A Criterion Study For Non-Singular Stress Concentrations With Size Effect

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ABSTRACT

In engineering applications, the non-singular stress concentrations can generally not be avoided. They often play an important role in structure designs. The simplest engineering strength criteria are in general not appropriate due to important size effect induced by stress gradients. In this paper, we present first a simple experimentation, which consists of a central-holed plate under uniaxial tensile loading, showing important size effect. Second, numerous criteria, including commonly used engineering criteria, crack initiation criteria based on the limit fracture mechanics, or cohesive criteria, were adapted to fit the experimental results. We found that most of these criteria, developed for cracks (-1/2 singularity) or for notches (weak singularities), are not suitable for the case of non-singular stress concentrations. On the other hand, the cohesive models show an excellent agreement with the experimental data. We believe that these models constitute physically reasonable criteria and can be used in both cases of singular and non-singular stress concentrations.

1 INTRODUCTION

Studies on criteria of material fractures are of important feature in applied mechanics. When stress concentrations exist in a structure, its strength is often different from that measured using a uniformly loaded specimen. Much attention has been focused on the cases when the stress concentrations present some singularities, such as those near a crack or a notch tip. However, in the case when the stress concentrations are not singular, much less studies have been dedicated. This is perhaps because of two raisons: first, the size effect of non-singular stresses have not been receiving major attention so far and second, one may believe that they are less dangerous than the singular ones.

The size effect study in solids mechanics is a newly developed domain with considerable importance in engineering applications (Bazant [1]). Not only this size effect is microscopically effective, but also it can occur in a macroscopic scale when stress concentrations exist. In this paper, we will demonstrate, from some simple experimental studies, that most of commonly used engineering criteria are not accurate enough due to the stress gradient and the size effect induced.

In order to give an objective evaluation of the existing strength criteria, we adapted numerous criteria, initially developed for different specific structural configurations, into the assessment of non-singular stress concentrations. These criteria comprise the commonly used engineering criteria such as the maximum stress criterion; crack onset criteria based on the Finite Linear Fracture Mechanics (FLFM), and cohesive force models. Special efforts have been made in developing criteria on the base of the cohesive models, which seems physically more reasonable comparing with other criteria. The specimen strengths predicted by applying these criteria were compared with test data. Finally, some discussions in this topic were advanced.

2 EXPERIMENTAL RESULTS

Central-holed dog-bone specimens were made from PMMA plate. The mechanical characteristics of used material are as follows: the elastic modulus E = 3000 Mpa, the Poisson ratio $\nu = 0.36$; the ultimate tensile stress σ_0=72 Mpa; the critical release energy rate G=290 J/m. The section of the specimens is of 10×30mm². Central holes are drilled with different diameters, namely, $d = 0.6$, 1,2, 2 and 3 mm. Specimens without holes were also prepared for comparison. These specimens were subjected to a uniaxial tension with a loading rate v = 10mm/s until the failure. Figure 1 plots the test results, which represent the remote broken stresses as function of the hole diameter.

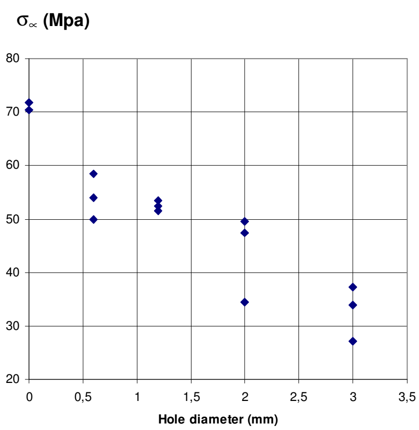

Figure 1: Broken stresses of PMMA plates as function of the hole diameter

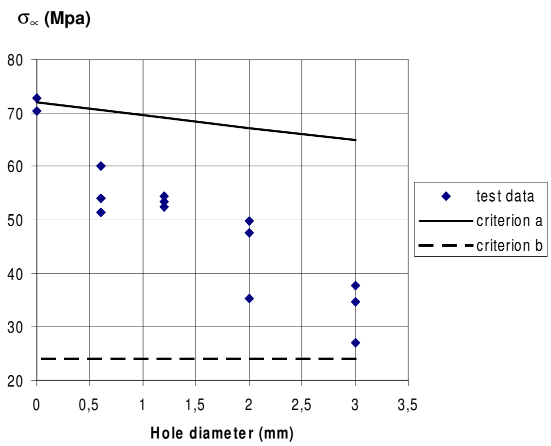

Figure 2: Broken stresses predicted by commonly used engineering criteria, comparison with test results

From Figure 1, important size effect can be observed within the chosen hole size range. It is seen that the strength of the central-holed specimens depends strongly on the hole diameter.

The stress distribution near a circular hole in an infinite plate under uniform remote uni-axial loading is well known. The tensile stress at the ligaments beyond the hole boundary is:

$$\sigma_{yy} = \frac{\sigma_\infty}{2}\left(2 + \frac{a^2}{r^2} + 3\frac{a^4}{r^4}\right) \tag{1}$$

where a is the hole radius, and σ_∞ is the uniform-distributed remote tensile stress. It is seen that the maximum tensile stresses occur at the hole boundary with $\sigma_{yy}(r = a) = 3\sigma_\infty$, independently of the hole diameter. In our tests, since the hole sizes are much smaller comparing with the characteristic specimen dimensions, we can consider that the theoretical results will accurately represent the stress distribution. Nevertheless we have carried out refined finite element analysis, which confirm this argument.

3 COMMONLY USED ENGINEERING CRITERIA

Here only the two mostly used engineering criteria are presented.

a: Net tensile stress criterion

In many engineering practice, the net section stress is calculated without considering any stress concentration. For the present central-holed specimens under pure tension, the net section stress σ_{net} is taken as the effective stress, namely,

$$\sigma_{net} = \frac{P}{S_{net}} \qquad (2)$$

where P is the resulting tensile force and S_{net} is the minimum net section of the specimen. This net stress is then, after applying a security factor k, compared with the ultimate stress σ_c.

b: Maximum tensile stress criterion

For a non-singular stress concentration, the maximum tensile stresses can be computed with accuracy by analytical or numerical analysis. This criterion states that the structure failure occurs if the maximum tensile stress reaches its critical value σ_c, measured with specimens under uniform loading.

The comparison between the broken stresses predicted by these two criteria and the test results is shown in Figure 2. It is shown that the net stress criterion predicts much higher failure strength in most of the cases. On the other hand, the maximum stress criterion is obviously too conservative.

4 CRITERIA BASED ON THE FINITE FRACTURE MECHANICS

The Finite Fracture Mechanics was founded on the basis of incremental Griffith criterion under the following form:

$$G = -\frac{\Delta\Pi}{\Delta S} > G_c \qquad (3)$$

where $\Delta\Pi$ is the variation of the structure potential energy when a crack increment surface ΔS occurs, G and G_c are respectively the energy release rate and its critical fracture value. This criterion differs from the initial Griffith criterion by its incremental (but not derivative) form in calculating G. This class of criteria enables us to consider problems of weak singularities due to notches, interfaces and so on. The main inconvenient is inherent in the fact that the incremental value ΔS is difficult to determine. Different schemes have been proposed. In the following, we will adapt these criteria to the present hole problem.

c: Modified McClintock Criteria

McClintoch [2] first proposed a strain criterion for crack growth in ductile materials. His criterion states that crack propagation occurs when the normal strain at a characteristic distance ρ_c measured from crack tip reaches its critical value. This distance is often related to the plastic zone size. Similar stress criterion was proposed by Ritchie et al. [3] which predicts the crack growth once the maximum opening stress $\sigma_{\theta\theta}$ at a characteristic distance ρ_c reaches a critical value σ_c, namely

$$\max \sigma_{\theta\theta}(r = \rho_c) \geq \sigma_c \qquad (4)$$

In the application of this criterion to the present problem, no physical argument is advanced for the determination of ρ_c. If our purpose is just to fit the experimental results, we can choose a suitable value of ρ_c such that the prediction curve agrees with the test data. Thus we found $\rho_c \approx 0.25$mm. The predicted fracture stresses for different hole sizes are plotted in Figure 3.

d: Incremental Griffith Criterion

The application of the incremental Griffith Criterion (3) is possible to the present case if one opens along the ligament a virtual crack issued from the hole boundary. In fact, the energy release rate at the opened crack tip can be calculated from the stress intensity factor, namely:

$$G = \frac{\left(1-\nu^2\right)K_I^2}{E} \qquad (5)$$

E is Young's modulus, ν is Poisson's ratio. Here only the mode I stress intensity factor K_I is concerned.

It should be pointed out that the use of this criterion needs to determine the virtual crack length ρ_c. The evaluation of this distance is questionable. Seweryn and Lukaszewicz [4] assumed that the two criteria, the maximum tensile stress criterion $\sigma_{\max} = \sigma_c$ (for the no-cracked specimen) and the energy release rate criterion $G = G_c$ (for a cracked specimen) are equivalent at this distance. These assumptions lead to write the following:

$$K_{Ic} = 1.122\sqrt{\pi\rho_c}\,\sigma_c \tag{6}$$

hence

$$\rho_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{1.122\sigma_c}\right)^2 \tag{7}$$

For the material used and in plane strain, we obtained ρ_c = 0.04875mm. To predict the broken stress, either maximum stress criterion or critical energy release rate criterion can be used. The first criterion is described by (4) with ρ_c = 0.04875mm. The predicted broken stresses are plotted in Figure 3. It is to note that if the critical energy release rate criterion is used, different broken stresses can be obtained. Therefore, this class of criteria is not self-consistent.

e: Non-local criterion

Different non-local schemes can be developed in assessment of size effect. Hereafter we adopt the non-local model developed by Moroz and Seweryn [5]. Other non-local models provide similar results. This criterion assumes that initiation of cracking occurs when the mean value of stress fracture factor *R* over a segment ρ_c reaches its critical value, thus in the present case

$$R = \frac{1}{\rho_c}\int_a^{a+\rho_c}\frac{\sigma(r)}{\sigma_c}dr = 1 \tag{8}$$

Seweryn and Lukaszewicz [4] suggested taking the Irwin characteristic length as ρ_c, namely:

$$\rho_c = \frac{1}{2\pi}\left(\frac{2K_{Ic}}{\sigma_c}\right)^2 \tag{9}$$

For the present material, ρ_c=0.1227mm. From (8), the broken stresses are readily be calculated and represented in Figure 3. Here again, this non-local model is not self-consistent.

f: Leguillon's criterion

The fracture criterion proposed by Leguillon [6] is self-consistent. This criterion states that fracture occurs if both the maximum tensile stress, at a point in the non-cracked configuration, and the energy release rate, at the virtual crack tip situated at the same point in the cracked configuration, reach simultaneously their critical values. The stress intensity factors for radial cracks of equal length emanating from a circular hole subjected to uniaxial tension were found by Bowie [7]. The mode I stress intensity factor for a crack length ρ_c is:

$$K_I = F\left(\frac{a+\rho_c}{a}\right)\sqrt{\pi(a+\rho_c)}\,\sigma_\infty \tag{10}$$

The function *F* is tabulated in Sih's handbook [8]. Then the remote broken stress is calculated by:

$$\sigma_\infty = \frac{K_{Ic}}{F\sqrt{\pi(a+\rho_c)}} \tag{11}$$

This critical load must equal to that calculated from the maximum tensile stress at the same point ρ_c. According to (1), (4) and (11), we can write:

$$2\sigma_c \Big/ \left(2 + \left(\frac{a}{a+\rho_c}\right)^2 + 3\left(\frac{a}{a+\rho_c}\right)^4\right) = \frac{K_{Ic}}{F\sqrt{\pi(a+\rho_c)}} \tag{12}$$

This relationship allows the determination of ρ_c then the fracture load for each hole size. The predicted broken stresses are plotted in Figure 3 for various hole diameters.

5 CRITERIA BASED ON THE COHESSIVE MODELS

Barenblatt [9] first proposed the idea of the cohesive zones at crack ends in order to assess strength of cracked materials. This idea can also be used to assess the material strength under non-singular stress concentrations. A Dugdale model and a Barenblatt model were developed but only the Dugdale model is presented here.

g: The Dugdale model

Dugdale [10] proposed a practical modal to study the crack strength in ideally plastic sheet. The application of the Dugdale model in the present problem can be described as follows. The uniformly distributed cohesive forces are taken to be the ultimate stress of the material. The length of the cohesive zone can be determined, according to the Dugdale concept, by considering the stress non-singularity at the ends of this zone. The cohesive surfaces separate when the energy release rate reaches the critical value of the material.

We develop in this work a rigorous Dugdale model to assess the strength of the central-holed specimens. For this purpose we regard the cohesive zone as a row of continually distributed dislocations. The stress field of an edge dislocation interacted with a hole was established by Dundurs and Mura [11]. By considering the force equilibrium, we can write:

$$\sigma_{dislocation} + \sigma_c + \sigma_\infty = 0 \tag{13}$$

where $\sigma_{dislocation}$ is the stress due to the continually distributed dislocations and has the following expression:

$$\sigma_{dislocation} = \int_a^{a+\rho_c} \sigma_1(b_y, s)ds + \int_{-a}^{-a-\rho_c} \sigma_2(b_y, s)ds \tag{14}$$

where σ_1 and σ_2 are respectively the tensile stress components at the ligament due to dislocations at the right and left sides in interaction with the circular hole, b_y is the Burger's vector of a dislocation. The explicit expression of σ_1 and σ_2 were found in Dundurs and Mura [11].

The stress intensity factor at the end of the cohesive zone must vanish; this condition permits to determine the parameter ρ_c. Then the release energy rate *G* can be evaluated by the formula

$$G = \delta\sigma_0 \tag{15}$$

where δ is the opening displacement of the virtual crack. Comparing the calculated G with its critical value of the material, the remote broken load can be evaluated. The results issued from this analysis are plotted in Figure 4.

6 DISCUSSIONS AND CONCLUSIONS

From the analyses above-mentioned, the following observations can be made:

1. Important size effects can be observed by the present tensile test on central-holed specimens, which can be used to evaluate different strength criteria;
2. For sufficiently small holes, the commonly used engineering criteria cannot provide suitable broken stress predictions. The net stress criterion predicts higher broken stresses because the stress concentration is not taken into account. On the other hand, the maximum stress criterion gives a too conservative prediction especially for small holes. When comparing this maximum tensile stress with the material strength, the hole size effect is not taken into consideration.
3. The criteria based on the finite fracture mechanics are capable to reproduce the tendency of the structure strength highlighted by test results. The principal inconvenient of these criteria is the difficulty to give an accurate estimation of the incremental length ρ_c for different stress

concentrations. Among these criteria, the Leguillon criterion is a self-consistent criterion with a physically reasonable estimation of ρ_c. Even though the accuracy is not yet satisfactory in the present case, it merits deeper investigations by its lucidity and coherence.

4. The cohesive models such like the Dugdale model present a remarkable advantage in assessment of the structure strength with non-singular stress concentrations. Since the concept of cohesive forces represents the atomic links in materials, this class of criteria is physically reasonable for most of the engineering materials. Moreover, accurate prediction of broken stresses for the present structure can be obtained. Since the cohesive models have been used successfully to study the singular stress concentrations, we believe that these models provide a unified method to assess structure strengths with both singular or non-singular stress concentrations.

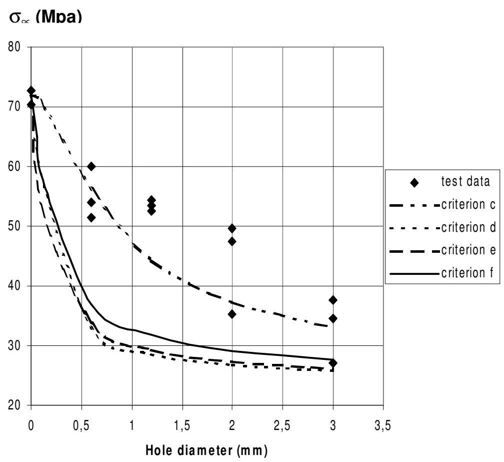

Figure 3: Broken stresses predicted by FFM criteria, comparison with test results

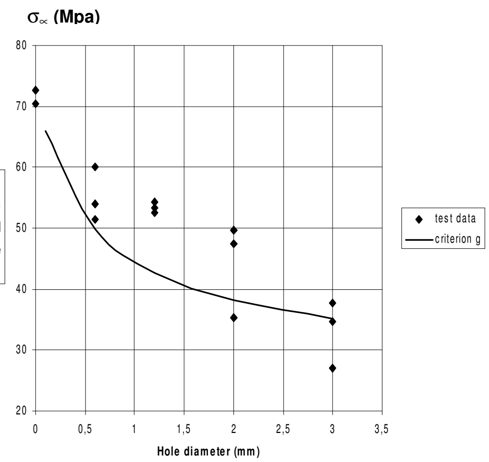

Figure 4: Broken stresses predicted by the Dugdale model, comparison with test results

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