugdale model; the first summand in relation (1) is singular solution part; assume that such τ_∞ is chosen for which $K_{III\infty} = \tau_\infty \sqrt{\pi l} > 0$, then $K_{III\Delta} < 0$.

When pre-fracture zones are described, two classes of solutions are possible:

$$K_{III} > 0, \tag{2}$$

$$K_{III} = 0. \tag{3}$$

The third class of solutions that corresponds to the inequality $K_{III} < 0$ for the total SIF is not considered. Thus, when equality (3) is implemented, the solution has no singular part; and when inequality (2) is implemented, the solution has two components. Deriving the second class of solutions (3) can be related to Khristianovich's hypothesis [10] for the absence of singularity at the imaginary crack tip [3, 4]. When studying the pre-fracture zone, the main attention was paid to solutions appropriate to the class (1), (3).

The Neuber-Novozhilov approach [6, 7] allows one to extend the class of solutions for structured media. According to Novozhilov's nomenclature, the criteria under study are called sufficient. Infinite stresses at the imaginary crack tip, as this is obvious from relations (1) and (2) are not admissible by the continuum criterion but do not contradict to discrete-integral criteria for structured material if the singular component of solution has integrable singularity. Substantiation of the Neuber-Novozhilov approach when formulating criteria is given in [11].

2. Sufficient fracture criterion for antiplane shear. Now we consider solids with regular structure. In what follows, the class of solutions (2) is considered. The sufficient discrete-integral strength criterion for the mode III has the form ($\Delta > 0, \omega > 0$)

$$\frac{1}{kr_0} \int_0^{nr_0} \tau_{yz}(x,0)dx \le \tau_m; \tag{4}$$

$$2w(x,0) \le \omega^*, \; -\Delta \le x \le 0. \tag{5}$$

here $\tau_{yz}(x,0)$ are shear stresses (1), which have the singular component (2) with integrable singularity; r_0 is the specific linear size of a regular microstructure of the original material (for example, r_0 is the grain diameter); n, k are integers ($n \ge k$); nr_0 is the averaging interval ($1 \le n \le 4$); $(n-k)/n$ is the damage of original material within the averaging interval ($1/2 \le (n-k)/n \le 1$); τ_m is the shear “theoretical” strength (yield strength) of granular material; $2w = 2w(x,0)$ is mutual displacement of imaginary crack flanks; $2w^*(-\Delta,0) = \omega^*$ is the critical displacement under which a structure in the pre-fracture zone nearest to the middle of a crack falls. It is naturally to measure the pre-fracture zone length Δ in units of the material structure length r_0. Further we assume that this specific linear size is not changed under inelastic material deformation. Undoubtedly, when necessary (4) or sufficient (4) – (5) criteria are used, the natural restriction should be imposed: the initial crack length should be more than the specific linear size, i.e., $2l_0 \geq r_0$ (as a rule, $l_0 \gg r_0$). Novozhilov’s sufficient strength criterion (4) – (5) permits the passage to the limit when the pre-fracture zone length vanishes, i.e., $\Delta \to 0$. In such a way, we obtain the Novozhilov’s necessary discrete-integral criterion [8]. Let stresses τ_∞^0 denote critical stresses obtained by the necessary criterion (4) and are appropriate to brittle material fracture. For $\Delta \to 0$, there is no displacement of imaginary crack flanks: lengths of the imaginary and initial cracks are coincident, i.e., for $\Delta = 0$, we have $2w(0) = 0$.

Consider the sufficient criterion. For critical fracture parameters $\tau_\infty^*, 2w^*, K_{III}^*$, and Δ^*, relations (4) and (5) transform into equalities (critical parameters are labeled with asterisk), critical stresses τ_∞^* correspond to material structure for the class of solutions (2). For the class of solutions (3), we introduce symbols $\tau_\infty^{**}, 2w^{**}, K_{III}^{**}$, and Δ^{**} for corresponding critical fracture parameters. It is obvious that

$$\tau_\infty^{**} > \tau_\infty^* > \tau_\infty^0, \quad 2w^{**} > 2w^* > 0, \quad K_{III}^{**} > K_{III}^* > K_{III}^0, \quad \Delta^{**} > \Delta^* > 0. \tag{6}$$

For critical lengths of imaginary cracks, the relations $l^* = l_0 + \Delta^*$ or $l^{**} = l_0 + \Delta^{**}$ are valid for corresponding classes of solutions.

Further we consider steadily increasing loading. When in the necessary (4) and sufficient (4) – (5) strength criteria strict inequalities are satisfied, advance of tips of both real and imaginary cracks does not occur. When stresses τ_∞ applied at infinity attain the magnitude τ_∞^0, i.e., $\tau_\infty = \tau_\infty^0$, the necessary criterion (4) of the Neuber-Novozhilov type [7, 8] is violated and formation of a pre-fracture zone begins. Under steadily increasing loading, when $\tau_\infty^0 < \tau_\infty < \tau_\infty^*$, steady growth of an imaginary crack takes place since its length increases by increase of pre-fracture zone lengths $2l = 2l_0 + 2\Delta$. For $\tau_\infty = \tau_\infty^*$, the structure nearest to the middle of a real crack falls if equality (5) for the first class of solutions (2) is violated.

Thus, relation (4) in the sufficient fracture criterion (4) – (5) governs the imaginary crack advance, and relation (5) governs the fracture of the first microstructure nearest to the middle of a real crack. Relation (5) corresponds to COD or CTOD from [12, 13].

Given in Fig. 1 are loading schemes (at the top) and stress fields ahead of real and imaginary cracks for the necessary fracture criterion (a) and the sufficient fracture criterion when the first (b) or the second (c) classes of solutions are implemented.

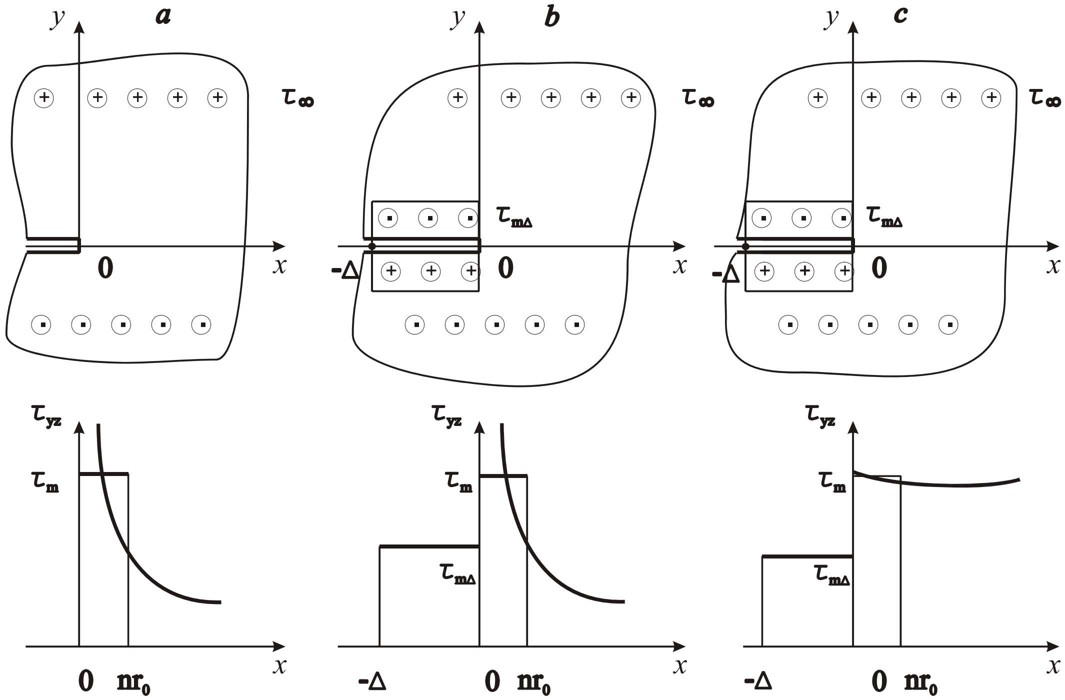

Fig. 1

3. Description of material properties in pre-fracture zone. The classical $\tau-\gamma$ diagram of material has, except the linear section of deformation $\tau = G\gamma$ for $0 \le \gamma \le \gamma_0$, the nonlinear section for $\gamma_0 \le \gamma \le \gamma^*$, where G is the shear modulus, γ_0 is the angle at which $\tau_m = G\gamma_0$, $\gamma^* = \gamma_1$ is the critical angle of deformation for material under study when the first class of solutions is implemented (2). Fig. 2 shows the initial $\tau-\gamma$ diagram of material (curve 1) and its simplest approximation (line 2) used for construction of critical fracture parameters. Location of the horizontal straight line 2 is defined from the condition of averaging on the nonlinear deformation section on $\tau-\gamma$ diagrams up to the point $\gamma^* = \gamma_1$ for the first class of solutions. In Fig. 2, the following notation is used: τ_m is the yield stress and $\tau_{m\Delta}$ are stresses acting according to the Leonov-Panasyuk-Dugdale model. In the general case, these stresses τ_m and $\tau_{m\Delta}$ can be distinct, i.e., $\tau_{m\Delta} \ne \tau_m$, the $\tau-\gamma$ diagram in Fig. 2 is appropriate to materials with significant strengthening after yield occurrence when $\tau_{m\Delta} > \tau_m$. Approximation of $\tau-\gamma$ diagram will be used further in discussion of the theory of critical distances [14, 15] when $\tau_{m\Delta} \ne \tau_m$.

We are coming now to estimation of a pre-fracture zone width. Equate the pre-fracture zone with the plasticity one. Assume that within the pre-fracture zone, the condition of yield is met [3, 4]

$$\tau_{xz}^2 + \tau_{yz}^2 = \tau_m^2 .$$

Making use of this condition, we obtain crude estimate of the plasticity zone radius r_p at the tip of a real crack [4, p. 242] if only singular components from (1) are kept

$$r_p = K_{III\infty}^2 /(2\pi\tau_m^2),\ K_{III\infty} = \tau_\infty \sqrt{\pi l_0}\ . \qquad (7)$$

The pre-fracture zone width at the tip of a real crack is a width of the plasticity zone at the tip of a real crack. Taking into account (7), we have

$$2r_p = l_0 \left(\tau_\infty / \tau_m\right)^2 .$$

Since, as a rule, $\tau_\infty / \tau_m \ll 1$, then $2r_p \ll l_0$. If values of $2r_p$ for the pre-fracture zone width and the critical strain $\gamma^* = \gamma_1$ for material under study are taken into account, critical displacements ω^* of imaginary crack flanks are estimated as follows

$$\omega^* = (\gamma^* - \gamma_0) l_0 \left(\tau_\infty / \tau_m\right)^2 . \qquad (8)$$

Mutual displacement of imaginary crack flanks $2w = 2w(x,0)$ in the pre-fracture zone for the class of solutions (2) can be given in the form [9]

$$2w(x,0) = \left(K_{III} / G\right)\sqrt{-x/(2\pi)}\ ,\ \ -\Delta \le x \le 0\ . \qquad (9)$$

Minor terms in relation (9) are omitted because further such fracture is considered when the pre-fracture zone length is essentially less than the crack length, i.e., $\Delta / l \ll 1$ and $l \approx l_0$.

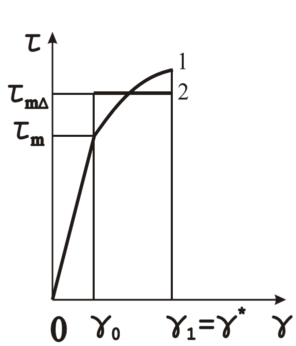

Fig. 2

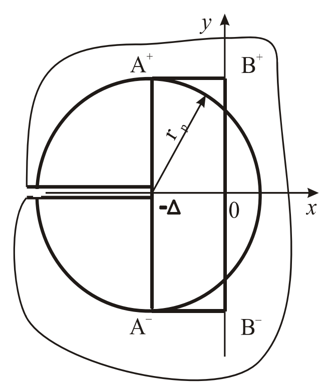

Fig. 3

Suggested modification of the Leonov-Panasyuk-Dugdale model, two-sheet solution is considered. The scheme elucidating the two-sheet solution is given in Fig. 3, the form of the plasticity zone corresponding to relation (7) is also given in the Figure. On the whole plane with a two-side cut, solution appropriate to linear fracture mechanics is defined; one or another solution corresponding to nonlinear fracture mechanics is defined only for the pre-fracture zone occupying the rectangle with the side $\Delta, 2r_p$. Vertexes of this rectangle are

$$A^+(-\Delta, r_p),\ A^-(-\Delta, -r_p),\ B^+(0, r_p),\ B^-(0, -r_p)\,.$$

Conditions of gluing solutions by stresses $\tau_{yz}^+(x,0), \tau_{yz}^-(x,0)$ for $-\Delta \le x \le 0$ and displacements $w(x,0)$ for $-\Delta \le x \le 0$ are essentially peculiar (signs plus and minus point upper and lower sides of cut upper and lower sides of rectangle):

$$\tau_{yz}^+\Big|_{-\Delta^+,0^+} = \tau_{yz}^+\Big|_{A^+,B^+},\ \tau_{yz}^-\Big|_{-\Delta^-,0^-} = \tau_{yz}^-\Big|_{A^-,B^-},\ w^+\Big|_{-\Delta^+,0^+} = w^+\Big|_{A^+,B^+},\ w^-\Big|_{-\Delta^-,0^-} = w^+\Big|_{A^-,B^-}\,.$$

Thus, the two-sheet solution takes place in the rectangle $A^+B^+B^-A^-$ from which a two-side cut along the segment $-\Delta \le x \le 0$ is excluded.

4. Critical fracture parameters (the first class of solutions). Inequalities of the fracture criterion (4) – (5) written for the critical parameters $\tau_\infty^*, 2w^*, K_{III}^*, \Delta^*$ are transformed into equalities. Specify the value of the component $\tau_{yz}(x,y)$ in relations (1) for the inner crack and its continuation at $y = 0$

$$\tau_{yz}(x,0) \cong K_{III} / \sqrt{2\pi x} + \tau_\infty\,. \tag{10}$$

This solution (10) allows one to take into account both singularity and the constant component for $x \to \infty$.

For the class of solutions (2), we consider SIF $K_{III\Delta}$ generated by constant stresses $\tau_{m\Delta} = const$ acting according to the modified Leonov-Panasyuk-Dugdale model at the end parts of an imaginary crack (see Fig. 2a). This SIF $K_{III\Delta}$ can be adopted from the reference book [16, p. 380-381] in the form

$$K_{III\Delta} = \tau_{m\Delta}\sqrt{\pi l}\left[1-\left(2/\pi\right)\arcsin\left(l_0/l\right)\right]. \tag{11}$$

After substitution of relations (8) – (11) into the sufficient fracture criterion (4) – (5) and some transformations, we obtain the system that involves one equality and one inequality

$$\frac{K_{III}}{k\sqrt{\pi}}\sqrt{\frac{2n}{r_0}} + \frac{n}{k}\tau_\infty = \tau_m,\quad \frac{K_{III}}{G}\sqrt{\frac{\Delta}{2\pi}} \le (\gamma^* - \gamma_0) l_0 \left(\frac{\tau_\infty}{\tau_m}\right)^2, \tag{12}$$
$$K_{III} = \tau_\infty\sqrt{\pi l} - \tau_{m\Delta}\sqrt{\pi l}\left[1-\left(2/\pi\right)\arcsin\left(l_0/l\right)\right]$$

When in relation (12), the equality is used, and in the second relation from (12), the inequality is implemented, we obtain the subcritical state of a system. When in both these relations from (12), equalities are implemented, then the system converts to the critical state. Whether critical and subcritical states will be stable yet to be elucidated.

For $\Delta \to 0$, we obtain critical stresses τ_∞^0 according to the necessary fracture criterion

$$\tau_\infty^0 = \tau_m\left(k^{-1}\sqrt{2nl_0/r_0} + n/k\right)^{-1}. \tag{13}$$

The suggested sufficient fracture criterion (12) for the class of solutions (2) has a physical meaning if the total SIF $K_{III} > 0$ for $K_{III\infty} > 0$, i.e.

$$1-\left[1-(2/\pi)\arcsin\left(l_0/l\right)\right]\tau_{m\Delta}/\tau_\infty > 0 . \qquad (14)$$

Restriction (14) corresponds to the situation when there is no crack flank overlap for the mode I fracture.

Now we simplify system (12). If $\Delta/l \ll 1$, with accuracy of the highest infinitesimal order, we have

$$\arcsin\left(1-\Delta/l\right) \cong \pi/2-\sqrt{2\Delta/l} \ \text{ at } \Delta/l \ll 1 .$$

Keeping summands with multipliers $\sqrt{\Delta/l}$ in system (12), we derive explicit expressions for the critical stresses τ_∞^* and the critical pre-fracture zone length Δ^* according to the sufficient fracture criterion

$$\frac{\tau_\infty^*}{\tau_m} = \left\{\frac{1}{k}\sqrt{\frac{2nl^*}{r_0}}\left[1-2\left(\frac{\gamma^*}{\gamma_0}-1\right)\frac{\tau_{m\Delta}}{\tau_m}\right]+\frac{n}{k}\right\}^{-1}, \quad \sqrt{\frac{\Delta^*}{l^*}} = \sqrt{2}\left(\frac{\gamma^*}{\gamma_0}-1\right)\frac{\tau_\infty^*}{\tau_m}. \qquad (15)$$

The passage to the limit $\tau_\infty^*/\tau_m \to \tau_\infty^0/\tau_m$ in system (15) for $\gamma^*-\gamma_0 \to 0$ is obvious as compared to (13). System (15) has meaning if

$$1-2\left(\gamma^*/\gamma_0-1\right)\tau_{m\Delta}/\tau_m > 0 . \qquad (16)$$

Restrictions (14) and (16) are conditions for existence of the class of solutions (2). Thus, for the first class of solutions, restriction (16) in the explicit form is derived using parameters characterizing both deformability and strength of a material based on approximation of the standard $\tau-\gamma$ material diagram. For $\tau_{m\Delta}/\tau_m = 1$, we have $\gamma_0 < \gamma^* < 1.5\gamma_0$ that corresponds to materials of the ceramic-types and high-strength alloys.

Under steadily increasing loading in the interval of loads $\tau_\infty^0 < \tau_\infty < \tau_\infty^*$, steady growth of the imaginary crack length takes place since $\tau_\infty(l_0+\Delta_1) < \tau_\infty(l_0+\Delta_2)$ in the system analogous to system (15), when $0 \le \Delta_1 \le \Delta_2 < \Delta^*$. Instability of the critical state of the crack of length l^* for $\tau_\infty = \tau_\infty^*$ is obvious from the first relation from (15): as the critical crack length increases up to the length $l^*+\delta$ for $\delta \to 0$, when $\delta > 0$, we have $\tau_\infty^*(l^*) < \tau_\infty^*(l^*+\delta)$. Thus, critical loads according to the necessary fracture criterion τ_∞^0 and the sufficient one τ_∞^* are lower and upper bounds of critical loads of the nonlinear system under consideration. Fig. 4 shows the ratio between critical loads according to sufficient and necessary fracture criteria $\tau_\infty^*/\tau_\infty^0$ for $\tau_{m\Delta}/\tau_m = 1$, $n = k = 1$, $l/r_0 \gg 1$ depending on the parameter γ^*/γ_0 characterizing deformability.

Sections with steady crack growth for $\tau_\infty^0 < \tau_\infty < \tau_\infty^*$ and $\Delta^* = \Delta^*(l_0)$ are of most interest since a peculiar trapping for propagating cracks occurs [17, 18, and 19].

Consider in more detail the steady behavior of the system for the following set of crack lengths $l_0, l_1 = l_0 + \Delta^*(l_0), l_2 = l_1 + \Delta^*(l_1), ..., l_{i+1} = l_i + \Delta^*(l_i), ...$ for $i = 1, 2, ...$ In Fig. 5, the "dinosaur's back" by Thomson [17] is shown. Here 1 and 2 are curves of critical loads τ_∞^0 and τ_∞^* obtained according to the necessary and sufficient fracture criteria for $\tau_{m\Delta} / \tau_m = 1$, $n = k = 1$, $l / r_0 \gg 1$, $\gamma^* / \gamma_0 = 4/3$. The forms of combs on "dinosaur's back" are defined by approximation of the $\tau - \gamma$ diagram of material under study; ascending parts of the comb are depicted with solid curves and descending ones are depicted with dashed lines. Lengths of pre-fracture zone for long cracks depend only on material deformability in this zone

$$\sqrt{\frac{\Delta^*}{r_0}} \approx \frac{\gamma^* / \gamma_0 - 1}{1 - 2\left(\gamma^* / \gamma_0 - 1\right)}.$$

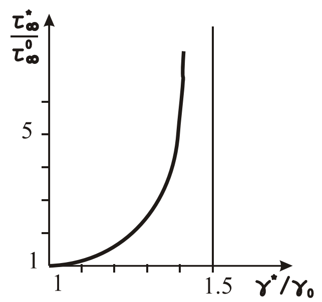

Fig. 4

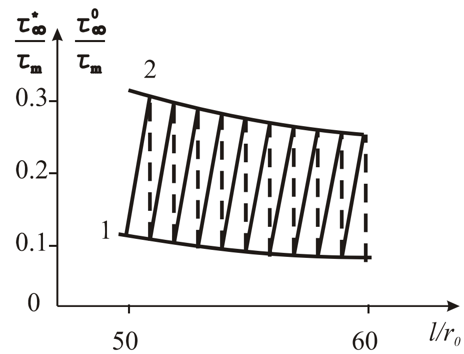

Fig. 5

Consider the critical SIF K_{III}^* for class of solutions (2)

$$K_{III}^* = \tau_\infty^* \sqrt{\pi l^*} \approx \tau_m \sqrt{\pi l^*} \left\{ \frac{1}{k} \sqrt{\frac{2nl^*}{r_0}} \left[1 - 2\left(\frac{\gamma^*}{\gamma_0} - 1 \right) \frac{\tau_{m\Delta}}{\tau_m} \right] + \frac{n}{k} \right\}^{-1}. \qquad (17)$$

The second part of approximated relation (17) corresponds to the restriction $\Delta^* / l^* \ll 1$. The critical SIF K_{III}^* (17) is derived in the explicit form using structural, strength and deformation material parameters (undoubtedly, restriction (16) is valid). In the framework of the modified Leonov-Panasyuk-Dugdale model, the critical SIF K_{III}^* is calculated trough characteristics of the $\tau - \gamma$ material diagram and it is not a material constant. Emphasize that for long cracks $l^* / r_0 \gg 1$ for $n = k = 1$, the approximate relationship for the critical SIF K_{III}^* takes the very simple form

$$K_{III}^* \approx \tau_m \sqrt{\pi r_0 / 2} \left[1 - 2\left(\gamma^* / \gamma_0 - 1\right) \tau_{m\Delta} / \tau_m \right]^{-1}. \qquad (18)$$

This notion (18) of critical SIF K_{III}^* is independent of the initial crack length l_0 for $l_0 / r_0 \gg 1$. For $\gamma^* \to \gamma_0$, the critical SIF K_{III}^* transforms to the critical SIF K_{III}^0 according to the necessary fracture criterion $K_{III}^0 \approx \tau_m \sqrt{\pi r_0 / 2}$.

5. Fracture assessment diagram. Consider limiting values of critical loads according to the necessary τ_∞^0 and sufficient τ_∞^* fracture criteria for short and long cracks. The simplest case is considered, when $\tau_{m\Delta}/\tau_m = 1$, $n = k = 1$, then we have

$$\begin{gathered}\tau_\infty^0 = \tau_m, \tau_\infty^* = \tau_m \quad for \quad l \to 0;\\ \tau_\infty^0/\tau_m = \sqrt{r_0/\left(2l_0\right)}, \tau_\infty^*/\tau_m = \sqrt{r_0/\left(2l^*\right)}\left[1-2\left(\gamma^*/\gamma_0-1\right)\right]^{-1} \quad for \quad l/r_0 \gg 1\end{gathered}. \quad (17)$$

The fracture assessment diagram for structured material is shown on a log-log-plot in Fig. 6, where curves 1 and 2 represent the dependences $\tau_\infty^0 = \tau_\infty^0(l)$ and $\tau_\infty^* = \tau_\infty^*(l^*)$ for $\gamma^*/\gamma_0 = 4/3$, respectively. The very simple form of the fracture assessment diagram is associated with notations (17): long cracks comply with linear elastic fracture mechanics (LEFM), but for short cracks, the fracture assessment curves deviate from the LEFM line towards τ_m [14]. For long cracks, when the strong inequality $l/r_0 \gg 1$ is implemented, the interval L of critical distances [14, 15] calculated by the rule is as follows

$$\left(K_{III}^0/\tau_m\right)^2/\pi \le L \le \left(K_{III}^*/\tau_m\right)^2/\pi; r_0/2 \le L \le \left(r_0/2\right)\left[1-2\left(\gamma^*/\gamma_0-1\right)\tau_{m\Delta}/\tau_m\right]^{-2}.$$

It is obvious that the upper bound of L, i.e., critical distances, can differ from the lowest bound by several times, sometimes by one order.

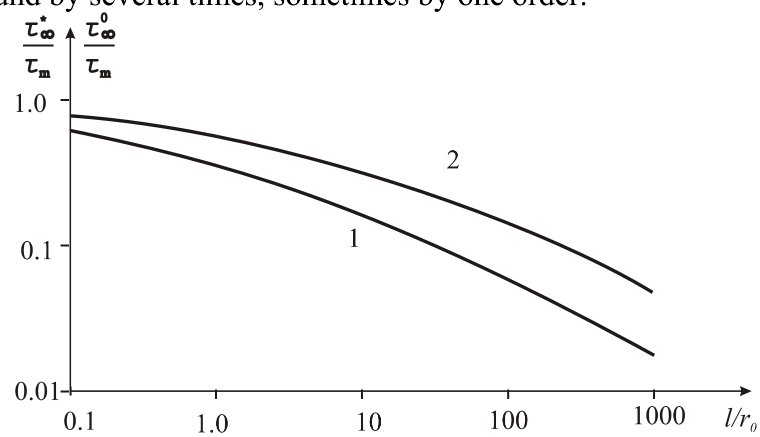

Fig. 6

6. Discussion. The Neuber-Novozhilov approach [7, 8] allows one to obtain the upper and lower bounds for critical loads [20]. These critical loads $\tau_\infty^0 = \tau_\infty^0(l)$ and $\tau_\infty^* = \tau_\infty^*(l^*)$ correspond to necessary and sufficient conditions of dividing specimens into parts. In the author's opinion these critical distances [14] depend both on the material's microstructure [15] and characteristics of the $\tau - \gamma$ material diagram ($\tau_{m\Delta} \ne \tau_m$).

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