Is the edge crack the most dangerous V-notch?

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Abstract

The determination of the failure load for brittle or quasi-brittle specimens containing a re-entrant corner has been faced by several Authors, whose approaches are available in the Scientific Literature. However, up to now, little attention has been paid to the presence of a minimum, i.e. an angle at which the critical load attains its minimum value. Even if the minimum was detected in several experiments, it was not highlighted or it was considered as a mere consequence of the scattering of experimental data. Restricting the analysis to a V-notched infinite slab under a remote tensile load, the problem is fully investigated in this paper. It is shown that a minimum, although imperceptible, is present. It means that the edge crack is not the most dangerous configuration, although the notch opening angle providing the minimum failure load tends to vanish for large notch depths as well as for very brittle materials.

1. Introduction

Is the edge crack the most dangerous V-notch? Most of the data presented in the literature, concerning experimental tests on V-notched brittle material specimens [1-3] with different notch opening angle ω, show a minimum on the evaluated failure loads which does not correspond to the edge crack case. In other words, it is observed that the failure load does not increase monotonically as ω increases, but it has a minimum in correspondence of a critical angle ω_c. Many examples could be given: three points bending (TPB) tests on V-notched PMMA specimens carried out by Carpinteri [1], for instance, show the minimum failure load for $\omega \approx 45°$; similar results ($\omega \approx 40°$) were obtained by Seweryn [2] testing double edge notched PMMA specimens under tensile load. Furthermore, TPB tests carried out on polystyrene specimens do not show a significant difference between the failure load for 60°-notch sample and 120°-notch sample, whereas the presence of a minimum between these two cases was suggested in the related analysis [3].
The problem of the critical notch angle is investigated in this paper restricting the analysis to a V-notched infinite slab under a remote tensile load (Fig. 1), i.e. when the notch is subjected to mode I loading.

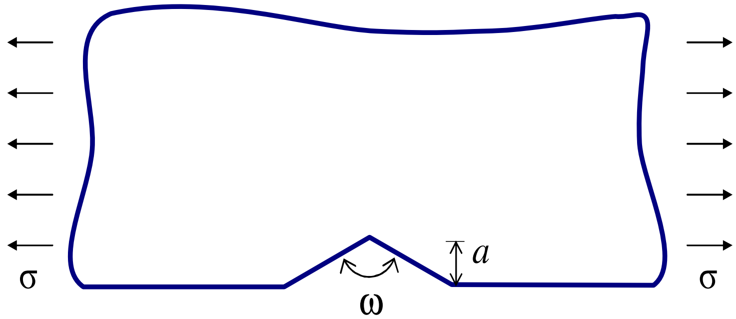

Figure 1. Infinite notched slab under remote tension

2. Infinite notched slab under tension

By means of dimensional analysis, it is possible to express the generalized stress intensity factor (SIF) K_I^* related to an infinite notched slab under remote tensile stress σ (Fig. 1) as [1]:

$$K_I^* = \beta(\omega)\sigma a^{1-\lambda(\omega)}, \tag{1}$$

where a is the notch depth and β, as well as λ, depend on the notch angle ω. The generalized SIF K_I^* is the coefficient of the dominant term of the stress field at the notch tip and, with brittle structural behaviour, it is expected to be the governing failure parameter. In other words, failure is supposed to take place whenever [1]:

$$K_I^* = K_{Ic}^*, \tag{2}$$

K_{Ic}^* being the generalized fracture toughness [1]. A theoretical justification of this fracture criterion (Eq. 2) may be also given in the framework of Finite Fracture Mechanics (FFM) [4], leading to the following general relationship

$$K_{Ic}^* = \xi(\omega)\frac{K_{Ic}^{2(1-\lambda)}}{\sigma_u^{1-2\lambda}}, \tag{3}$$

where function ξ(ω) depends on the fracture criterion used and K_{Ic} and σ_u are the fracture toughness and tensile strength, respectively.
Inserting Eqs. (1) and (3) into Eq. (2), yields:

$$\frac{\sigma_f}{\sigma_u} = \frac{\xi(\omega)}{\beta(\omega)}\alpha^{\lambda(\omega)-1}, \tag{4}$$

where σ_f is the remote stress at failure and

$$\alpha = a \frac{\sigma_u^2}{K_{Ic}^2} \tag{5}$$

is the dimensionless notch depth.
Let us now recall the brittleness number definition:

$$s = \frac{K_{Ic}}{\sigma_u \sqrt{a}}, \tag{6}$$

which is a non-dimensional quantity, introduced by Carpinteri [5,6]: brittle structural behaviours are generally expected for low brittleness numbers. Note that in the present case, i.e. infinite slab, the characteristic structural size corresponds to the notch depth a, the only relevant size in the problem. Since $\alpha = 1/s^2$, Eq. (4) can be rewritten equivalently as:

$$\frac{\sigma_f}{\sigma_u} = \frac{\xi(\omega)}{\beta(\omega)} s^{2(1-\lambda(\omega))}. \tag{7}$$

3. The functions λ(ω), β(ω) and ξ(ω)

The values of λ can be directly evaluated by solving the Williams' eigenvalue problem [7] (Fig. 2):

$$\lambda \sin \omega = \sin \lambda (2\pi - \omega). \tag{8}$$

On the other hand, in order to compute the β-function values the generalized SIF K_I^* for unit load and notch depth is required (Eq. (1)). Note that for the extreme cases, i.e. crack and flat edge (0° and 180°, respectively), these values are known [8]. In fact:

- $\omega = 0°$, $K_I^* = K_I = 1.12 \sqrt{(\pi a)} \Rightarrow \beta(0°) = 1.12 \sqrt{\pi} \cong 1.99$,
- $\omega = 180°$, $K_I^* = \sigma \Rightarrow \beta(180°) = 1$.

Observe that, since the plate is infinite, considering a notch opening angle equal to 180° is equivalent to considering an un-notched slab.
For the intermediate angles, a Finite Element Analysis (FEA) has been performed, by using the LUSAS ® code, every 30° in the interval 0°-180° of the geometry under examination. By means of the H-integral [9], the relative SIFs and, consequently, the values of β have been evaluated: the resulting curve, obtained by means of spline interpolation, is shown in Fig. 3.

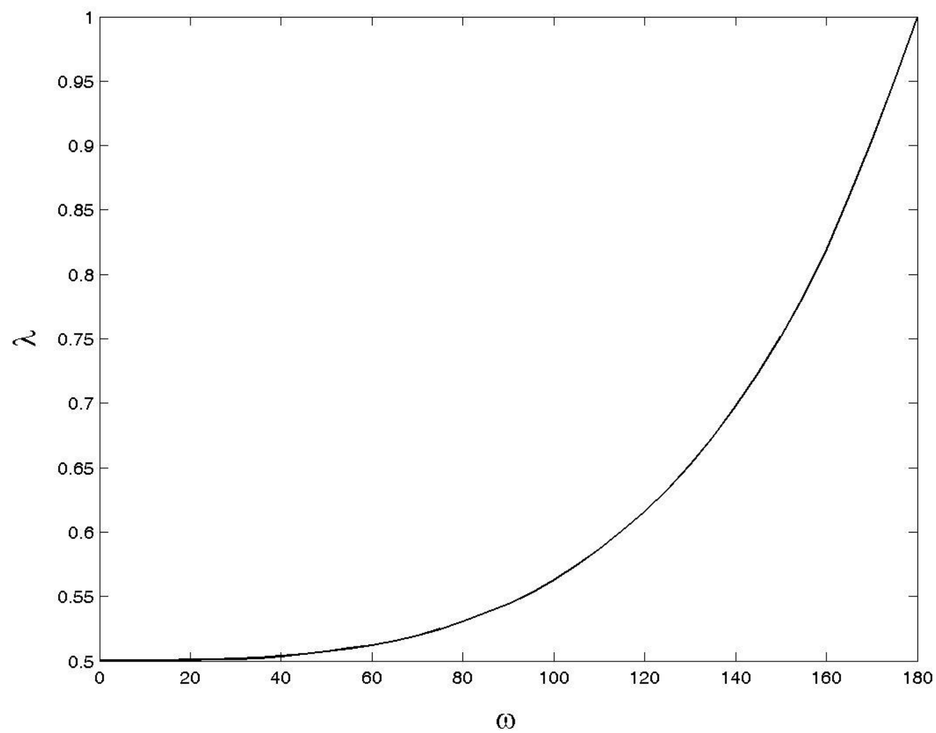

Figure 2. λ-function vs. notch opening angle ω

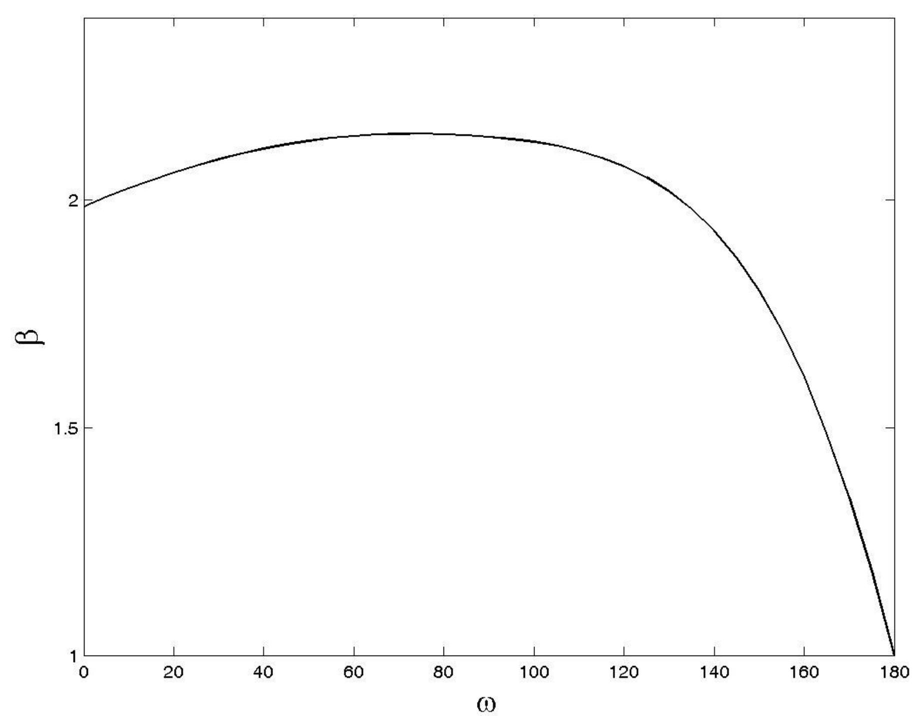

Figure 3. β-function vs. notch opening angle ω

Although more sophisticated criteria may be used [3,10,11,12], only the average stress criterion will be considered herein for the sake of simplicity. Hence the following expression for function ξ holds [2,13]:

$$\xi(\omega) = \lambda(\omega) 4^{1-\lambda(\omega)}. \quad (9)$$

Since ξ is equal to 1 for ω=0° and 180° (Fig. 4), Eq. (3) shows that the generalized fracture toughness is intermediate between strength and toughness, as regards the physical dimensions.

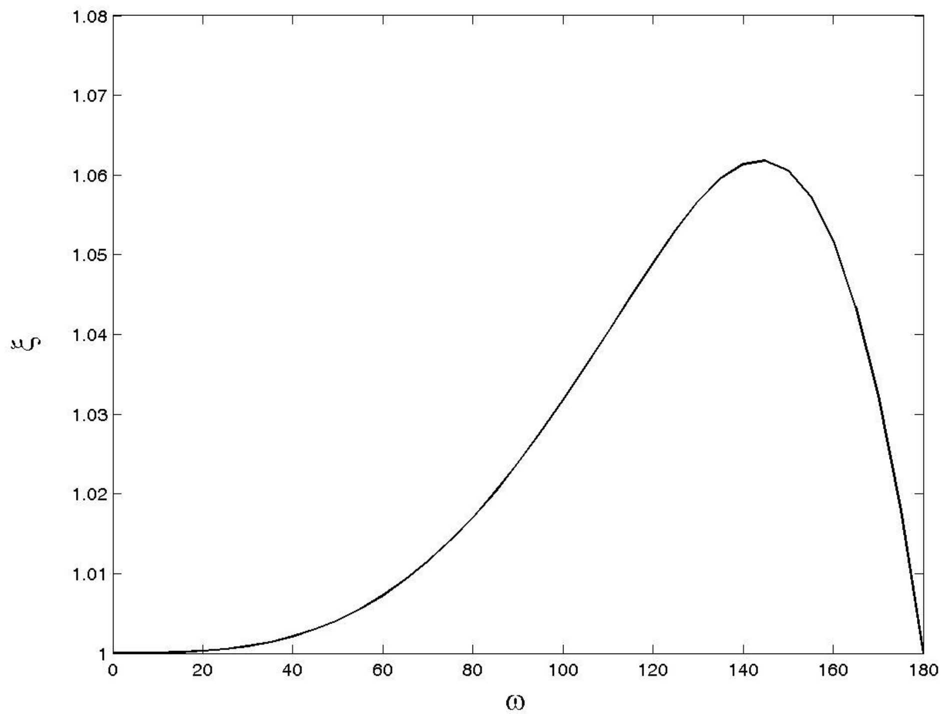

Figure 4. ξ-function vs. notch opening angle ω

4. Notch sensitivity and critical notch opening angle

Once the λ, β and ξ-functions are known, the results connected with Eq. (7) can now be fully investigated. Note that Eq. (7) can be analyzed from two different points of view, i.e. either by varying *s* and keeping ω fixed, or by varying ω and keeping *s* fixed.
In the former case, let us start by considering just three geometries: ω = 0°, 120° and 180°. That is a cracked, a notched and a flat geometry. The dimensionless failure loads vs. the dimensionless notch depths are plotted in Fig. 5. The two curves corresponding to ω = 0° and 180° (thick and thin line, respectively) intersect each other at

$$a_0 \cong 0.25\left(\frac{K_{\mathrm{Ic}}}{\sigma_{\mathrm{u}}}\right)^2 \Rightarrow \alpha_0 \cong 0.25. \tag{10a}$$

Therefore, a_0 is a characteristic of the material. It is evident from Fig. 5 that the failure load provided by the stress intensification, for $a < a_0$, inconsistently exceeds the material tensile strength σ_u. In this case, failure occurs whenever $\sigma_f = \sigma_u$. In other words, the structure is insensitive to cracks shorter than a_0 [14]. On the other hand, for $a > a_0$, the cracked geometry provides failure stresses lower than σ_u.

If also a notched slab with $\omega = 120°$ is taken into account ($\lambda \cong 0.616$, $\xi \cong 1.049$, $\beta \cong 2.073$), the corresponding curve (dashed line, Fig. 5) intersects the two curves previously considered ($\omega = 180°$ and $0°$) respectively at:

$$a_1 \cong 0.16\left(\frac{K_{\mathrm{Ic}}}{\sigma_{\mathrm{u}}}\right)^2 \Rightarrow \alpha_1 \cong 0.16, \tag{10b}$$

$$a_2 \cong 0.93\left(\frac{K_{\mathrm{Ic}}}{\sigma_{\mathrm{u}}}\right)^2 \Rightarrow \alpha_2 \cong 0.93, \tag{10c}$$

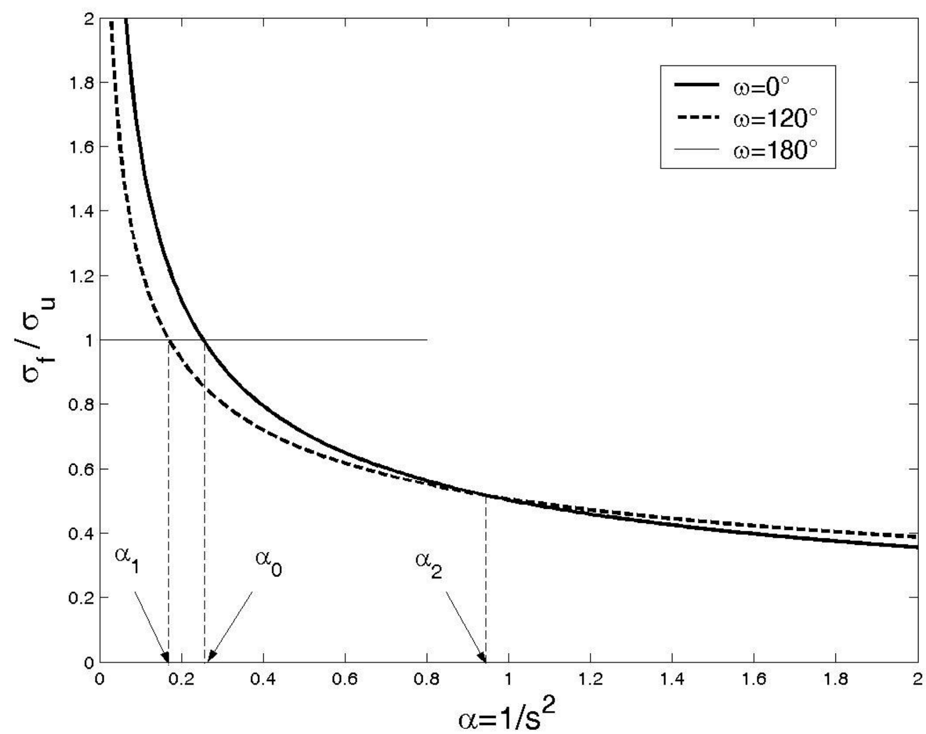

Figure 5. Dimensionless failure load vs. dimensionless notch depth for $\omega = 0°$ (thick line), 120° (dashed line) and 180° (thin line).

Three situations can hence be met (Fig. 5):

- If $0 < a < a_1$, the minimum failure load is provided by $\omega = 180°$ i.e. the flat geometry. In this case the structure is said to be insensitive to cracks and notches.
- If $a_1 < a < a_2$, the minimum failure load is provided by $\omega = 120°$.
- If $a > a_2$, the crack ($\omega = 0°$) is the most dangerous geometry.

Two important considerations emerge from this simple example: (*i*) since $a_1 < a_0$, the structure may be insensitive to cracks and, at the same time, sensitive to notches; (*ii*) for certain depths, a notch may be more dangerous than a crack.
Now let us consider the latter case, i.e. let us plot Eq. (7) by varying ω and keeping s fixed. The results are shown in Fig. 6: it is clear that there exists always a critical notch angle ω_c (i.e. corresponding to the minimum failure stress), whose position moves from 0° to 180° as the brittleness number s increases: in other words the larger are the values of s (i.e. the larger the fracture toughness and/or the smaller the tensile strength and/or the smaller the notch depth), the larger are the critical values expected. This result is in good agreement with experimental data presented in the literature [1-3].

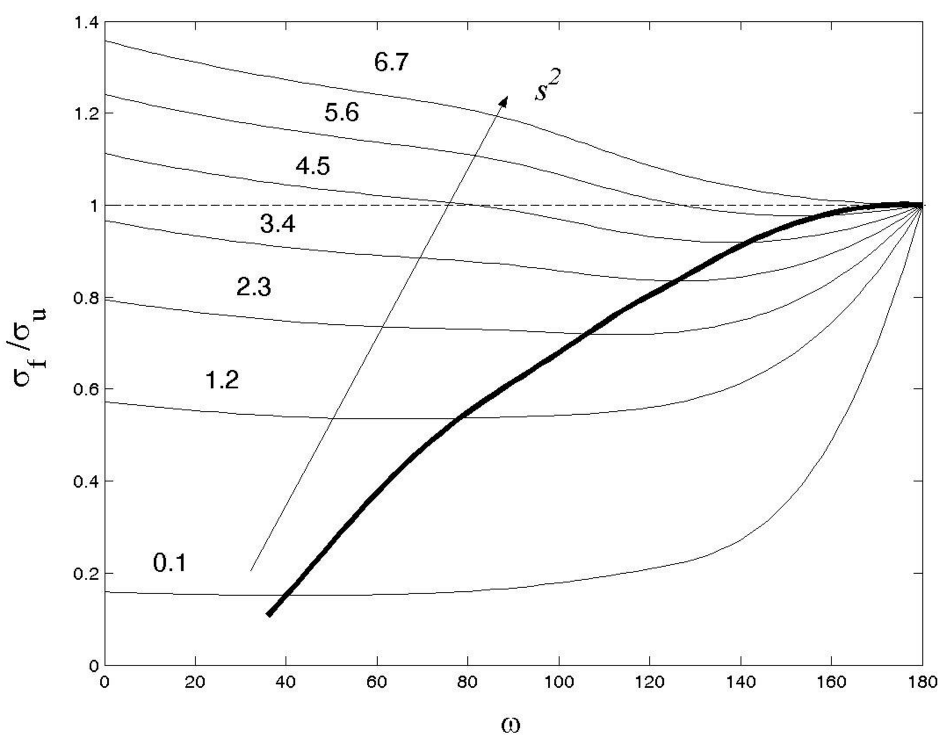

Figure 6. Dimensionless failure load vs. opening angle ω for increasing values of the brittleness number s. The thick line represents the locus of the minima.

Eventually, the determination of a critical notch angle may be formalized by deriving Eq. (4) with respect to ω and imposing the stationarity condition:

$$\frac{d(\sigma_f/\sigma_u)}{d\omega} = \alpha^{1-\lambda(\omega)}\left(\lambda'(\omega)\frac{\xi(\omega)}{\beta(\omega)}\ln\alpha + \frac{\xi'(\omega)\beta(\omega)-\xi(\omega)\beta'(\omega)}{\beta^2(\omega)}\right) = 0\,. \tag{11}$$

where the prime denotes the derivative with respect to the notch opening angle ω. Hence the following relationship is obtained:

$$\alpha = \exp\left[\frac{1}{\lambda'(\omega)}\left(\frac{\beta'(\omega)}{\beta(\omega)} - \frac{\xi'(\omega)}{\xi(\omega)}\right)\right]\Bigg|_{\omega=\omega_c}\,. \tag{12}$$

By evaluating the derivatives λ', β' and ξ', the inverse of Eq. (12) is plotted in Fig. 7. It provides the value of the critical notch angle ω_c for a given α or *s* value. Note again that, consistently with the previous analysis, ω_c depends − through *s* − both on the material and the geometry and that large critical notch angles correspond to large brittleness numbers.

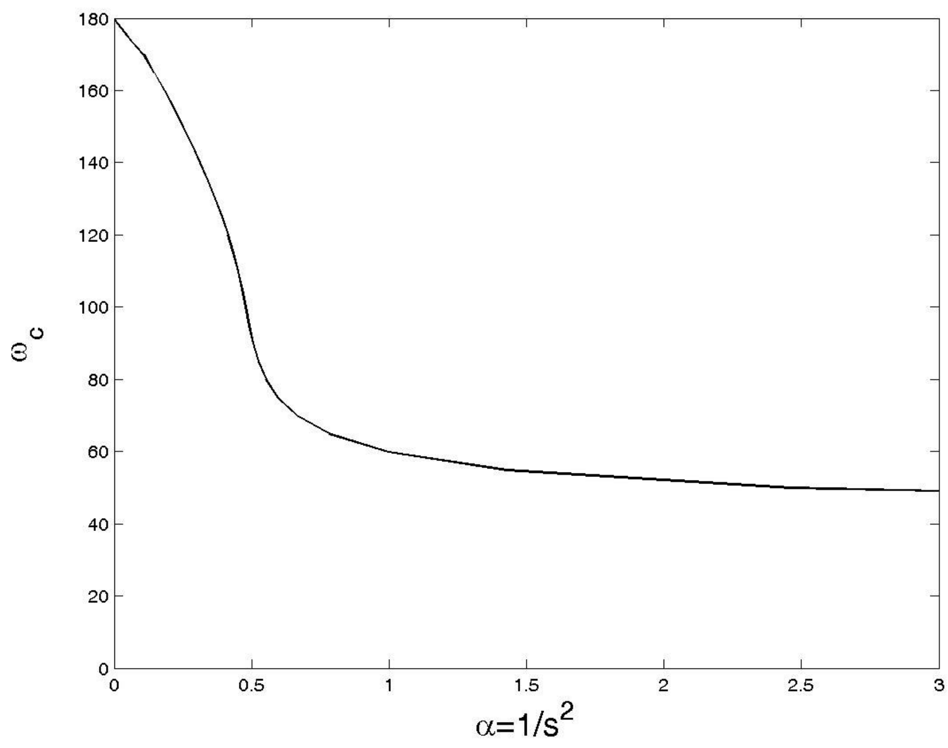

Figure 7. Critical notch opening angle ω_c vs. dimensionless notch depth.

5. Conclusions

The problem of the most dangerous notch angle i.e., the angle providing the minimum failure load, in an infinite notched slab under remote tension is investigated in this paper. It is shown that the critical value depends on the brittleness number: the edge crack is thus not the most dangerous configuration. This result is supported by several experimental data presented in the literature.

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