Improvement of Slice Synthesis Methodology to Estimate Stress Intensity Factor of an Embedded Crack in a finite body

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Abstract
The limit of applicability of the former published method for an embedded cack in a finite body is investigated by comparing with the conventional stress intensity factor formula. The error of estimated stress intensity factors by published slice synthesis methodology increases for a decreasing distance of the crack tip from the back surface of the body. The reason is considered that the former published slice synthesis methodology conform the crack opening behavior of an embedded crack obtained analytical solution in an infinite cracked body to the ones obtained by the slice synthesis methodology.
The improved slice synthesis methodology to increase accuracy of stress intensity factor solutions in a finite body is proposed by incorporating a correction coefficient into the formulation.
The validity of improved slice synthesis methodology is confirmed by comparing the stress intensity factor solutions with ones by finite element analyses.

1. Introduction

Fatigue life estimation for three dimensional cracks, e.g. a part-through surface crack and an embedded crack is important because most fatigue crack shapes found in welded built-up structures are three dimensional crack morphologies and these cracks are located in stress concentration regions. Conventional fatigue life estimation for welded built-up structures is based on the combination of S-N curves with hot spot stress and cumulative damage rules [1]. However, this method contains the following serious weaknesses.

1. S-N curves based approach cannot give the fatigue crack growth history.
2. The transferability of fatigue life obtained by S-N curves to in-service structures has not established [2]. The relation between fatigue crack length found in in-service structures and fatigue life obtained by S-N curves is not defined clearly.

On the other hand, fatigue life predictions based on fracture mechanics are performed in order to overcome the weaknesses of the conventional S-N curves approaches. Most of fatigue life assessments based on fracture mechanics cannot quantitatively evaluate the retardation and the acceleration of crack propagation, because of insufficient consideration of fatigue crack opening / closing behavior caused by crack wake.

Toyosada et.al [3] proposed the procedures of fatigue crack growth simulation for three dimensional crack problems by applying numerical fatigue crack opening /

closing simulation for two dimensional crack problems with the equivalent distributed stress method which enables the representation of the stress-strain field at a reference point of three dimensional cracks for a through thickness crack. Stress intensity factor of analysis objects should be given even though above mentioned method is applied.

Calculation procedures of stress intensity factor for through thickness cracks under arbitrary stress distributions are generally straight-forward, while for three-dimensional cracks such as surface cracks and embedded cracks are more complicated. The slice synthesis methodology with weight function methods to calculate stress intensity factors for a three dimensional crack was developed [4] [5]. The applied method has advantages for modeling and CPU time comparing with other numerical calculation method, e.g. finite element analyses. The application of slice synthesis methodology is expanding to perform numerical calculation of a crack opening profile and plastic zone length for a semi-elliptical surface cracks in an elastic-perfectly plastic body under monotonic loading [6] [7]. By referring published achievements related to the slice synthesis methodology, it is expected that the application of slice synthesis methodology enables to construct the numerical simulation of a surface or an embedded crack growth caused by fatigue.

Although the slice synthesis methodology had already been proposed, quantitative investigations of the applicability seem to be insufficient. Especially, the back surface effect for stress intensity and crack opening behavior is ignored because most of previous researches treat the problems which a surface crack locates in the plate with semi finite thickness or an embedded crack locates in an infinite body. In this study, the authors make it clear the applicable limits of published slice synthesis methodology and propose the modification method in order to take into account the back surface effect for stress intensity and crack opening behavior in three dimensional cracks. In this research, embedded crack morphology is highlighted as three dimensional crack shapes.

2. The slice synthesis methodology

The slice synthesis methodology with weight function to calculate stress intensity factors has potential as a usable calculation method. Calculation procedure for stress intensity factors is explained below. The detail of this methodology was described in the reference [4].

The analysis object is an embedded elliptical crack shown in Fig.1 and crack surface is divided by two series of orthogonal slices shown in Fig.2. The slice parallel to *y-z* plane is called basic slice whose crack length is $2a_x$, while parallel to *z-x* plane is spring slice and crack length is $2b_y$. Basic and spring slices are restrained by the spring whose stiffness is k_a and k_b respectively. The spring stiffness k_a enables it to have a value from zero to infinity according to the dimensional ligament area R_a. k_b also enables it to have a value from zero to infinity. That is, R_i tends to zero, k_i tends to zero, while R_i tends to infinity, k_i tends to infinity. Dimensional ligament areas R_i are defined as follows.

$$R_a = 4t(c-b)/b^2 \, , \; R_b = 4c(t-a)/b^2 \tag{1}$$

The basic slice is subjected to arbitrary stress distribution $\sigma(x,y)$ and undetermined spring force $P(x,y)$, while the spring slice is subjected to spring force $P(x,y)$. The schematic illustration of two slices is shown in Fig.3. The spring force $P(x,y)$ is the cohesive force which has a role in fulfilling the boundary conditions toward neighboring basic slice deformation behavior.

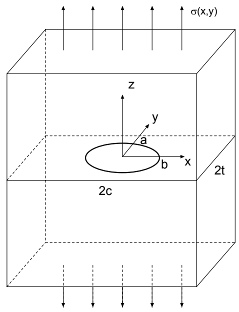


Fig.1 An embedded elliptical crack subjected to arbitrary stress distribution

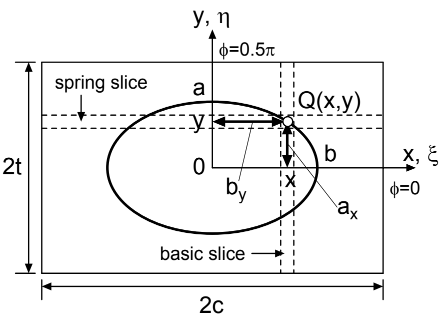


Fig.2 A coordinate system for an embedded elliptical crack

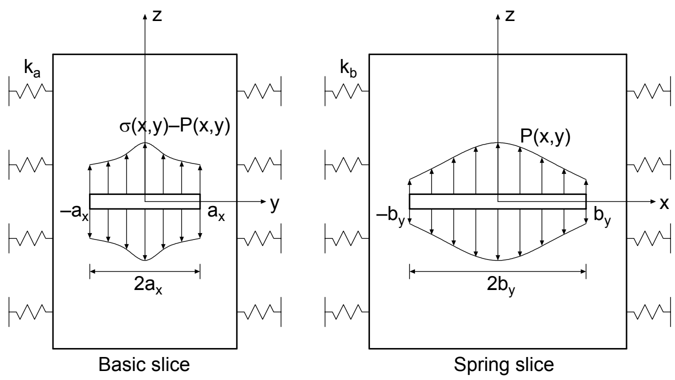


Fig.3 Schematic illustration of basic slice and spring slice

By applying the weight functions, the stress intensity factors for the basic slice and spring slice on reference point $Q(x,y)$ shown in Fig.2 are calculated as follows,

$$\begin{aligned} K_a(a_x) &= \int_0^{a_x} [\sigma(x,\eta) - P(x,\eta)] w_a(R_a, a_x, t, \eta) d\eta \\ K_b(b_y) &= \int_0^{b_y} P(\xi, y) w_b(R_b, b_y, c, \xi) d\xi \end{aligned} \tag{2}$$

w_a and w_b are weight functions for basic slice and spring slice respectively. The weight functions w_i are calculated by combination of two limiting conditions shown in Eq.(3). R_i tends to zero, w_i tends to free boundary condition w_i^{free}, while R_i tends to infinity, w_i tends to fixed boundary condition $w_i^{collinear}$.

$$
\begin{aligned}
&w_i = w_i^{collinear} + T(R_i)\left(w_i^{free} - w_i^{collinear}\right) \\
&T(R_i) = 1.05^{-R_i}
\end{aligned}
\tag{3}
$$

Weight functions for the basic slice are given by Eq. (4) [7] in this research. Weight function for spring slice is given by replacing t, a_x, and y with c, by and x in Eq. (4).

$$
\begin{aligned}
w_a^{collinear} &= \frac{2}{\sqrt{2t}}\sqrt{\tan\frac{\pi a_x}{2t}}\frac{\cos(\pi y/2t)}{\sqrt{\sin^2(\pi a_x/2t) - \sin^2(\pi y/2t)}} \\
w_a^{free} &= \frac{2}{\sqrt{2t}}\left\{1 + 0.297\sqrt{1-(y/a_x)^2}\left(1-\cos\left(\frac{\pi a_x}{2t}\right)\right)\right\} \\
&\quad \times\left\{\sqrt{\tan\frac{\pi a_x}{2t}}\bigg/\sqrt{1-\left(\frac{\cos(\pi a_x/2t)}{\cos(\pi y/2t)}\right)^2}\right\}
\end{aligned}
\tag{4}
$$

Crack opening displacement profile (COD profile) along the a_x, by are given by,

$$
\begin{aligned}
V_a(x, y, a_x) &= \frac{2}{E_a}\int_y^{a_x} K_a(\alpha) w_a(R_a, \alpha, t, y) d\alpha \\
V_b(x, y, b_y) &= \frac{2}{E_b}\int_x^{b_y} K_b(\beta) w_b(R_b, \beta, c, x) d\beta
\end{aligned}
\tag{5}
$$

where E_a is Young's modulus and E_b is given by Eq. (6) [3].

$$
E_b = \left(\frac{\Phi}{1-\nu^2}\right)\frac{b}{a}E_a \text{, for } a/b \le 1 \tag{6}
$$

Φ is the complete elliptic integral of the second kind and ν is Poisson ratio.

On the reference point *Q(x,y)* illustrated in Fig.2, Crack opening displacement of basic slice is equal to spring slice.

$$
\begin{aligned}
&\int_y^{a_x}\left\{\int_0^{\alpha}\sigma(x,\eta) w_a(R_a, \alpha, t, \eta) d\eta\right\} w_a(R_a, \alpha, t, y) d\alpha \\
&= \int_y^{a_x}\left\{\int_0^{\alpha} P(x,\eta) w_a(R_a, \alpha, t, \eta) d\eta\right\} w_a(R_a, \alpha, t, y) d\alpha \\
&\quad + \frac{E_a}{E_b}\int_x^{b_y}\left\{\int_0^{\beta} P(\xi, y) w_b(R_b, \beta, c, \xi) d\xi\right\} w_b(R_b, \beta, c, x) d\beta
\end{aligned}
\tag{7}
$$

The spring force $P(x,y)$ is expressed as polynomial approximation.

$$P(x,y)=\sum_{i=1}^{19} e_i \left(\frac{x}{b}\right)^{k_i}\left(\frac{y}{a}\right)^{l_i} \quad (8)$$

e_i is unknown coefficients. k_i and l_i in Eq. (8) are selected and shown in Table 1. The other combination of polynomials was applied in the references [4] [5]. If some COD evaluation points are chosen more than polynomial approximation order, unknown coefficients e_i are determined by using the least-square fitting. In this study, 35 points are chosen as COD evaluation points according to the references [4] [5].

Table 1. Values of exponents' k_i and l_i in Eq. (8).

i	1	2	3	4	5	6	7	8	9	10
k_i	0	1/3	0	1/3	1/2	0	1/2	1	0	1
l_i	0	0	1/3	1/3	0	1/2	1/2	0	1	1

i	11	12	13	14	15	16	17	18	19
k_i	2	0	2	3	0	3	4	0	4
l_i	0	2	2	0	3	3	0	4	4

3. Investigation of the applicability of slice synthesis methodology

In order to investigate the applicable limits of the slice synthesis methodology, comparison of stress intensity factors under uniform tension by applying the slice synthesis methodology with conventional formula developed by Newman and Raju [9] are performed. Total analytical cases are a/b = 0.2, 0.5, 0.8 and 1.0, a/t = 0.2, 0.5 and 0.8, b/c = 0.05, 0.2 and 0.5.

The comparison results are shown in Table 2. The error in Table 2 is defined as follows.

$$Error\,[\%]=\left|\left(F_{NR}-F_{SSM}\right)/F_{NR}\right|\times 100\,. \quad (9)$$

F_{NR} and F_{SSM} represent the magnification factor for stress intensity factor obtained by conventional formula [8] and slice synthesis methodology, respectively.

In Table 2, the difference of stress intensity factor between slice synthesis methodology and conventional formula is increasing according to the increase of dimensionless crack depth a/t. It is considered that the ignorance of back surface effect for stress intensity factor cause the increase of error. Eq.(6) is derived from the matching condition of crack opening displacement in basic and spring slices with analytical solution in infinite bodies [10] at the same reference points. The effect of side surface is not investigated because applied conventional formula is valid when b/c is less or equal to 0.5. As a result, it is concluded that the applicability limit of published slice synthesis methodology is $a/b \le 1.0, a/t \le 0.4, b/c \le 0.5$ in the case that allowable error is less than 5%.

Table 2 Error of stress intensity factor solutions between slice synthesis methodology and the reference solutions.

	Error at *x=b* and *y=0* in Fig.2 [%]											
a/b	0.2			0.5			0.8			1.0		
b/c	0.05	0.2	0.5	0.05	0.2	0.5	0.05	0.2	0.5	0.05	0.2	0.5
a/t=0.2	0.65	0.62	0.00	1.16	1.15	1.74	1.27	1.37	2.86	1.28	1.45	3.41
a/t=0.5	0.78	0.48	4.62	2.38	2.69	0.33	3.42	3.52	2.60	3.61	3.61	3.40
a/t=0.8	1.45	0.39	8.00	4.42	6.21	0.72	7.59	8.87	5.16	8.41	9.29	6.48

	Error at *x=0* and *y=a* in Fig.2 [%]											
a/b	0.2			0.5			0.8			1.0		
b/c	0.05	0.2	0.5	0.05	0.2	0.5	0.05	0.2	0.5	0.05	0.2	0.5
a/t=0.2	1.00	1.12	0.20	1.45	1.36	0.82	1.52	1.44	1.41	1.51	1.46	1.67
a/t=0.5	0.59	2.63	2.00	3.57	5.03	1.98	4.46	5.02	2.91	4.55	4.74	2.97
a/t=0.8	3.16	0.89	7.11	5.52	10.3	4.06	9.00	12.2	7.30	9.82	12.0	7.67

4. Improvement of the slice synthesis methodology in order to consider the back surface effect

Many modified methods are possible to improve the accuracy of stress intensity factor computation by applying the slice synthesis methodology. In this research, modification of E_b in Eq. (6) is performed by referring Table 2. That is, a correction coefficient γ is incorporate into Eq. (6).

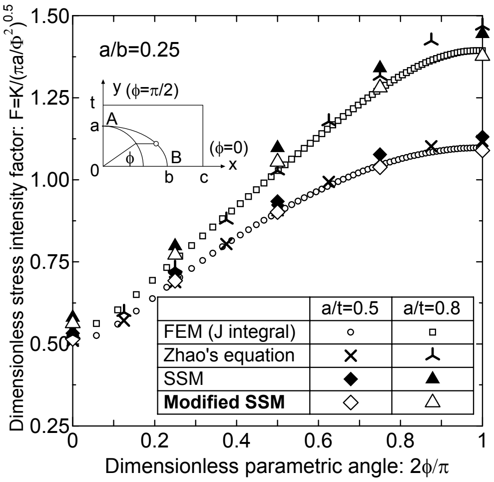
$$E_b = \gamma \left(\frac{\Phi}{1-\nu^2} \right) \frac{b}{a} E_a \text{ , for } a/b \le 1 \tag{10}$$

The value of γ is identified heuristically to minimize the larger value of stress intensity factor error at the minor axis tip (*x=0* and *y=a* in Fig.2) or the major axis tip (*x=b* and *y=0* in Fig.2).

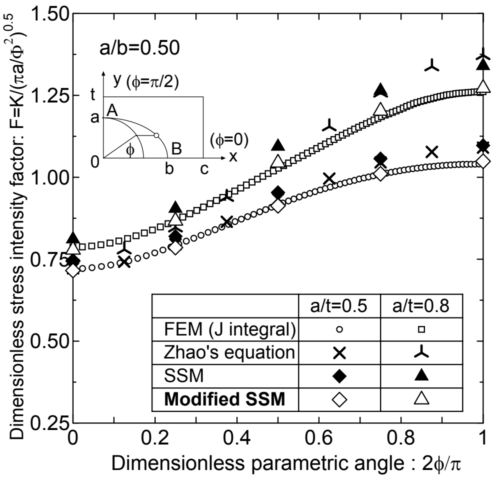
Comparison of stress intensity factor and crack opening profiles by applying the slice synthesis methodology with a correction coefficient γ and finite element analyses is performed. MSC. Marc (ver. 2005r3) is used to the FE analyses. Representative comparison results are shown in Figs.4 and 5. All of the geometrical conditions of crack shapes listed in Table 2 are applied in order to investigate the validity of proposed improving method. The value of stress intensity factors by applying other calculation methods are plotted in Fig.4. Zhao's equation in Fig.4 means the conventional calculation formula for the stress intensity factor at arbitrary parametric angle. This equation is proposed in the reference [4]. J integral method is applied in the computation of stress intensity factor by FEM.

It is concluded that the incorporating a correction coefficient into Eq. (6) enables to achieve the accuracy improvement of the slice synthesis methodology.

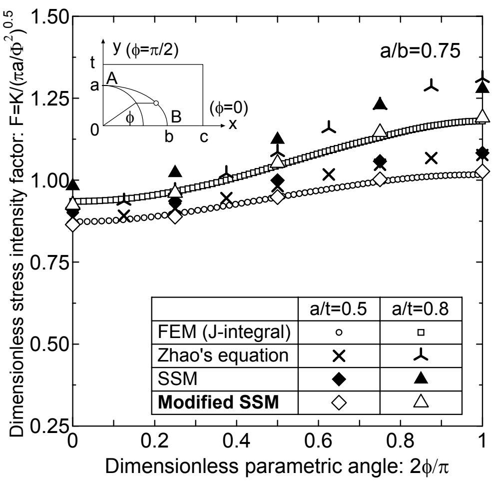
Database of correction coefficients γ for crack shape conditions shown in Table 2 are created in this research. A correction coefficient γ for an arbitrary crack shape condition can be computed by extracting the data from this database.



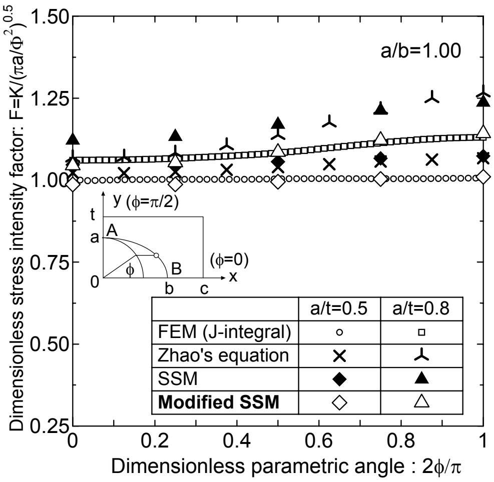
a) *a/b*=0.25, *a/t*=0.5 and 0.8



b) *a/b*=0.50, *a/t*=0.5 and 0.8

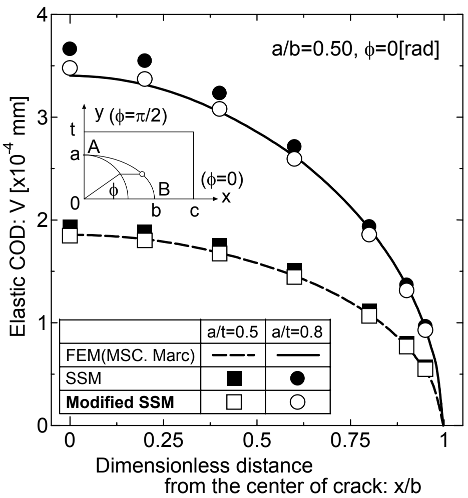


c) *a/b*=0.75, *a/t*=0.5 and 0.8

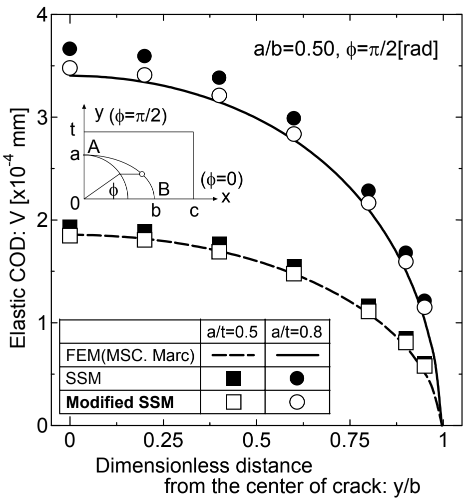


d) *a/b*=1.00, *a/t*=0.5 and 0.8

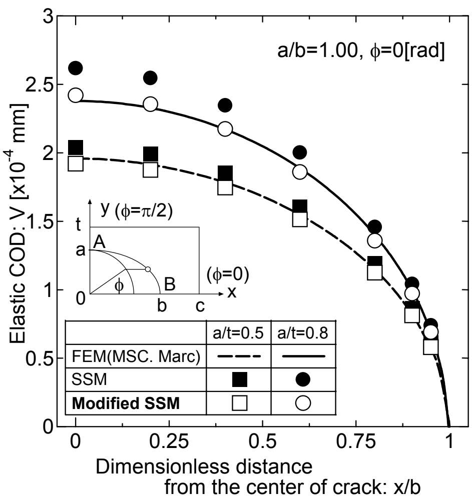
Fig.4 Comparison of the stress intensity factors by applying modified slice synthesis methodology (SSM) with other methods.



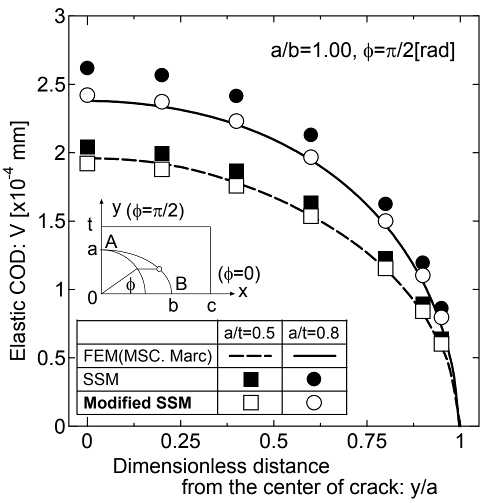
a) a/b=0.50, ϕ=0[rad]



b) a/b=0.50, $\phi=\pi/2$[rad]



c) a/b=1.00, ϕ=0[rad]



d) a/b=0.50, $\phi=\pi/2$[rad]

Fig.5 Comparison of crack opening profiles between modified slice synthesis methodology (SSM) and former published method and FEM.

5. Concluding remarks

The limit of applicability of the slice synthesis methodology with weight function methods to compute stress intensity factors for an embedded crack is investigated by comparing of stress intensity factors obtained from a conventional formula based on finite element analyses. From these calculation results, recommended regions for the application of former published slice synthesis methodology are conducted. Improvement method of the slice synthesis methodology in order to consider the back surface effect is proposed. By incorporating a correction coefficient into Eq. (6), the accuracy improvement of the slice synthesis methodology is achieved.

Future challenge for the improvement of slice synthesis methodology is to collect correction factors in Eq. (6) for wide range of crack shape morphologies and application of propose method for a part-through surface crack in finite size body. Moreover, construction of three dimensional shaped fatigue crack growth simulation based on strip yield model with weight functions and improved slice synthesis methodology are expected.

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