FRACTURE STATISTICS OF CERAMICS – A SHORT OVERVIEW

R. Danzer[1,2], T. Lube[1], P. Supancic[1,2]
[1] Institut für Struktur- und Funktionskeramik, Montanuniversität Leoben, A-8700 Leoben, Austria
[2] Materials Center Leoben, A-8700 Leoben, Austria

ABSTRACT

For more then 60 years Weibull statistics is widely used to describe the strength distribution of ceramics. It is still the backbone in the mechanical design process of ceramic components. Weibull statistics is based on weakest link theory and on the assumption that fracture in ceramics starts at sparsely distributed flaws. But it can be shown that the Weibull statistics is not generally valid: it is a special case out of a class of more general distribution functions occurring only for a special set of material conditions and in a limited parameter space.

In this paper examples for the deviation of fracture statistics from Weibull statistics are discussed. They may occur in materials having bi- or multi- modal flaw distributions, inhomogeneous material properties, an increasing crack resistance curve, a too high density of dangerous flaws or in loading situations with steep stress gradients. Of course these examples are not complete.

This analysis is made by theoretical modelling. It is an intrinsic consequence of the statistical behaviour of fracture that experimental results show large measuring uncertainties if they are not based on very large samples (containing several thousand specimens). But due to the high machining costs of ceramic specimens the size of samples is - in general – very small (a typical sample contains 30 specimens). Using Monte Carlo simulations it can be shown that in this case a proper distinction between different distribution functions is not possible.

An important consequence of the fracture statistics of brittle materials is the size effect on strength, which also depends on the type of fracture statistics. Therefore a way out of the dilemma mentioned above is to measure the strength of specimens of different size, what allows the determination of the strength distribution with a reasonable experimental effort in a reliable way in a wide range of parameters.

1 INTRODUCTION

In ceramic materials fracture - in general – originates at flaws, which are distributed in the material or on its surfaces; Davidge [1], Munz et al. [2], Wachtman [3]. The size, orientation and size distribution of these flaws are responsible for the scatter of the strength in ceramic materials; Jayatilaka et al. [4]: In a homogeneous stress field (as in a tensile test) the "largest" flaw in the specimen controls the strength (for simplicity but without loss of generality for the following conclusions effects arising from the orientation of flaws are neglected). In general flaws are described to behave as cracks. Then the Griffith/Irwin failure criterion, can be used, Munz et al. [2], and the strength is proportional to the fracture toughness K_{IC} and inverse proportional to the square root of the crack length a: $\sigma_f = K_{Ic} / \sqrt{\pi a}$. The geometric factor is assumed to be one: $Y \approx 1$. The scatter of the strength results from the scatter of the length of the fracture causing flaws, Danzer [5]. In consequence the strength distribution (probability of failure, F, as function of stress, σ) depends on the distribution of the flaw sizes, a. F increases with increasing stress (at a higher stress smaller defects can cause failure and smaller defects occur more frequently) and the mean strength decreases with the volume (it is more likely to find a large flaw in a large volume than in a small volume), Danzer et al. [6].

In principle the fracture statistics can only be measured using a large sample (containing several thousands of specimens or more). For obvious reasons, this can hardly be done. The experimental efforts necessary to find the appropriate strength distribution can be reduced to a large extent, if its mathematical structure is known. Then only some material parameters instead of the entire distribution curve have to be determined. This is the motivation for the theoretical work on fracture statistics.

Weibull was the first who proposed a statistical theory of brittle fracture Weibull [7, 8]. His fundamental assumption was the weakest link hypothesis, i.e. the specimen fails, if its weakest volume element fails. Using some empirical arguments necessary to obtain a simple and good fitting of his experimental data, he derived the so-called Weibull distribution function, which - in its simplest form and for an uniaxial homogenous and tensile stress state and for specimens of the volume, V, - is given by:

$$F(\sigma, V) = 1 - \exp\left[-\frac{V}{V_0}\left(\frac{\sigma}{\sigma_0}\right)^m \right] \quad . \qquad (1)$$

The Weibull modulus m is a measure for the scatter of strength data: the distribution is the wider the smaller m is. σ_0 is a characteristic strength value and V_0 the chosen normalising volume. It should be noted, that in almost all experimental studies on the strength distribution of ceramics, it is claimed that the data are Weibull distributed. But this is not necessarily true, because - for that type of statement - the sample size is too small. This point will be discussed in section 3.

In the past, a significant amount of research was directed towards giving Weibull's theory a more fundamental basis. The paper of Kittl et al. [9] gives a good overview on the former developments. Freudenthal [10] and Danzer [5] showed that, for a homogenous and brittle material and if the flaws do not interact, the probability of failure only depends on the number of critical flaws, N_c, occurring in a specimen of size and shape, S,

$$F_S(\sigma) = 1 - \exp[-N_{c,S}(\sigma)] \quad , \qquad (2)$$

where $N_{c,S}(\sigma)$ denotes the mean number of critical flaws in a large set of specimens (i.e. the value of expectation). In the following, without loss of generality, the specimen size and shape will be replaced by the specimen volume. Jayatilaka et al. [4] demonstrated in their noteworthy paper, that, for a brittle and homogeneous material, the distribution of sizes (and orientations) of the flaws causes the distribution of the strength data and that a Weibull distribution of strength will be observed for flaw populations where the density of flaws decreases monotonically with size. Danzer et al. [6, 11, 12] extended their ideas to flaw populations with any size distribution and to specimens with an inhomogeneous flaw population Danzer et al. [13]. Again it was necessary to assume, that a specimen fails if any single one flaw initiates fracture, and that there is no interaction between flaws (the weakest link hypothesis). The function $N_{c,V}(\sigma)$ is obtained by integrating the local density of destructive flaws

$$n_c(\sigma, \bar{r}) = \int_{a_c(\sigma)}^{\infty} g(a, \bar{r})\,\mathrm{d}\,a \qquad (3)$$

over the specimen's volume: $N_{c,V} = \int n_c \mathrm{d}V$. For simplicity and without loss of generality it has been assumed that the size and orientation of a flaw can be described by the single variable (the effective flaw length, a); Jayatilaka et al. [4]. The frequency distribution density of flaw lengths, $g(a)$, may depend on the position vector, $\bar{r}$. A local fracture criterion (e.g. the Irwin/Griffith criterion) correlates stress amplitude and flaw length: Since the critical flaw length, $a_c(\sigma)$. depends on the magnitude of the applied stress, so do the value of n_c and also of $N_{c,S}$. For homogeneous materials and for flaw populations with relative frequencies that decrease according to a negative power of their (effective) radius, a, a Weibull distribution is expected to occur, Jayatilaka et al. [4].

In this paper it is shown that - as a consequence of the ideas presented above – the Weibull modulus may depend on the stress and the volume, i.e. the strength distribution is not a Weibull distribution, Danzer et al. [6, 11, 13]. Monte Carlo simulations are made to perform "virtual" strength tests and to give a sound basis for the discussion whether such effects can experimentally be observed with a sound significance or not; Danzer et al. [6, 14]. Finally the size effect on strength is discussed.

2 FLAW DISTRIBUTIONS AND FRACTURE CRITERION

2.1 Weibull distribution

If the fracture toughness is independent on the crack advance the simple Griffith/Irwin failure criterion applies for many ceramic materials: $K = \sigma\sqrt{\pi a} \geq K_{Ic}$ (K: stress intensity factor) and the lower integration limit (critical crack size) in eq. 3 is: $a_c \sim \sigma^{-2}$. It is often claimed that the relative frequency of sparsely distributed flaws decreases with their size, what can – in general - approximately be described by an inverse power law: $g(a) \sim a^{-b}$. b is a number and the integral, eq. 3 is $n_c \sim \sigma^{-2(b-1)}$. For a homogeneous tensile stress state it holds: $N_{c,V} \sim V\sigma^{2(b-1)}$. This corresponds exactly to the Weibull distribution, eq. 1, with $m = 2(b-1)$, Jayatilaka et al. [4]. Typical values for m are between 10 and 20 and for b between 6 and 11, which indicates a very steep decrease of relative frequency with increasing crack length. This simple evaluation shows, that the Weibull distribution describes the situation in materials with a special (but often occurring) flaw size distribution and failure criterion.

2.2 Unimodal flaw size distribution

In general there must exist a lower and an upper bond for the existence of flaws: Very small flaws will heal out and large flaws cannot be larger than the specimen size. Therefore there also must exist a most frequent flaw size. The behaviour described in section 2.1 can only occur on the right hand side of this maximum, in a size interval, where the inverse power law is an appropriate approximation of the flaw size distribution, $g(a)$. The upper bond of the flaw size will cause a lower bond for the strength (which can be very small but still exists) and the lower bond of flaw sizes will cause an upper bond for the probability of failure (which depends on the volume, it can be near one but will never reach one). In other words the "Weibull modul" gets stress dependent and the fracture statistics is not longer the Weibull statistics, eq. 1; Danzer et al. [11, 13].

The Weibull distribution is a good approximation for critical flaws being in the size interval (i.e. in the stress interval) were the inverse power law describes $g(a)$. It is still assumed that flaws are sparsely distributed and that the Griffith/Irwin failure criterion is applicable.

2.3 Bi- and multi- modal flaw size distributions.

Flaws are inhomogeneities in the microstructure, which result from the processing of the machining of the specimens. Examples in ceramics are inorganic or organic inclusions, hard or hollow agglomerates, badly sintered grain boundaries, large grains or cracks arising from the machining. It is obvious that each individual flaw population will have its typical size distribution. This has strict consequences on the fracture statistics. Let us discuss them on the example of a two modal population of sparsely distributed volume flaws. The first population (I) should have a flaw size distribution, which causes a Weibull distribution and the second population (II) should be narrow peaked (have a lower bond, a_l, and an upper bond, a_u, opening only a narrow window for possible flaw sizes: $a_l \leq a \leq a_u$); Danzer et al. [6, 14]. The number of critical defects is the sum of the defects of both populations: $N_c = N_{c,I} + N_{c,II}$ and each number can be evaluated separately. For the first population the stress dependency is: $N_{c,I} \propto V \cdot \sigma^{m_I}$. For the second population there exist three stress intervals: (a) $\sigma \leq K_{ic}/(\pi a_u)^{1/2}$, (b) $K_{ic}/(\pi a_u)^{1/2} \leq \sigma K_{ic}/(\pi a_l)^{1/2}$, and (c) $K_{ic}/(\pi a_l)^{1/2} \leq \sigma$. In these regions the contribution of population II to $N_{c,S}$ is (a) $N_{c,II} = 0$, (b) $0 \leq N_{c,II} \leq N_{c,II,\max}$ and (c) $N_{c,II} = N_{c,II,\max} = V \cdot \int_{a_l}^{a_u} g_{II} da$ respectively. That means that – compared with the situation, where only population (I) exists (the Weibull case) the probability of failure (and the Weibull modul) is not altered in region (a), strongly altered in region (b) and almost not altered in region (c): defects have only a significant influence on the probability of failure in that stress interval, which corresponds to the size interval of their occurrence.

Of course similar results can be expected for other types of bi- modal and multi- modal flaw populations. It should be noted that structures in the strength distribution belong to structures in the flaw size distribution and vice versa ($a \sim \sigma^{-2}$). In these cases the strength distribution is not a Weibull distribution.

2.4 Ceramics with increasing crack resistance curve

If the fracture toughness of a ceramic increases with increasing crack length (R-curve behaviour) some stable crack growth before fracture is possible: $a \to a_0 + \Delta a$. Then the Griffith/ Irwin fracture criterion has to be supplemented by the condition $\partial K / \partial a \geq \partial R / \partial a$; R being the fracture toughness, which depends on the crack extension, Δa; Munz et al. [2]. In this case the strength is: $\sigma_f = (K_{Ic,0} + \Delta K_{Ic})/(\pi(a_0 + \Delta a))^{1/2}$. As well as the increase in fracture toughness, ΔK_{Ic}, also the crack extension, Δa, depend on the shape of the R-curve but also on the length of the starter crack, a_0. It is obvious that the R-curve behaviour leads to a homogenisation of the critical crack length in the material, Danzer et al. [13]. This causes a stress dependence of the Weibull modulus (it increases) and the strength is not longer Weibull distributed.

2.5 Other effects influencing the Weibull modulus and the strength distribution

There exist many other effects, which may affect the Weibull modulus and the shape of the strength distribution. In this paper only examples can be mentioned.

Such an effect is claimed for materials with a non-homogeneous microstructure, Danzer et al. [13], and for materials containing residual stresses, Danzer et al. [14]. Fett et al. [15] all observed strong deviations of the Weibull distribution, if specimens are loaded with steep stress gradients, as they occur due to contact loading or thermal shock loading. Zimmermann et al. [16] analysed the case of a crack in front of a pore (such defects are often observed in ceramics produced by pressing and sintering spray dried powders) and observed that for that case the size effect on strength disappears. Similar behaviour is observed by Lu et al. [17, 18] for materials containing flaws of such high densities that interaction between flaws gets possible.

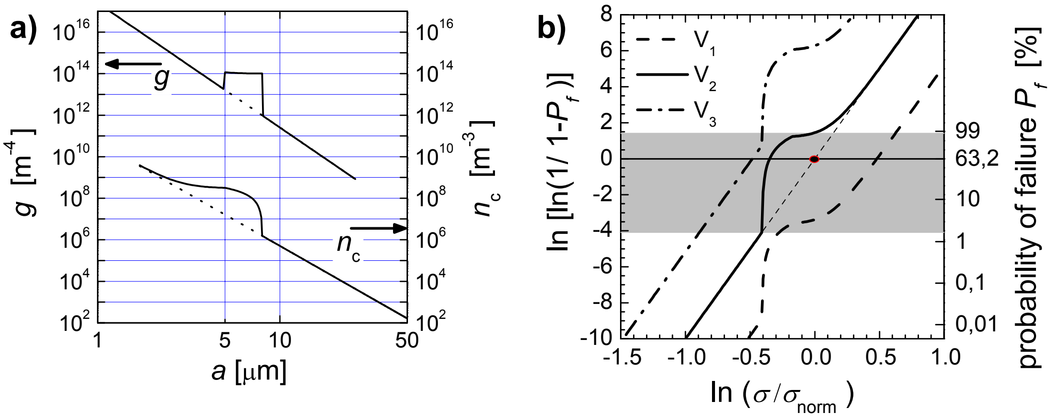
3 MONTE CARLO SIMULATIONS - SAMPLE AND POPULATION

Monte Carlo simulation techniques can be used to simulate experiments in order to judge how precise a sample can describe the population: Random numbers between 0 and 1 are diced. The number of random numbers is equal to the size of the sample. Then - for a given strength distribution (the population) - the corresponding strength values are determined (virtual strength testing). These data are then analysed in the usual way in order to find the corresponding fracture statistics (the sample), Danzer et al. [6, 14].

Standards recommend to test at least 30 specimens if the parameters of the Weibull statistics are to be determined, ENV843-5 [19]. Due to the high specimen costs more specimens are not tested in general. For modern ceramic materials the Weibull modulus is – in general – between 10 and 20. With the Monte Carlo techniques described above it can easily be shown that in that parameter range and testing only 30 specimens no clear distinctions between different statistics (e.g. Weibull, normal, log- normal) can be made, Lu et al. [20]. In the relevant parameter range a clear distinction between different statistical distribution functions would make the testing of several thousand specimens necessary, Danzer et al. [11]. Under these conditions it is clear that a Weibull distribution can be adjusted to any small sample. But it should be recognised, that this does not necessarily mean, that the population is a Weibull distribution.

4 SIZE EFFECT ON STRENGTH

For obvious reasons it is not possible to test several thousand of specimens to determine a fracture statistics. A way out of this dilemma is the testing of specimens of different size. In the fracture statistics, eq. 2, the only variable is the mean number of critical flaws, $N_{c,V}(\sigma)$, which depends on the specimen size (volume V) and the applied stress, σ. Therefore the variables σ and V are exchangeable: the same number of flaws may occur in a large volume at a low stress or in a small volume at a high stress level. Instead of testing many small specimens to get some failures at low stresses, a few specimens with a large volume can be fractured. Using eq. 1 or eq. 2 the behaviour of a few large specimens can be translated into the behaviour of many small specimens. Details can be found in Danzer et al. [6].



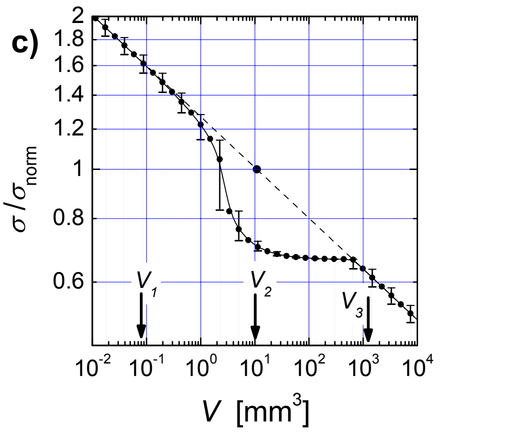


Fig. 1: Bi modal flaw population; one population is narrow peaked:
a) relative frequency distribution and density of critical defects,
b) strength distributions for tensile specimens with the volume $8\cdot10^{-2}$, 10 and $1{,}25\cdot10^{3}$ mm^3 respectively (the shaded area is the interval of failure probabilities for a sample containing 30 specimens) and
c) characteristic strength versus specimen volume. The scatter bars refer for the 90 % confidence intervals of samples containing 30 specimens.

Fig. 1 gives an illustrative example. Shown is the type of bi modal distribution discussed in section 2.3. (a) shows the relative frequency distribution of the flaw size and the corresponding density of critical defects (see eq. 3) and (b) shows the corresponding strength distribution functions (eq. 2) for three types of tensile specimens with different volume. Since structures in the distribution function refer to defect populations of a typical size, they belong also to a typical strength value ($a_c \sim \sigma^{-2}$). In (c) the corresponding strength for a failure probability of 63 % is plotted versus the volume. Confidence intervals refer to 90 %. The shaded area in (b) is the measuring interval (for the failure probability) for a sample size of 30. It is interesting to note, that, in this case, the peaked flaw population can only be recognised, if the specimens of medium size are tested (b, c). It is obvious from Fig. 1 that the fracture statistics is an image of the flaw size distribution and that both can be determined with reasonable effort by testing specimens of different size.

5 CONCLUSIONS

- The most important consequence of the fracture statistics of brittle materials is the size effect on strength, which must be taken into account in ceramics design.
- The fracture statistics reflects the size distribution of flaws in the material.
- The fracture statistics is not always a Weibull statistics, but this can not be decided on the basis of a small sample containing about 30 specimens.
- With reasonable experimental effort the relevant fracture statistics can be determined using a sample, which has (small) sub samples with specimens of different size.

6 LITERATURE

[1] R. W. Davidge: Mechanical Behaviour of Ceramics, Cambridge University Press, Cambridge, (1979).

[2] D. Munz, T. Fett: Ceramics: Mechanical properties, failure behaviour, materials selection, Springer Verlag, Berlin, (1999).

[3] J. B. Wachtman: Mechanical Properties of Ceramics, John Wiley & Sons, Inc., New York, (1996).

[4] A. D. S. Jayatilaka, K. Tustrum: Materials Science 12 (1977) 1426.

[5] R. Danzer: J. Eur. Ceram. Soc. 10 (1992) 461.

[6] R. Danzer, T. Lube, P. Supancic: Z. f. Metallkde. 92 (2001) 773.

[7] W. Weibull: A statistical theory of strength of materials, Royal Swedish Institute for Engineering Research, Stockholm, (1939).

[8] W. Weibull: Journal of Applied Mechanics 18 (1951) 293.

[9] P. Kittl, G. Diaz: Res Mechanica 24 (1988) 99.

[10] A. M. Freudenthal, in: Liebwitz (Ed.) Fracture, Academic Press, New York, (1968).

[11] R. Danzer, T. Lube, in: K. Niihara (Ed.) Ceramic Materials, Components for Engines, Japan Fine Ceramics Association, Tokyo, (1998) 683.

[12] R. Danzer, T. Lube, in: R.C. Bradt, D.P.H. Hasselmann, D. Munz, M. Sakai, V.Y. Shevchenkov (Eds.), Fracture Mechanics of Ceramics, Plenum Publishing Corp., New York, (1996) 425.

[13] R. Danzer, G. Reisner, H. Schubert: Zeitschrift für Metallkunde 83 (1992) 508.

[14] R. Danzer, P. Supancic, T. Lube: Ceramic Engineering and Science Proceedings 24 (2003) 497.

[15] T. Fett, E. Ernst, D. Munz, D. Badenheim, R. Oberacker: J. Eur. Ceram. Soc. 23 (2003) 2031.

[16] A. Zimmermann, J. Rödel: J. Am. Ceram. Soc. 82 (1999) 2279.

[17] C. Lu, R. Danzer, F. D. Fischer: Fracture Mechanics of Ceramics 14 (in press).

[18] C. Lu, R. Danzer, F. D. Fischer: Journal of the European Ceramic Society (in press).

[19] ENV843-5: in Advanced Technical Ceramics, Monolithic Ceramics; Mechanical Tests at Room Temperature, Part 5- Statistical Analysis, (1997) 41.

[20] C. Lu, R. Danzer, F. D. Fischer: Physical Review E 65 (2002) 1.