TOUGHNESS MEASUREMENT IN BRITTLE MATERIALS FROM DIGITAL IMAGE CORRELATION

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ABSTRACT

From the analysis of digital images taken at different stages of loading, the propagation of a crack is studied in a silicon carbide ceramic. A sandwiched-notched SiC beam is subjected to three-point flexure. Digital images of the beam side are taken by using a long distance microscope allowing for a pixel size less than 2 μm at different stages of loading when the crack grows. A novel measurement technique of displacement fields based on intercorrelation is introduced. It consists in the identification of the optimum correlation between pairs of images based on a reduced number of 2D displacement fields, namely a rigid body motion, a uniform strain and the two singular mode I and II crack displacements. The amplitude of each of these different components is obtained once a crack geometry has been introduced. The same algorithm allows for a quality estimate of the residual image difference. Based upon the latter, the crack tip position is optimized. The amplitudes of the two crack fields give a direct access to the Crack Opening Displacement (COD) in both modes. Knowing the elastic properties of SiC, the toughness is extracted from the previous analysis. The interest of the new method presented here lies in the *direct* identification of kinematically admissible fields, and this with a resolution in the amplitude of the displacement well below $1/100^{\text{th}}$ pixel, *i.e.* about 10 nm in our case with no specific surface preparation, plain visible light and common digital image acquisition techniques.

1 INTRODUCTION

Toughness measurement is a necessity to use brittle materials in all kinds of mechanically demanding applications. However, for brittle materials, this measurement is experimentally a difficult task. Among other difficulties, the design of a test where crack propagation is controlled often requires a numerical modeling to estimate the stress intensity factor, and the preparation of a sharp crack tip is an experimental challenge. These remarks motivate the development of techniques that allow for the identification of the stress intensity factor (provided the elastic properties are known) or the crack opening displacement. Digital image correlation techniques (Sutton *et al.*, 1983), inasmuch as they offer full-field measurements, are clearly very appealing in this context. However, for ceramic materials, focusing on the close neighborhood of the crack tip to capture the singular components of the field, requires the measurements of submicrometric displacement amplitudes that appear at first sight as precluding the use of plain optics since the light wavelength appears as a natural limitation of direct displacement measurements.
The novel technique, which is introduced herein, consists in having a direct estimate of the displacement field projected onto a set of a few reference *mechanical* fields, using intercorrelation maximization. The use of full-field measurements allows one to break through the previously mentioned wavelength limitation, and to achieve about ten-nanometer resolution. This approach makes toughness measurements very versatile, allowing for complex test geometry, and yet direct local measurement of crack tip characteristics.

2 EXPERIMENTAL PROCEDURE

A detailed presentation of the material and test procedure can be found in (Forquin *et al.,* 2004). Only the most salient points are recalled. The chosen test is a sandwiched beam (Nose and Fujii, 1988; Marshall *et al.*, 1991; Pancheri *et al.*, 1998) whose core is the silicon carbide sample to be studied, and the steel facings provide the necessary rigidity for the crack propagation to be controlled in a three-point flexural test as shown in Fig. 1. A notch is machined along the tensile loaded face of the SiC sample, and a preliminary loading is applied so that a crack is initiated from the notch. In this presentation, the first stage is analyzed. Once a sharp crack is formed, the steel beams are removed and a classical three-point flexural test is subsequently carried out on a *cracked* sample.

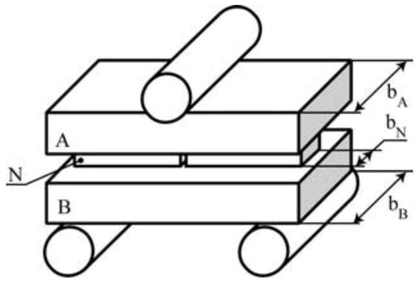

Figure 1: The ceramic-notched sample (N) is sandwiched between two compliant beams (A and B) and subjected to three-point flexure.

Digital images of the sample face are taken with a long-distance microscope (Questar QM100) that allows for a high resolution (*e.g.*, 1.85 µm per pixel) in comfortable conditions (*i.e.*, focal distance about 20 cm) with respect to the testing machine. A brightfield illumination is used. No specific surface preparation is performed since the natural surface roughness provides the necessary speckle inhomogeneity of the images. An example of two stages of cracking is shown in Fig. 2. Bare eye examination of the picture does not allow for the detection of the crack front.

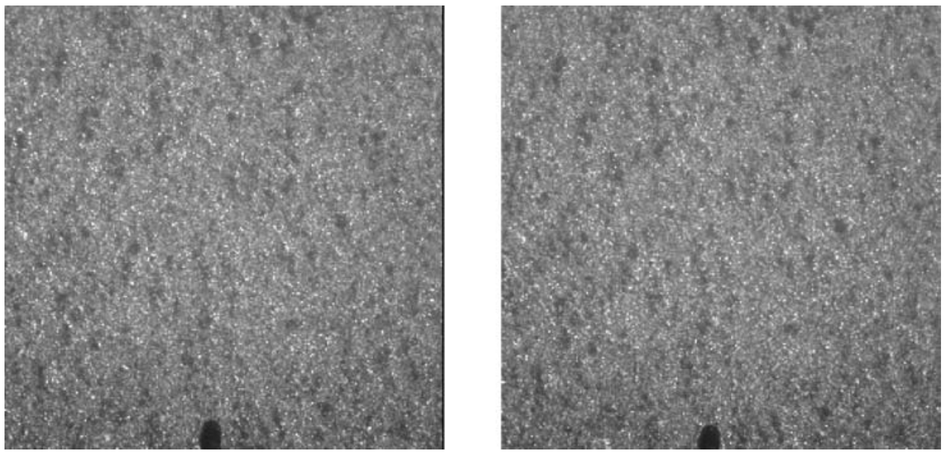

Figure 2: Two images (1008 × 1016 pixels, 1 pixel = 1.85 µm) of the ceramic sample face prior to cracking (left) and after propagation (right). The initial notch is visible at the center of the lower edge of the images.

In (Forquin *et al.*, 2004), a conventional digital image correlation technique is used to have access to the displacement field. In contrast to the approach followed here, the displacement field was analyzed for interrogation windows extracted from the entire image of size 64 × 64 pixels, and where a translation is looked for (Hild *et al.*, 2002). This approach was already able to resolve the presence of a crack and to measure the COD profile.

3 INTERCORRELATION TECHNIQUE

The reference image is represented as a graylevel function f of the spatial coordinates $\mathbf{x}$, while the "deformed" image is called $g(\mathbf{x})$. It is assumed that no image distortion other than that due to the elastic (and crack) displacement takes place so that f and g are related by

$$g(\mathbf{x}) = f(\mathbf{x}+\mathbf{u}(\mathbf{x})).$$

The displacement field, $\mathbf{u}(\mathbf{x})$, is moreover assumed to be well captured by a reduced set of relevant displacement fields, $\mathbf{\Psi}_i(\mathbf{x})$. In the present case, 8 fields are considered, namely, 2 for in-plane translations, 1 for in-plane rotation, 3 for homogeneous in-plane strains, and 2 crack fields (mode I and II). The amplitudes a_i such that

$$\mathbf{u}(\mathbf{x}) = a_i\, \mathbf{\Psi}_i(\mathbf{x})$$

are looked for. To identify those fields, the general principles discussed by Wagne *et al.* (2002) are followed so that the unknown amplitudes are solutions of the linear system

$$M_{ij}\, a_j = b_i$$

where

$$M_{ij} = \iint [\nabla f(\mathbf{x}) \otimes \nabla f(\mathbf{x})] \cdot\cdot [\mathbf{\Psi}_i(\mathbf{x}) \otimes \mathbf{\Psi}_j(\mathbf{x})]\, d\mathbf{x}$$

$$\boldsymbol{b}_i = \iint [f(\mathbf{x}) - g(\mathbf{x})] \nabla f(\mathbf{x}) \cdot \mathbf{\Psi}_i(\mathbf{x})\, d\mathbf{x}.$$

This approach results from a direct minimization of a first order expansion of the functional

$$\varepsilon(\mathbf{u}) = \iint [\, f(\mathbf{x}+\mathbf{u}(\mathbf{x})) - g(\mathbf{x})]^2\, d\mathbf{x}.$$

As a side result, a direct estimate of the global quality of the determination based on the minimized objective function ε is obtained. Moreover, since the latter is a spatial integral, the spatial distribution of the residues that contribute to the remaining error can be obtained as well.

The same scheme can also be used iteratively after a correction of the image based on the translation part of the displacement field, which can easily be taken into account through a subpixel correction based on Fourier interpolation of the image. Last, let us also note that the two images, f and g, can be smoothed by using a Fourier low-pass filtering to achieve a gross determination of the displacement field (based on fewer components of the displacement) in the first steps of the procedure. Once this displacement has been corrected for, finer details are restored and more degrees of freedom are introduced, until final convergence.

Throughout this procedure, the position of the crack has to be postulated. Since a quality estimate can easily be computed, it may be used to determine the relevance of the crack field displacement in the images difference. As the guessed crack tip comes closer to its actual position, the error estimator ε is expected to decrease. Such a procedure is used to optimize the crack tip position.

4 PERFORMANCE OF THE CORRELATION ALGORITHM

To evaluate the performance of the proposed procedure, the left (8-bit) picture of Fig. 1 is first artificially moved by using the shift/modulation property associated to Fourier transforms. For interrogation windows of size 256×256 pixels, a maximum difference between prescribed and measured displacements of $8/1000^{th}$ pixel is obtained (average less than $3/1000^{th}$ pixel). A classical correlation algorithm applied to the same picture would yield a maximum difference of $2/100^{th}$ pixel and an average of the order of $1/100^{th}$ pixel, hence the uncertainty is reduced by a factor 3 by using the proposed scheme.

Second, the same picture is artificially deformed by performing a linear interpolation to compute each graylevel. For strain levels equal to $\pm 10^{-3}$, a relative error less than 2% is found for all examined cases (i.e., strain in one direction alone, and both directions simultaneously) with a *single* interrogation window. For a strain equal to 5×10^{-4}, a relative error of 3% is obtained and for a strain equal to 2.5×10^{-4}, a relative error less than 10% is achieved. These values are barely achieved by using conventional techniques applied to the *whole* 1 Mpixel picture by using *at least* 49 interrogation windows.

5 RESULTS

Figure 3 shows the two components of the displacement field obtained by utilizing the above procedure. A mean vertical translation of the order of 20 µm (*i.e.*, 11 pixels) between the two images is resolved in two iterations. The mean longitudinal strain over the entire image is estimated to be of order 4×10^{-4}. Finally the mode I COD is obtained with a maximum displacement discontinuity of the order of 0.25 pixel on the edge of the interrogation window. Using the elastic properties of SiC, an estimate of the toughness equal to 2.8±0.2 MPa $m^{1/2}$ is obtained, quite consistent with known values of the order of 2.85±0.15 MPa $m^{1/2}$ (Forquin *et al.*, 2004).

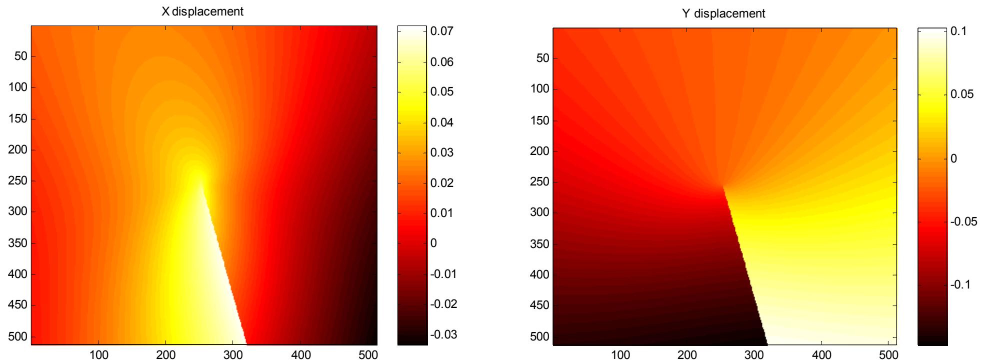

Figure 3: Estimated displacement components along the vertical (left) and horizontal (right) direction for a 512×512-pixel interrogation window.

As a side result, we also observe that in spite of the lateral crack propagation, the mode II amplitude is very small. For the optimum crack tip location, the mode II amplitude is of order 0.014 MPa $m^{1/2}$, i.e. two orders of magnitude smaller than the mode I stress intensity factor. In terms of crack tip location, Fig. 4 shows the objective function for a uniform sampling over a restricted zone where the crack tip is positioned. It can be seen that a good accuracy is achieved in the direction perpendicular to the crack orientation (about 15 pixels or 30 µm). In contrast, along the crack direction, the crack tip position is more difficult to position precisely (uncertainty about 30 pixels or 60 µm).

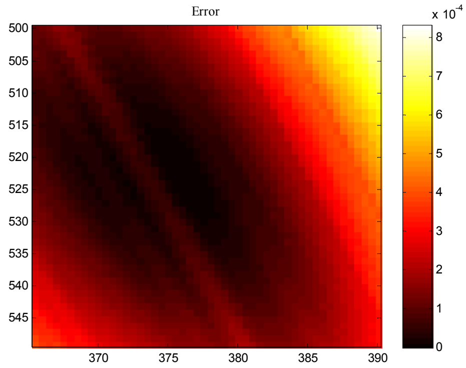

Figure 4: Residual error (above the minimum) for different crack tip positions. Note the scale in pixels, and the highly anisotropic character of this error function.

These uncertainties have a significant impact on the estimates of the stress intensity factors (of order 0.2 and 0.1 MPa $m^{1/2}$ for mode I and II, respectively), and hence on the toughness (Fig. 5).

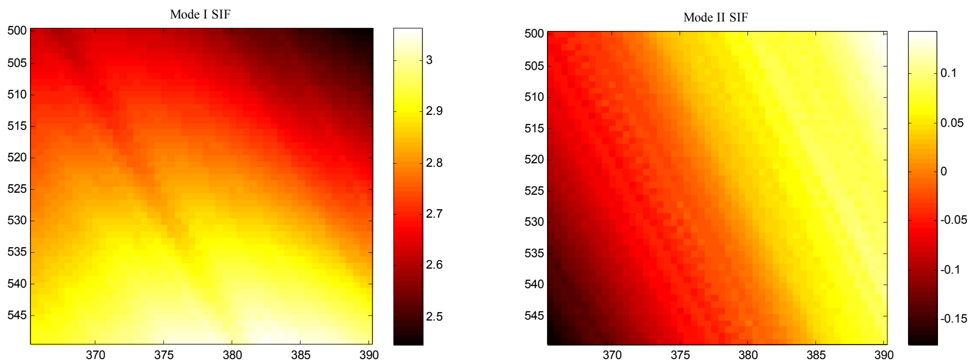

Figure 5: Mode I (left) and Mode II (right) stress intensity factors (in MPa $m^{1/2}$) for different crack tip positions.

6 SUMMARY

A novel image correlation algorithm is introduced to measure displacement fields over a limited set of test functions. This approach is applied to the problem of crack detection and toughness measurement on a silicon carbide ceramic where the crack discontinuity is a fraction of the pixel size. In spite of this small amplitude, the algorithm gives access to a precise determination of the toughness.

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