Contact Interaction of Interlaminar Crack Faces under Harmonic Loading

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1 Introduction

Generally, fracture mechanics problems for cracked solids under dynamic loading can be solved using advanced numerical methods, since the analytical solutions are limited to a relatively small number of idealized model problems corresponding to very special geometrical configurations and loading conditions. Previously, a considerable amount of work was devoted to cracked homogeneous materials [1–3]. The case of inter-component cracks received much less attention due to the substantial complications which arise in numerical solution of such problems. Primarily, the publications concerning crack fracture in composite materials are focused on static loading. However, understanding the mechanism of dynamic fracture in composite materials becomes more and more important with the increased use of composites in modern engineering, where the components are frequently subjected to dynamic loadings, see, for example [4–9].

In paper [10], the system of boundary integral equations for the general case of an interface crack between two dissimilar elastic materials under dynamic loading was derived. In papers [11–13], the derived integral system was solved numerically by the method of boundary elements for the case of a penny-shaped interface crack under normally incident tension-compression wave. The distributions of displacements and tractions were computed for several typical materials of half-spaces. It was shown that with decreasing frequency of the loading the dynamic solution tends to the static one, and the obtained numerical results are in a very good agreement with the analytical static solution [4, 5, 8]. This study is devoted to the problem for an interface crack under harmonic external loading taking the opposite crack faces contact interaction into account. The system of boundary integral equations, derived in [10], is modified in order to increase the stability and the accuracy of the solution, and decrease the computation time. The numerical solution is obtained for the normally incident tension-compression wave. The distribution of the displacements and tractions at the bimaterial interface and the surface of the crack are analysed for the normally incident tension-compression wave. The dynamic stress intensity factors (opening and shear modes) are also computed as functions of the frequency of the incident wave and properties of the upper and the lower half-spaces. The results are compared with those obtained neglecting the contact interaction.

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2 Problem statement

Let us consider a crack located at the bimaterial interface under external dynamic loading. For this purpose, we investigate an unbounded elastic solid which consists of two dissimilar homogeneous isotropic half-spaces $\Omega^{(1)}$ and $\Omega^{(2)}$. The interface between the half-spaces, Γ^{*}, acts as the boundary $\Gamma^{(1)}$ for the upper half-space, and the boundary $\Gamma^{(2)}$ for the lower half-space. The boundaries $\Gamma^{(1)}$ and $\Gamma^{(2)}$ differ by the opposite orientation of their outer normal vectors. Henceforth, the superscript (1) refers to the upper half-space and the superscript (2) refers to the lower half-space. We assume that surfaces $\Gamma^{(m)}$ ($m = 1,2$) consist of the infinite parts $\Gamma^{(m)*}$, which form the bonding interface Γ^{*}, and the finite parts $\Gamma^{(m)\mathrm{cr}}$, which form the crack surface Γ^{cr}, see Fig. 1.

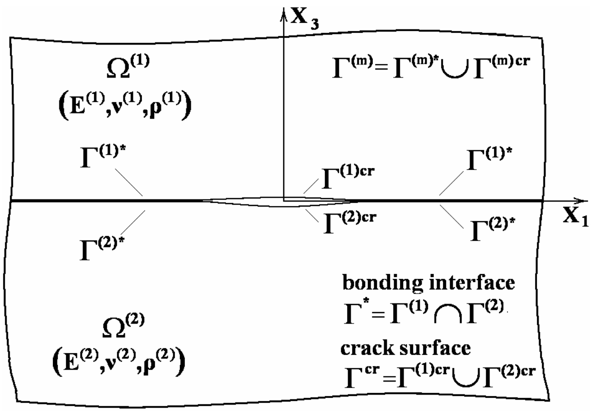

Figure 1. An interface crack between two half-spaces

In the absence of body forces, the stress-strain state of both domains is defined by the dynamic equations of the linear elasticity for the displacement vector $\mathbf{u}^{(m)}(\mathbf{x},t)$ (the Lamé equations)

$$(\lambda^{(m)} + \mu^{(m)})\operatorname{grad}\operatorname{div}\mathbf{u}^{(m)}(\mathbf{x},t) + \mu^{(m)}\Delta\mathbf{u}^{(m)}(\mathbf{x},t) = \rho^{(m)}\partial_t^2\mathbf{u}^{(m)}(\mathbf{x},t), \quad \mathbf{x}\in\Omega^{(m)}, \quad t\in \mathrm{T} = [0,\infty), \tag{1}$$

where Δ is the Laplace operator, $\lambda^{(m)}$ and $\mu^{(m)}$ are the Lamé elastic constants, $\rho^{(m)}$ is the specific material density.

It was assumed that there are no initial displacements of the points of the body, in other words the body is strainless at the initial moment. The following conditions of continuity for displacements and stresses are satisfied at the bonding interface:

$$\mathbf{u}^{(1)}(\mathbf{x},t) = \mathbf{u}^{(2)}(\mathbf{x},t), \quad \mathbf{p}^{(1)}(\mathbf{x},t) = -\mathbf{p}^{(2)}(\mathbf{x},t), \quad \mathbf{x}\in\Gamma^{*} = \Gamma^{(1)}\cap\Gamma^{(2)}, \quad t\in\mathrm{T}, \tag{2}$$

where the known traction vectors on the crack surface, caused by the external loading, are given as

$$\mathbf{p}^{(1)}(\mathbf{x},t)=\mathbf{g}^{(1)}(\mathbf{x},t),\quad \mathbf{p}^{(2)}(\mathbf{x},t)=\mathbf{g}^{(2)}(\mathbf{x},t),\quad \mathbf{x}\in\Gamma^{(m)cr},\quad t\in\mathrm{T}.$$

The Sommerfeld radiation-type condition, which provides a finite elastic energy of an infinite body, is also imposed at infinity on the vector of displacements.

3 Integral equations

The components of displacement field in the upper and lower half-spaces $\Omega^{(m)}$ in terms of boundary displacements and tractions can be represented using the Somigliano dynamic identity [1–3, 10, 13, 14]:

$$u_j^{(m)}(\mathbf{x},t)=\int_{\mathbf{T}}\int_{\Gamma^{(m)}}(p_i^{(m)}(\mathbf{y},\tau)U_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)-u_i^{(m)}(\mathbf{y},\tau)W_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau))d\mathbf{y}d\tau,\quad \mathbf{x}\in\Omega^{(m)},\quad t\in\mathrm{T}, \tag{3}$$

where $\mathbf{x}$ is the point of observation and $\mathbf{y}$ is the point of loading. Here the integral kernel $U_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)$ is the Green fundamental displacement tensor [1–3]. The integral kernel $W_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)$ can be obtained from $U_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)$ by applying the following differential operator [1, 14] :

$$P_{ik}\left[\bullet,(y)\right]=\lambda n_i(y)\frac{\partial[\bullet]}{\partial y_k}+\mu\left[\delta_{ik}\frac{\partial[\bullet]}{\partial\mathbf{n(y)}}+n_k(y)\frac{\partial[\bullet]}{\partial y_i}\right]. \tag{4}$$

Applying the differential operator (4) to the Somigliano dynamic identity (3) we obtain the components of the traction vector in terms of boundary displacements and tractions for both half-spaces:

$$p_j^{(m)}(\mathbf{x},t)=\int_{T}\int_{\Gamma^{(m)}}(p_i^{(m)}(\mathbf{y},\tau)K_{ij}^{(m)}(x,y,t-\tau)-u_i^{(m)}(y,\tau)F_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau))\mathrm{d}\mathbf{y}\mathrm{d}\tau,\quad \mathbf{x}\in\Omega^{(m)},\quad t\in\mathrm{T}. \tag{5}$$

For the limiting case $\mathbf{x}\to\Gamma^{(m)}$, taking into account the assumed relatively smooth distribution of traction on regular surfaces $\Gamma^{(1)}$ and $\Gamma^{(2)}$, the following representation of the traction vector at the interface can be obtained from Eq. (5):

$$\frac{1}{2}p_j^{(m)}(\mathbf{x},t)=\int_{\mathrm{T}}\int_{\Gamma^{(m)}}(p_i^{(m)}(\mathbf{y},\tau)K_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)-u_i^{(m)}(\mathbf{y},\tau)F_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau))\,d\mathbf{y}\,d\tau, \tag{6}$$

where $\mathbf{x} \in \Gamma^{(m)}$, $t \in \mathrm{T}$.

Finally, the system of boundary integral equations takes the form:

$$\frac{1}{2} g_j^{(1)}(\mathbf{x},t) = \int\limits_{\mathrm{T}} \int\limits_{\Gamma^{(1)\mathrm{cr}}} (g_i^{(1)}(\mathbf{y},\tau) K_{ij}^{(1)}(\mathbf{x},\mathbf{y},t-\tau) - u_i^{(1)}(\mathbf{y},\tau) F_{ij}^{(1)}(\mathbf{x},\mathbf{y},t-\tau))\, d\mathbf{y}\, d\tau$$
$$+ \int\limits_{\mathrm{T}} \int\limits_{\Gamma^*} (u_i^*(\mathbf{y},\tau) F_{ij}^{(1)}(\mathbf{x},\mathbf{y},t-\tau) - p_i^*(\mathbf{y},\tau) K_{ij}^{(1)}(\mathbf{x},\mathbf{y},t-\tau))\, d\mathbf{y}\, d\tau, \quad \mathbf{x} \in \Gamma^{(1)\mathrm{cr}}, \tag{7}$$

$$\frac{1}{2} g_j^{(2)}(\mathbf{x},t) = \int\limits_{\mathrm{T}} \int\limits_{\Gamma^{(2)\mathrm{cr}}} (g_i^{(2)}(\mathbf{y},\tau) K_{ij}^{(2)}(\mathbf{x},\mathbf{y},t-\tau) - u_i^{(2)}(\mathbf{y},\tau) F_{ij}^{(2)}(\mathbf{x},\mathbf{y},t-\tau))\, d\mathbf{y}\, d\tau$$
$$- \int\limits_{\mathrm{T}} \int\limits_{\Gamma^*} (u_i^*(\mathbf{y},\tau) F_{ij}^{(2)}(\mathbf{x},\mathbf{y},t-\tau) - p_i^*(\mathbf{y},\tau) K_{ij}^{(2)}(\mathbf{x},\mathbf{y},t-\tau))\, d\mathbf{y}\, d\tau, \quad \mathbf{x} \in \Gamma^{(2)\mathrm{cr}}, \tag{8}$$

$$\int\limits_{\mathrm{T}} \int\limits_{\Gamma^{(1)\mathrm{cr}}} g_i^{(1)}(\mathbf{y},\tau) K_{ij}^{(1)}(\mathbf{x},\mathbf{y},t-\tau)\, d\mathbf{y}\, d\tau = \frac{1}{2} p_j^*(\mathbf{x},t) + \int\limits_{\mathrm{T}} \int\limits_{\Gamma^{(1)\mathrm{cr}}} u_i^{(1)}(\mathbf{y},\tau) F_{ij}^{(1)}(\mathbf{x},\mathbf{y},t-\tau)\, d\mathbf{y}\, d\tau$$
$$- \int\limits_{\mathrm{T}} \int\limits_{\Gamma^*} (u_i^*(\mathbf{y},\tau) F_{ij}^{(1)}(\mathbf{x},\mathbf{y},t-\tau) - p_i^*(\mathbf{y},\tau) K_{ij}^{(1)}(\mathbf{x},\mathbf{y},t-\tau))\, d\mathbf{y}\, d\tau, \quad \mathbf{x} \in \Gamma^*, \tag{9}$$

$$\int\limits_{\mathrm{T}} \int\limits_{\Gamma^{(2)\mathrm{cr}}} g_i^{(2)}(\mathbf{y},\tau) K_{ij}^{(2)}(\mathbf{x},\mathbf{y},t-\tau)\, d\mathbf{y}\, d\tau = \frac{1}{2} p_j^*(\mathbf{x},t) + \int\limits_{\mathrm{T}} \int\limits_{\Gamma^{(2)\mathrm{cr}}} u_i^{(2)}(\mathbf{y},\tau) F_{ij}^{(2)}(\mathbf{x},\mathbf{y},t-\tau)\, d\mathbf{y}\, d\tau$$
$$+ \int\limits_{\mathrm{T}} \int\limits_{\Gamma^*} (u_i^*(\mathbf{y},\tau) F_{ij}^{(2)}(\mathbf{x},\mathbf{y},t-\tau) - p_i^*(\mathbf{y},\tau) K_{ij}^{(2)}(\mathbf{x},\mathbf{y},t-\tau))\, d\mathbf{y}\, d\tau, \quad \mathbf{x} \in \Gamma^*. \tag{10}$$

We also introduced the new variables, $u_i^*(\mathbf{x},t)$ and $p_i^*(\mathbf{x},t)$:

$$u_i^*(\mathbf{x},t) = u_i^{(1)}(\mathbf{x},t), \quad p_i^*(\mathbf{x},t) = -p_i^{(1)}(\mathbf{x},t), \quad \mathbf{x} \in \Gamma^{(1)*}, t \in \mathrm{T},$$
$$u_i^*(\mathbf{x},t) = u_i^{(2)}(\mathbf{x},t), \quad p_i^*(\mathbf{x},t) = p_i^{(2)}(\mathbf{x},t), \quad \mathbf{x} \in \Gamma^{(2)*}, t \in \mathrm{T}.$$

Note that in the present study the form of the resulting boundary integral equations system (7)–(10) differs significantly from the corresponding integral system in [9–11]. The resulting system of boundary integral equations becomes simpler, it does not contain integral kernels $U_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)$ and $W_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)$. The further algebraic manipulations (i.e. summation and subtraction of the corresponding integral equations) are also unnecessary, therefore the system of boundary integral equations does not contain residuals of all integral kernels and the corresponding matrix of linear algebraic equations becomes sparser. For example, in the case of the collocation method with a piecewise continuous approximation used in this study, the number of non-zero elements of the matrix of linear algebraic equations decreases by $8N_{out}(N_{in}-1)$ in 2D case and by $16N_{out}N_{in}-12N_{out}$ in 3D case, where N_{out} and N_{in} are the numbers of boundary elements located on the bonding interface and crack surface respectively. Consequently the total number of integrals over boundary elements, required to be computed in order to solve the problem, decreases even further, by $16N_{out}(N_{in}+N_{out})$ in 2D case and by $36N_{out}(N_{in}+N_{out})$ in 3D case, which results in the significant acceleration of the numerical solution of the problem. On average, the solution time decreases by 40–50%. The stability of the obtained solution also increases, especially for the higher frequencies of external loading and for the cases with considerable difference between mechanical properties of adjoining half-spaces.

For the case of harmonic loading with the frequency $\omega = 2\pi/T$, which is considered in the paper, tractions and displacements are harmonic functions and can be presented as follows:

$$f(\bullet,t) = \mathrm{Re}(f(\bullet)e^{i\omega t}), \qquad f(\bullet) = \frac{\omega}{2\pi}\int_0^T f(\bullet,t)e^{-i\omega t}dt.$$

Then the system of boundary integral equations (7)–(10) can be rewritten as:

$$\begin{aligned} &\frac{1}{2}g_j^{(1)}(\mathbf{x}) - \int_{\Gamma^{(m)\mathrm{cr}}} g_i^{(1)}(\mathbf{y})K_{ij}^{(1)}(\mathbf{x},\mathbf{y},\omega)\,d\mathbf{y} = -\int_{\Gamma^{(m)\mathrm{cr}}} u_i^{(1)}(\mathbf{y})F_{ij}^{(1)}(\mathbf{x},\mathbf{y},\omega)\,d\mathbf{y} \\ &+\int_{\Gamma^*}(u_i^*(\mathbf{y})F_{ij}^{(1)}(\mathbf{x},\mathbf{y},\omega) - p_i^*(\mathbf{y})K_{ij}^{(1)}(\mathbf{x},\mathbf{y},\omega))d\mathbf{y}, \quad \mathbf{x}\in\Gamma^{(1)\mathrm{cr}}, \end{aligned} \tag{11}$$

$$\begin{aligned} &\frac{1}{2}g_j^{(2)}(\mathbf{x}) - \int_{\Gamma^{(m)\mathrm{cr}}} g_i^{(2)}(\mathbf{y})K_{ij}^{(2)}(\mathbf{x},\mathbf{y},\omega)\,d\mathbf{y} = -\int_{\Gamma^{(m)\mathrm{cr}}} u_i^{(2)}(\mathbf{y})F_{ij}^{(2)}(\mathbf{x},\mathbf{y},\omega)\,d\mathbf{y} \\ &-\int_{\Gamma^*}(u_i^*(\mathbf{y})F_{ij}^{(2)}(\mathbf{x},\mathbf{y},\omega) - p_i^*(\mathbf{y})K_{ij}^{(2)}(\mathbf{x},\mathbf{y},\omega))d\mathbf{y}, \quad \mathbf{x}\in\Gamma^{(2)\mathrm{cr}}, \end{aligned} \tag{12}$$

$$\begin{aligned} &\int_{\Gamma^{(1)\mathrm{cr}}} g_i^{(1)}(\mathbf{y})K_{ij}^{(1)}(\mathbf{x},\mathbf{y},\omega)\,d\mathbf{y} = \frac{1}{2}p_j^*(\mathbf{x}) + \int_{\Gamma^{(1)\mathrm{cr}}} u_i^{(1)}(\mathbf{y})F_{ij}^{(1)}(\mathbf{x},\mathbf{y},\omega)\,d\mathbf{y} \\ &-\int_{\Gamma^*}(u_i^*(\mathbf{y})F_{ij}^{(1)}(\mathbf{x},\mathbf{y},\omega) - p_i^*(\mathbf{y})K_{ij}^{(1)}(\mathbf{x},\mathbf{y},\omega))\,d\mathbf{y}, \quad \mathbf{x}\in\Gamma^*, \end{aligned} \tag{13}$$

$$\int_{\Gamma^{(2)cr}} g_i^{(2)}(\mathbf{y}) K_{ij}^{(2)}(\mathbf{x},\mathbf{y},\omega)\, d\mathbf{y} = \frac{1}{2} p_j^*(\mathbf{x}) + \int_{\Gamma^{(2)cr}} u_i^{(2)}(\mathbf{y}) F_{ij}^{(2)}(\mathbf{x},\mathbf{y},\omega)\, d\mathbf{y}$$
$$+ \int_{\Gamma^*} (u_i^*(\mathbf{y}) F_{ij}^{(2)}(\mathbf{x},\mathbf{y},\omega) - p_i^*(\mathbf{y}) K_{ij}^{(2)}(\mathbf{x},\mathbf{y},\omega))\, d\mathbf{y}, \quad \mathbf{x} \in \Gamma^*. \tag{14}$$

The integral kernels $W_{ij}^{(m)}(\mathbf{x},\mathbf{y},\omega)$, $K_{ij}^{(m)}(\mathbf{x},\mathbf{y},\omega)$ and $F_{ij}^{(m)}(\mathbf{x},\mathbf{y},\omega)$ are obtained from the fundamental Green tensor by applying the differential operator (4). Note that due to the presence of non-integrable singularities in the integral kernels $F_{ij}^{(m)}(\mathbf{x},\mathbf{y},\omega)$, whose rank exceeds the dimension of the integration region, the corresponding hypersingular integrals of the system of boundary integral equations (11)–(14) are treated in the sense of the Hadamard finite part [14, 15, 16].

4 Contact interaction of crack faces

In reality, under dynamic loading the opposite crack faces interact with each other, significantly changing the stress and strain fields near the crack tips. However, since the area of interest is hidden in the solid, the direct observation and measurement of the contact characteristics is impossible. The nature of the contact interaction between two opposite crack surfaces is very complex. Under deformation of the material, the contact area changes in time. It is unknown beforehand and must be determined as a part of solution. The complexity of the problem is further compounded by the fact that the contact behaviour is very sensitive to the material properties of two contacting surface, their textures and topologies, frequency, magnitude and direction of the external loading in relation to the contact [13]. Taking these effects into account will make the contact crack problem highly non-linear. Thus considering the crack closure effect is the natural next stage of this research.

Under the external dynamic loading the opposite crack faces move with respect to each other and the corresponding displacement is given by the discontinuity vector $\Delta\mathbf{u}(\mathbf{x},t) = \mathbf{u}^{(1)}(\mathbf{x},t) - \mathbf{u}^{(2)}(\mathbf{x},t)$. The contact interaction results in the appearance of the contact force $\mathbf{q}(\mathbf{x},t)$ in the contact region.

In order to include contact interaction into consideration, the Signorini constraints must be imposed for the normal components of the contact force and the displacement discontinuity vectors

$$\Delta u_n(\mathbf{x},t) \ge 0, \quad q_n(\mathbf{x},t) \ge 0, \quad \Delta u_n(\mathbf{x},t) q_n(\mathbf{x},t) = 0, \quad \mathbf{x} \in \Omega,\, t \in [0;T]. \tag{15}$$

The constraints ensure that there is no interpenetration of the opposite crack faces (the first constraint in Eq. (15)); the contact force is unilateral (the second constraint in Eq. (15)); and the contact forces are absent if there is a non-zero opening of the crack (the third constraint in Eq. (15)).

Additionally we assume that the interaction satisfies the Coulomb friction law:

$$\left|q_{\tau}(\mathbf{x},t)\right| < k_{\tau} q_n(\mathbf{x},t) \Rightarrow \frac{\partial \Delta u_{\tau}(\mathbf{x},t)}{\partial t} = 0; \quad (16)$$

$$\left|q_{\tau}(\mathbf{x},t)\right| = k_{\tau} q_n(\mathbf{x},t) \Rightarrow \frac{\partial \Delta u_{\tau}(\mathbf{x},t)}{\partial t} = -\frac{q_{\tau}(\mathbf{x},t)}{\left|q_{\tau}(\mathbf{x},t)\right|}\left|\frac{\partial \Delta u_{\tau}(\mathbf{x},t)}{\partial t}\right|, \quad \mathbf{x} \in \Omega,\ t \in [0;T], \quad (17)$$

where k_{τ} is the friction coefficient. The Eq. (16) means that the opposite crack faces remain immovable with respect to each other in the tangential plane as long as they are held by the friction force. However, as soon as the magnitude of the tangential contact forces reaches a certain limit, depending on the friction coefficient and the normal contact forces (see Eq. (17)), the crack faces begin to slip.

If the contact interaction of crack faces is taken into account, the resulting process is a steady-state periodic process, but not a harmonic one. As a result, components of the stress-strain state cannot be represented as a function of coordinates multiplied by an exponential function. According to [15], all components of solution can be expanded into the Fourier series

$$f(\bullet,t) = \frac{f_{0,\cos}(\bullet)}{2} + \sum_{k=1}^{\infty}\left(f_{k,\cos}(\bullet)\cos(\omega_k t) + f_{k,\sin}(\bullet)\sin(\omega_k t)\right),$$

where $\omega_k = 2\pi k / T$, and Fourier coefficients are given as

$$f_{k,\cos}(\bullet) = \frac{\omega}{2\pi}\int_0^T f(\bullet,t)\cos(\omega_k t)dt, \quad f_{k,\sin}(\bullet) = \frac{\omega}{2\pi}\int_0^T f(\bullet,t)\sin(\omega_k t)dt.$$

The considered contact problem is non-linear and requires an iterative solution procedure. During the iterative process, the Fourier coefficients will change from one iterative step to the next until the distribution of the displacements and contact force vectors satisfying the constraints (15) – (17) will be found.

5 Numerical results

The piecewise-constant approximation of the known and unknown functions was used to solve the problem numerically [11–13]. Note that the solution is symmetric with respect to the centre of the crack and the Sommerfeld radiation-type condition (3) is satisfied at the infinity, i.e. the displacements at the interface decrease gradually with increase in the distance to the crack.

As a numerical example let us consider a linear crack with length of $2R$ under the normally incident time-harmonic tension-compression wave of the unit intensity. The properties of bimaterial are: $E^{(1)} = 207\text{GPa}$, $E^{(2)} = 70\text{GPa}$; $\nu^{(1)} = 0.25$, $\nu^{(2)} = 0.35$; $\rho^{(1)} = 7800\,\text{kg}/\text{m}^3$, $\rho^{(2)} = 2700\,\text{kg}/\text{m}^3$. On Figure 2 – 4 the normal displacements and contact forces are given at the crack surface for $k_2^{(1)}R = 0.1$.

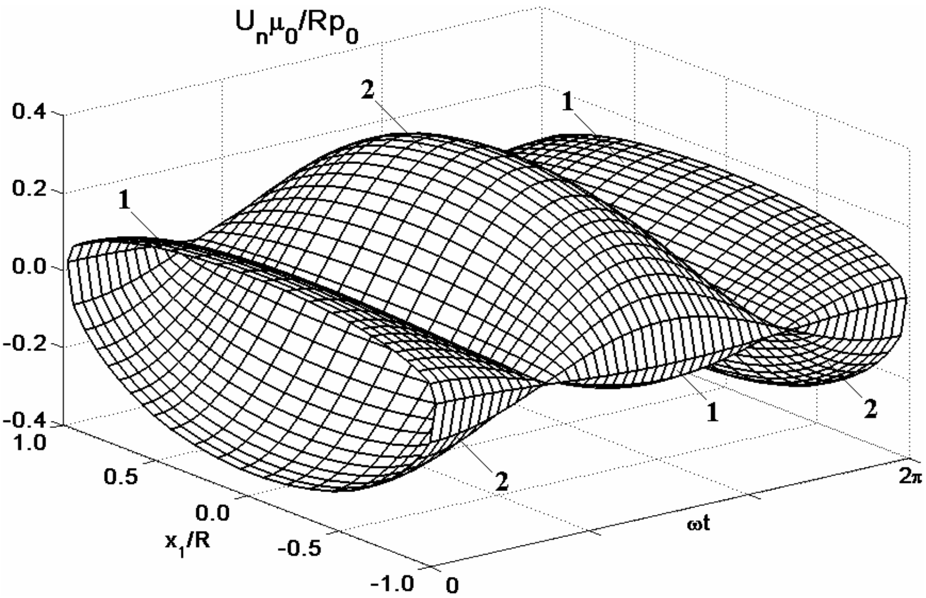

Figure 2. Normal displacements neglecting the contact interaction

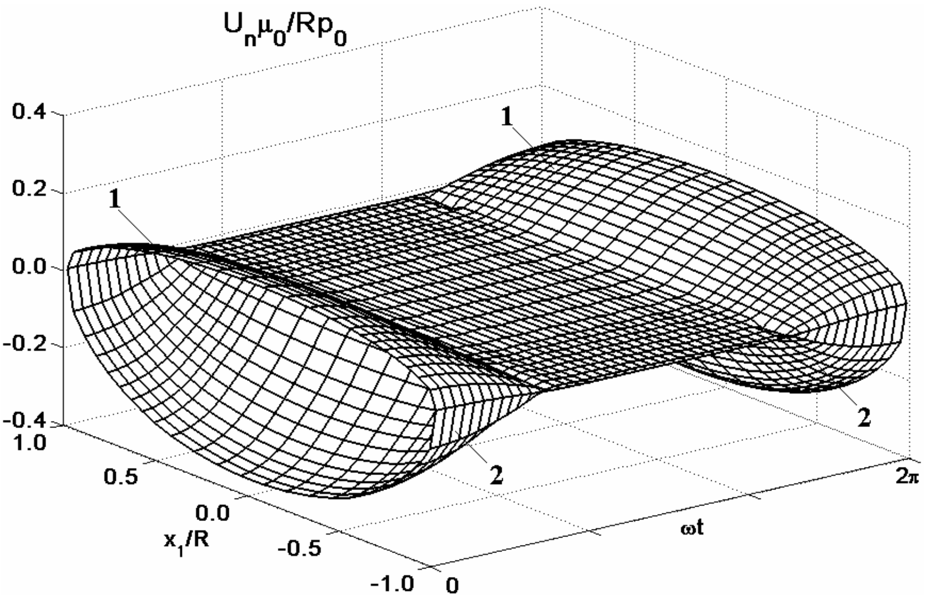

Figure 3. Normal displacements taking the contact interaction into account

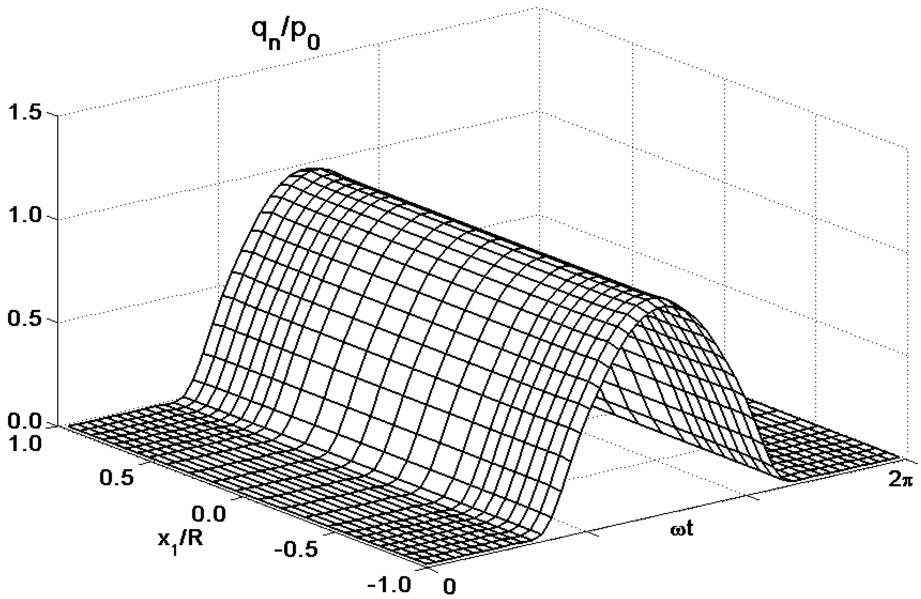

Figure 4. Normal contact forces at the crack surface

It is obvious that the contact interaction of crack faces changes the solution both quantitatively and qualitatively. The unilateral Signorini constraints (15) are satisfied on the surface of the crack during the period of oscillation. There is no interpenetration of the opposite crack faces and normal contact forces are unilateral. With the rise in the wave number the distribution of forces and displacements become much more complicated. A similar situation is observed for shear components of the solution.

Note that contact forces were normalized by the stress amplitude of the incident wave, p_0; and displacements were normalized by the factor $2\mu_0 / Rp_0$, where μ_0 was specified as follows [8]:

$$\mu_0 = \mu_1 \frac{1-\gamma_2}{1+\kappa_1}, \quad \gamma_2 = \left(\frac{a_1}{2} - a_2\right),$$
$$a_1 = \frac{\mu_1 - \mu_2}{\mu_1 + \kappa_1\mu_2}, \; a_2 = \frac{\kappa_1\mu_2 - \kappa_2\mu_1}{2(\mu_2 + \kappa_2\mu_1)}, \; \kappa_i = 3 - 4\nu_i .$$

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