The UniGrow Fatigue Crack Growth Model for Spectrum Loading

S. Mikheevskiy[1], *G. Glinka*[1]
[1]*University of Waterloo, Department of Mechanical Engineering,*
Waterloo, ON, Canada, N2L 3G1

1. Introduction

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The presence of a crack or any other discontinuities can significantly reduce the fatigue life of a component or structure. The crack may be initiated by fatigue, be a manufacturing defect, caused by an impact, or similar event (e.g., a thermal shock.). However, in most engineering cases, the initial size of a crack or discontinuity is not large enough to cause catastrophic failure. Once a crack is present in a structure, it tends to grow under the influence of cyclic loading until its critical size is reached. At this point, fracture occurs.
Many fatigue crack growth studies available in literature have been carried out under constant amplitude loading, otherwise referred to as steady state conditions. As a result, constant amplitude fatigue crack growth data is in general repeatable and well understood. The problem of predicting fatigue crack growth life becomes increasingly more complex when the loading spectrum is not constant amplitude. This is commonly referred to as *variable-amplitude* or *spectrum loading* and produces what is known as *memory effects* or *load-history effects*.
The UniGrow fatigue crack growth model initially proposed by Glinka and Noroozi [1] is based on the elastic-plastic stress-strain material response at the crack tip region. This model can be also related to the group of so-called 'residual stress based models', according to the Skorupa's classification [2]. Residual or compressive stresses ahead of the crack tip can either delay or accelerate subsequent fatigue crack growth depending upon the current crack length and load history.
The present work is a modified version of the UniGrow fatigue crack growth model, and makes it applicable to all kinds of variable amplitude loading spectra. The modifications include a very important feature called the 'memory effect'.

2. Basics of the UniGrow model

A brief description of the UniGrow fatigue crack growth model proposed by Glinka and Noroozi is provided in this section.
According to Neuber's [3] micro-support concept, a real material can be considered as a set of elementary particles or material blocks of a finite elements size ρ^*. The same idea was also proposed by Forsyth [4] based on the microscopic observations of fatigue crack front. It leads to the idea of modeling the fatigue crack as a sharp notch with a tip radius ρ^* within the continuum mechanics framework. Therefore, the usual notch analysis techniques can be used in order to define stresses and strains ahead of the crack

tip. It should be noted that the stress value in each finite element is the average value of the stress acting over its length (Fig.1).

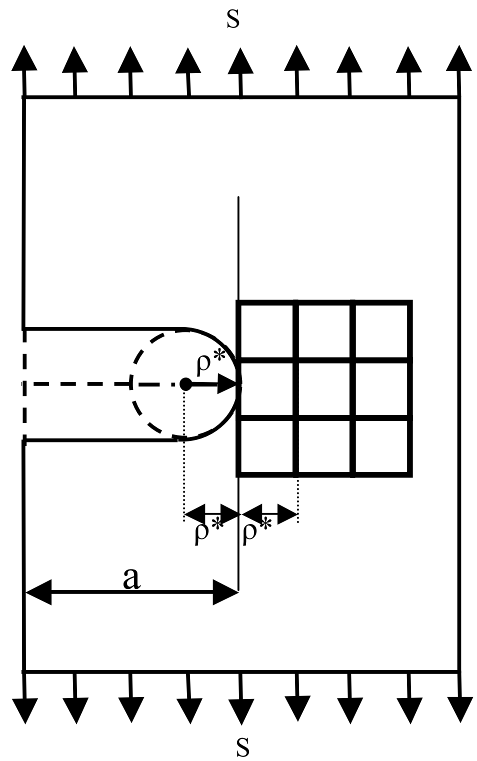

Figure 1: The crack tip geometry and the discrete material model

The following assumptions and computational rules are the basics behind the UniGrow fatigue crack growth model.

- The material consists of elementary blocks of a finite dimension ρ^*.
- The fatigue crack is regarded as a notch with the tip radius ρ^*.
- The analysis is based on Ramberg-Osgood [5] (cyclic) and Manson-Coffin [6] (fatigue) material properties.
- The number of cycles to fail the material over the distance ρ^* can be obtained using Smith-Watson-Topper [7] damage parameter and Manson-Coffin fatigue curve.

$$D = \sigma_{\max} \frac{\Delta\varepsilon}{2} = \frac{\left(\sigma'_f\right)^2}{E}\left(2N\right)^{2b} + \sigma'_f \varepsilon'_f \left(2N\right)^{b+c} \qquad (1)$$

- The fatigue crack growth rate can be expressed as:

$$\frac{da}{dN} = \frac{\rho^*}{N} \qquad (2)$$

Based on these assumptions a fatigue crack growth expression was derived in the form of

$$\frac{da}{dN} = C\left(K_{\max,tot}^{p}\Delta K_{tot}^{1-p}\right)^{m}. \tag{3}$$

A similar expression has been proposed by Walker [8], with the only difference being that the applied values of maximum stress intensity factor and the stress intensity range were used instead of total ones. Total and applied stress intensity parameters differ only by the amount of the residual stress intensity factor, K_r, corresponding to the compressive residual stresses in the crack tip region, σ_r.

$$\begin{aligned} K_{\max,tot} &= K_{\max,appl} + K_r \\ \Delta K_{tot} &= \Delta K_{appl} + K_r \end{aligned}. \tag{4}$$

Therefore, the correct estimation of residual stresses produced by all previous loading cycles and corresponding residual stress intensity factor becomes one of the most important (and complicated) parts of the UniGrow model.

3. Modifications of the UniGrow model for a variable amplitude loading

The fatigue crack growth depends on several factors. These include the load, geometry and crack size. All of these factors are combined into one parameter called “the stress intensity factor”. Therefore a method for calculating the stress intensity factors must be determined. The weight function technique [9] was shown to be one of the easiest and most versatile ways of determining the stress intensity factor for a variety of load and geometry configurations. Based on this method the required stress intensity factor can be determined by taking the integral of the product of a stress function multiplied with an appropriate weight function over the crack length. It is therefore important to know the stress field in the crack tip region.

It can be noted that each cycle of a loading spectrum produces a qualitatively similar type of near the crack tip stress field. The UniGrow fatigue crack growth model uses the multiaxial Neuber [10] rule to determine the actual crack tip stresses induced by a loading cycle. Let us consider several consecutive reversals of the arbitrary variable amplitude loading history shown in Fig. 2.

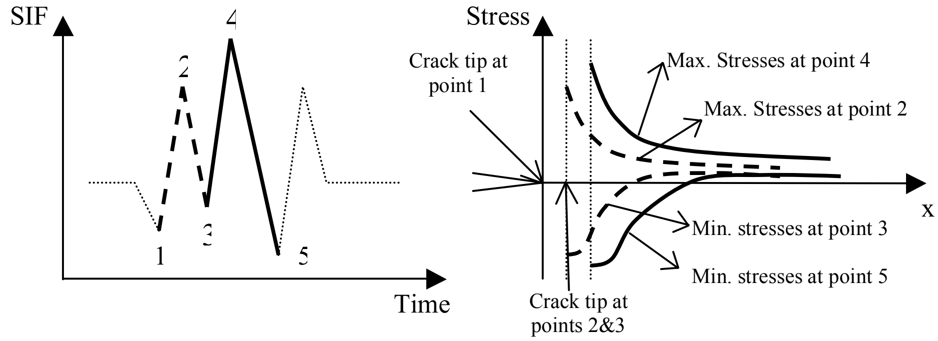

Figure 2: Schematic stress field corresponding to the different points of the loading history

Application of the tensile load reversal from point 1 to point 2 may extend the fatigue crack by a certain increment, Δa. Therefore, the maximum stresses corresponding to the maximum load of level 2 must be associated with the new crack tip position. On the other side, the fatigue crack does not grow during the unloading reversal from load level 2 to load level 3, thus the static notch analysis can be used to determine the minimum stresses corresponding to the minimum load level 3. Next load reversal (3 - 4) may again propagate the fatigue crack and a new compressive minimum stress field can be created at the load level 5. However, it is important to understand that the actual fatigue crack growth increment due to load reversal (3 - 4) is less than it could be under the applied range of stress intensity factor due to the presence of the compressive (residual) stresses left behind the crack tip.

Based on the experimental observations of fatigue crack behaviour under variable amplitude loading it was concluded that the residual stress intensity factor for the current cycle is not only a function of the residual stress field ahead of the crack tip induced by the last cycle, but can also depend on the residual stress fields produced by the preceding cycles of the loading history. Therefore this effect has to be taken into account while calculating fatigue crack growth increment induced by the current load cycle. It is also necessary to define when the effect of the previous cycle (or cycles) can be neglected due to the fact that the crack tip has propagated out of its zone of influence.

Based on the available experimental data [11] a new methodology for obtaining the residual stress intensity factor representing the current load cycle has been proposed. Four rules have been formulated that allow for the determination of the residual stress intensity factor required for subsequent estimation of the instantaneous fatigue crack growth rate and crack increments. According to the proposed methodology all crack tip stress distributions induced by previous cycles have to be combined into one resultant minimum stress field influencing current fatigue crack growth rate.

- First, only the compressive part of the crack tip stress field corresponding to the minimum load affects the fatigue crack growth rate
- Secondly, if the compressive part of the minimum stress distribution induced by the current loading cycle is completely inside of

the previous resultant minimum stress field, the material does not “feel” it and the current minimum stress distribution should be neglected.

- Thirdly, if the compressive part of the minimum stress distribution of the current loading cycle is fully or partly outside of the previous resultant minimum stress field they should be combined. 4).

- The fourth rule states that, each minimum stress distribution should be included into the resultant one only when the crack tip is inside of its compressive stress zone. In other words, when the crack tip has propagated across the entire compressive stress zone of given minimum stress field it should be excluded from the subsequent fatigue crack growth analysis.

All four rules are schematically explained in Fig. 3.

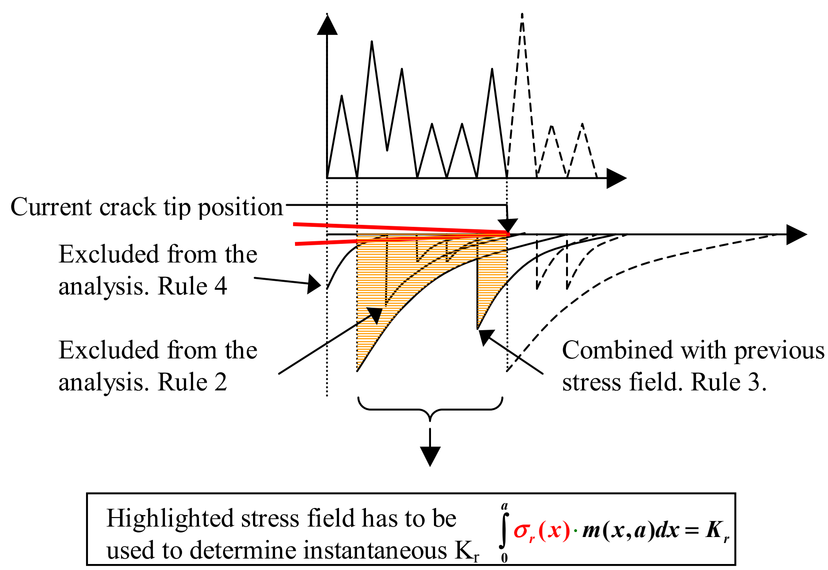

Figure 3: Compressive residual stress fields induced by subsequent load cycles of a spectrum loading

4. Fatigue crack growth under spectrum loading – Predictions vs. Experiments

In order to verify whether the model is capable of predicting fatigue crack growth under variable amplitude loading spectra the calculated fatigue crack growth results were compared with experimental data. The comparison between them shows the level of accuracy of the proposed UniGrow fatigue crack growth model and its ability to predict the fatigue crack life for a variety of materials and loading spectra.

4.1 Step-wise loading (Al 2024-T3 alloy)

The step-wise loading spectra were used in order to determine the load-interaction effect. Central through crack specimens made of aluminium alloy 2024-T3 were used in order to generate fatigue crack growth data. Experimental fatigue crack growth data for constant and variable amplitude loading was taken from the paper by A. Ray [12]. The predicted and experimental crack length vs. number of cycles (a-N) data sets are shown in Fig. 4-5 for two different loading spectra (applied stress is given in ksi, 1 ksi = 6.894759 MPa).

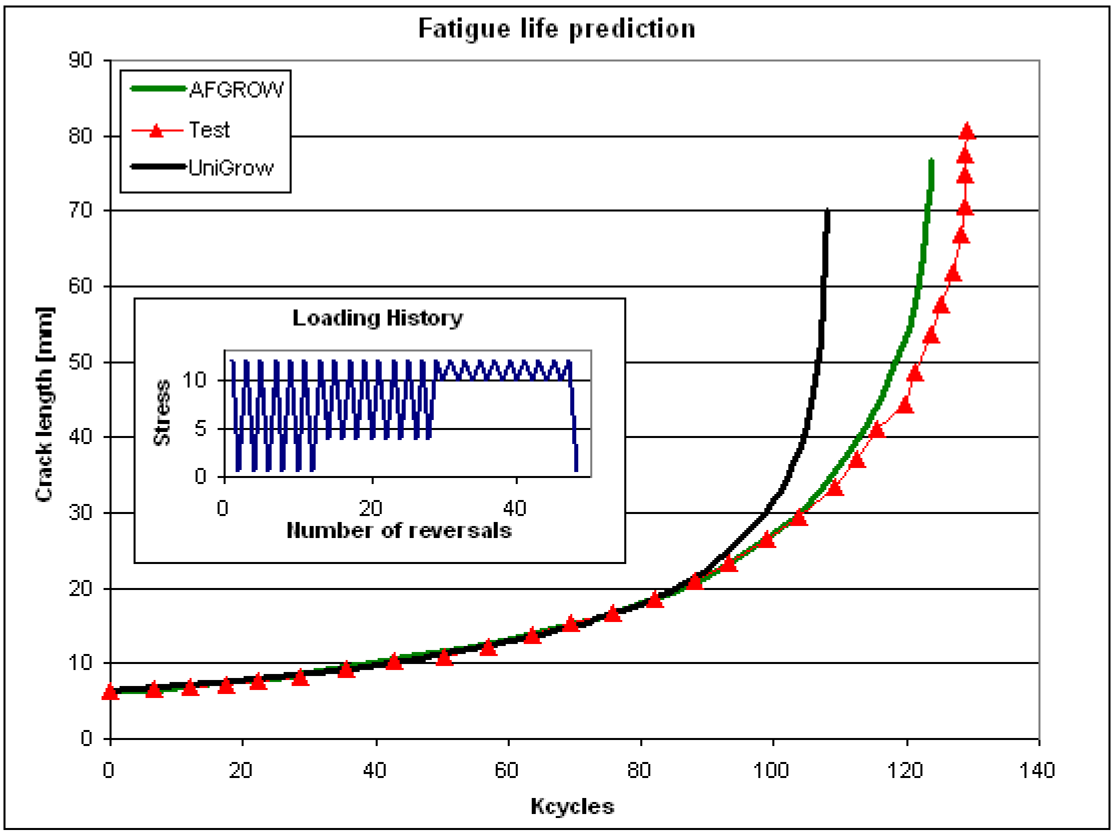

Figure 4: Fatigue life prediction; Al 2024-T3, Constant S_{max}, decreasing ΔS

For comparison the AFGROW fatigue life prediction for the same loading spectrum is also presented in Fig. 4-5. As it can be noted the proposed fatigue crack growth model returns a fatigue life similar to experimental one.

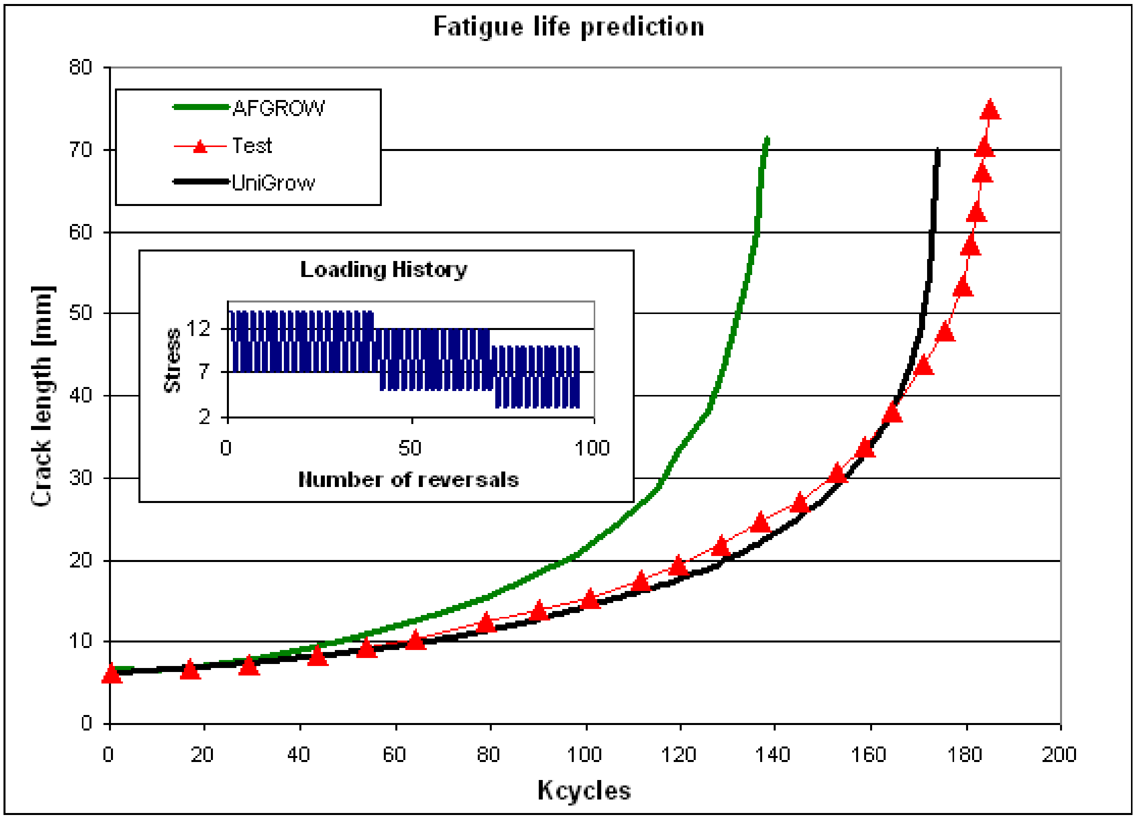

Figure 5: Fatigue life prediction; Al 2024-T3, decreasing S_{max}, constant ΔS

4.2 Variable amplitude loading spectra (Al 7010 – T73651 material)

The fatigue crack growth analysis was carried out for the real helicopter material and loading spectrum. So-called "The Helicopter Damage Tolerance Round-Robin Challenge" data provided by Dr. P E Irving was used in the analysis.
The selected geometry (used successfully in the "ROBUST" crack growth UK research work), is a flanged plate with a central lightening hole. This is representative of many features found in a helicopter lift frame. Crack growth was initiated from a corner defect, a=b=2.0 mm on the inner edge of the hole (Fig. 6).

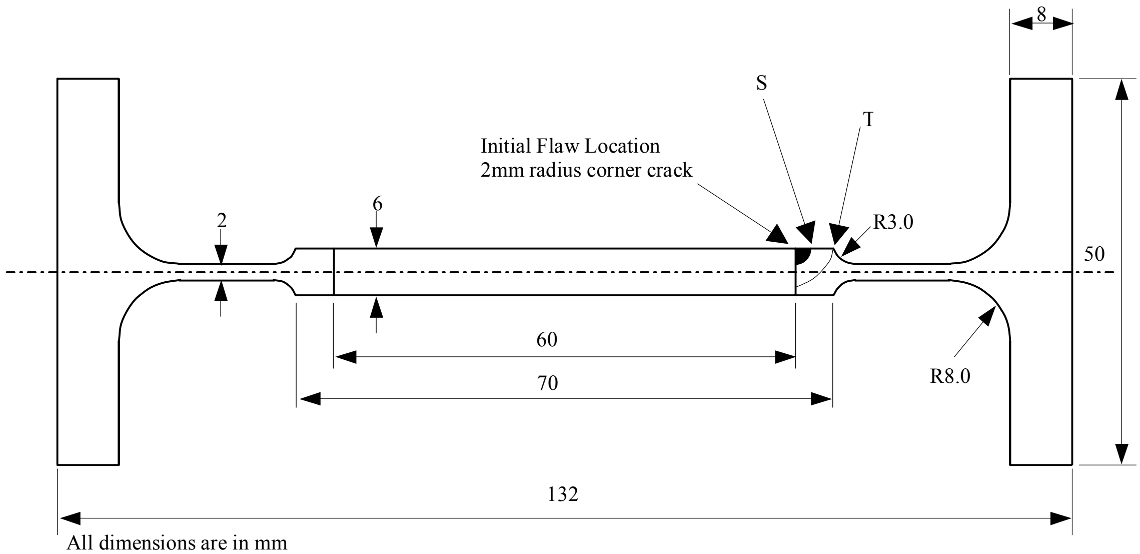

Figure 6: Corner crack in the flanged plate with a central lightening hole

The corresponding stress intensity factor solution derived during the UK Robust programme was provided to all participants of Round-Robin Challenge.

Al 7010 T73651 aluminium alloy was selected. Comprehensive constant amplitude fatigue crack growth rates, covering threshold to near failure, at four stress ratios between 0.9 and 0.1 were supplied and used as input data

The recently developed standard helicopter loading spectrum (ASTERIX) was used in the fatigue crack growth analysis. ASTERIX has been derived from real strain data measured on a helicopter lift frame. The spectrum is made from 140 sorties representing 190.5 hours of flight. The sequence of manoeuvres in each sortie is fixed. The total number of cycles in its complete form is 3.67×10^5 cycles.

The fatigue crack life predicted based on the UniGrow fatigue crack growth model and the experimental measurements are shown in Fig. 7. Both the final fatigue life and the fatigue crack growth ‘a vs. N’ profiles are quantitatively and qualitatively well simulated by the proposed model. The predicted crack length nicely matches the experimental data in the interval from 2 to 7 mm where the crack remains of a corner type. Some deviation from the experimental data was observed when the crack propagated through the thick part of the specimen and became of an edge type. However, that deviation can probably come from the fact that the provided stress intensity factor solution is not absolutely accurate in the region where crack changes from corner to edge type.

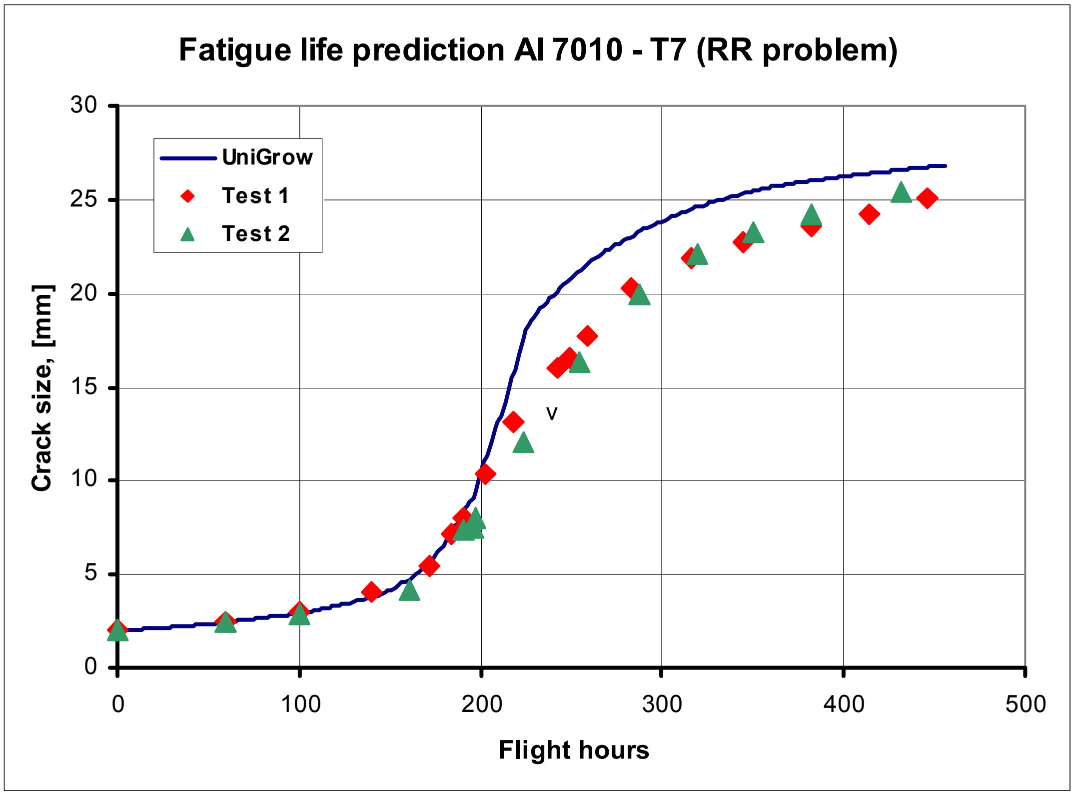

Figure 7: Fatigue life prediction for Al 7010 T73651 material, corner crack emanating from the hole

5. Conclusions

The UniGrow fatigue crack growth model based on the analysis of the elastic-plastic stress/strain behaviour in the crack tip region has been modified in order to make it applicable to a wide variety of variable amplitude loading spectra. The load-interaction effect occurring in the case of variable amplitude loading was simulated by accounting for residual compressive stresses produced by the reverse plastic deformation in the crack tip region.

It was concluded that the instantaneous fatigue crack growth rate depends not only on the residual stresses produced by the previous loading cycle, but depend on all stress fields generated by the previous loading history. Based on this observation, several rules have been established in order to combine residual stresses fields generated by all preceding loading cycles into one resultant minimum stress field which affects the current fatigue crack growth rate.

Experimental verification of the proposed methodology was carried out using two materials (Al 2024-T3 alloy and Al 7010 T73651 alloy) and two different types of loading spectra (Step-wise and General variable amplitude). The comparison between experimental and predicted data sets clearly shows the ability of the UniGrow fatigue crack growth model to simulate the load – interaction effects.

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