T-STRESS AND MODE I FATIGUE CRACK GROWTH.

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ABSTRACT

Fatigue crack growth is dependent on material properties and loading parameters. Critical areas of industrial components are often subjected to complex stresses, either because multiaxial loads are applied or because residual stresses are present in the material. In plane stresses are taken into account through mode I, II and III stress intensity factors. Out of plane stresses are usually not included in fatigue crack growth models. However, experimental evidences of the influence of out-of plane stresses on fatigue crack growth are reported in the literature. This paper is centered on the role of the T-stress during mode I fatigue crack growth.

The effect of a T-stress on fatigue crack growth is studied through its effect on crack tip blunting. As a matter of fact, fatigue crack growth is characterized by the presence of striations on the fracture surface, which implies that the crack grows by a mechanism of blunting and re-sharpening. From a mechanical point of view, this means that forward and reverse plasticity occurs within the crack tip plastic zone. The presence of a T-stress modifies significantly the evolution of plastic deformation at crack tip and thus plastic blunting.

In the present study, crack tip blunting is a global scalar variable, which is calculated using the finite element method as the average value of the permanent displacement of the crack faces over the whole K-dominance area. Experimental measurements have also been conducted using cross-correlation of images taken at various stress intensity factors, in order to measure the displacement field in the K-dominance area.

A yield stress intensity factor is defined for the cracked structure, as the stress intensity factor for which crack tip blunting exceeds a given value. The variation of the yield stress intensity factor as a function of the T-stress was studied. It is found that the T-stress can modify very significantly the yield point of the cracked structure. Secondly it is found that the yield limit in a (T,K_I) plane is not-dependent on the crack length,

A yield criterion is proposed for the cracked structure. This criterion is an extension of the Von-Mises yield criterion to the problem of the cracked structure. As a matter of fact, the present yield criterion is based on the elastic shear energy within the K-dominance area, as calculated using the Westergaard stress functions. The proposed criterion matches almost perfectly the results stemmed from the FEM.

Finally the evolution of the yield limit of the cracked structure in a (T,K_I) plane was studied for various loading scheme. These results allow developing a plasticity model at the global scale for the cracked structure taking into account the effect of the T-stress.

1 INTRODUCTION

Critical areas of industrial components are often subjected to complex stresses, either because multiaxial loads are applied or because residual stresses are present in the material. In plane stresses are taken into account through mode I, II and III stress intensity factors. However, out of plane stresses are usually not taken into account in fatigue crack growth criterion though there are experimental evidences of their effect in the literature.

The role of out of plane stresses is examined in the present paper through the effect of the T stress and in mode I only. In linear elastic fracture mechanics, the T-stress is the first second order term in the asymptotic development of σ_{xx} at crack tip. If remote biaxial loads (Sx and Sy) are applied on a central through thickness crack lying in the plane (x,z), the mode I stress intensity factor and the *T*-stress are as follows: $K_I = Sy\sqrt{\pi a}$ and $T = S_x - S_y$. Under pure remote biaxial loadings, the T-stress is zero. It is worth to mention that under uniaxial loading $S_x=0$, a long crack with a length a_1 and a short crack with a length $a_2=a_1/\alpha$, subjected to the same stress intensity factor K_I, are subjected to different T stresses, i.e. $T_2 = T_1\sqrt{\alpha}$. Therefore the T-stress is expected to be a suitable parameter for the description of the behavior of mechanically short cracks [1].

The role of the T-stress can be studied using biaxial tests or using specimens with different evolutions of T with crack length. It was shown by various authors that the T-stress can have a very significant effect on fatigue crack growth [3-6]. For instance, Tong et al [2] show that the fatigue crack growth of a crack in a CT sample is up to ten times higher than that measured in SENT or CCT samples, with the same thicknesses. The author argued that this difference is due to the T-stress, positive in CT samples and negative in CCT and SENT samples. However, the effect of the T-stress appears to depend also on the stress ratio employed in the experiments. For instance, if the stress ratio is negative, the crack growth rate was found to decrease when T increases. The same effect is also found for high positive values of R. However, when the stress ratio is zero, the effect is opposite [3-5]. Empirical models have also been proposed in the literature [5,6] for taking into account the T-stress. For example, Howard et al. [6] proposed a corrected Paris law on the basis of their experimental results as follows : $da/dN = C.\exp\left(-0.14.\left(1+T/S_Y\right)\right).(\Delta K)^{3.3}$.

The general conclusion of this introduction is that the role of the T-Stress appears to be very significant in the literature but that the underlying mechanisms need to be clarified.

2 METHOD

Our first task was to define a fracture mechanics state variable which would be suitable to measure crack tip plasticity. The crack tip blunting ρ was shown to be a suitable variable [7]. It is defined as the average value, of the permanent part of the displacement of the crack faces, i.e. the part of the displacement of the crack faces that would remain after unloading elastically the structure. Crack tip blunting is calculated as follows using the FEM. The displacement of the crack faces is calculated under elastic u_{ye} and elastic-plastic u_{yep} conditions (Fig. 1). The difference u_{yp} between u_{yep} and u_{ye} is the part of the displacement that would remain after unloading elastically the cracked structure. The average value of u_{yp} is calculated over a distance δ well above the dimension of the crack tip plastic zone, and the average value is defined as the plastic blunting at crack tip ρ (Fig.1). The same approach is also applied to experimental measurement of the displacement using cross-correlation of images.

Once plastic blunting is defined, it can be plotted versus the applied stress intensity factor (Fig.2). In Fig 2 have also been added the mean square distance between $u_{yp}(r)$ and ρ over the distance δ as an error bar on ρ, and the error associated with the asymptotic development at crack tip in elasticity as an error bar on K_I.

It appears in Fig. 2, that there is a large domain within which the plastic deformation at crack tip is not significant. A criterion Δρ was chosen in order to define precisely this domain for various loading conditions. At the first loading, the “elastic” domain for the cracked structure is between $K_I=0$ and $K_I=K_{IY}$. Then, forward and reverse plasticity occurs and the existence of residual stresses within the cyclic plastic zone causes a displacement of that “elastic domain” which is now bounded by $K_I=K_I^+$ and $K_I=K_I^-$ (Fig. 2)

First of all, it was checked that the obtained results obey the principle of similitude. Calculations where performed with the same mesh refinement at crack tip, and the same averaging zone δ, but with different crack lengths. It is shown that the results are not dependent on the crack length. In Fig. 3, for instance is plotted the evolution of crack tip blunting versus the stress intensity factor for T=0 and for various crack lengths, the discrepancy between each curve and the average one is always below 5% of the average one.

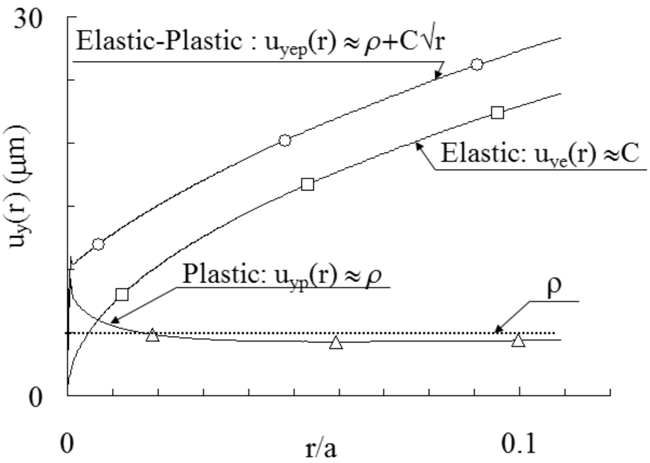

Figure.1. Displacement of the crack faces, as calculated using the finite element method for a stationary crack. 2D. Plane strain. CCT specimen.

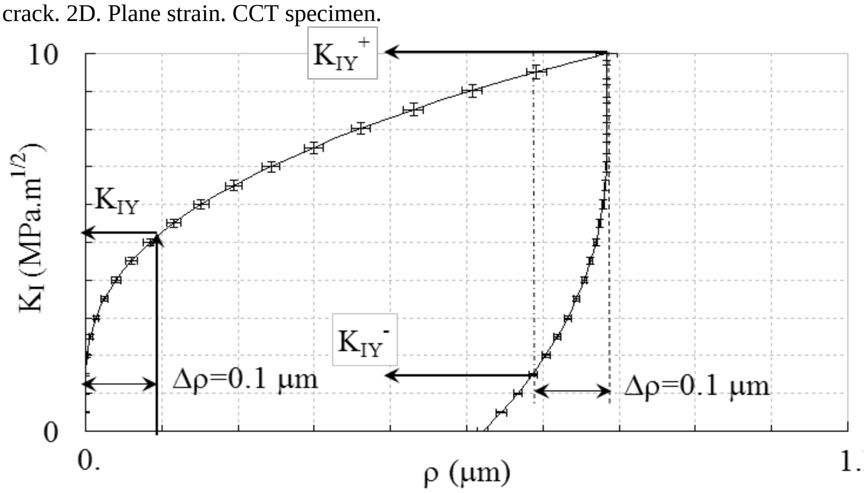

Figure.2. Evolution of plastic blunting at crack tip, as defined in Fig. 1, during the loading and unloading of the cracked structure. Definition of the yield criterion $\Delta\rho$ and of the initial yield point K_{IY} and cyclic elastic domain (K_I^- and K_I^+). 2D. Plane strain. CCT specimen. Material: E=200 GPa, ν=0.3. Re=100 MPa, H=100 MPa

3 RESULTS AND CRITERION

Once the yield point of the cracked structure (K_{IY}) is defined (Fig. 2), it can be determined for various values of the T-stress. This allows to plot the frontier of the elastic domain for the cracked structure in a (T,K_I) plane (Fig. 4). It was checked that this curve is independent of the crack length.

In Fig. 4, the yield point of the cracked structure appears to increase when the T stress increases, up to a maximum around T=0, then the yield point decreases again when the T stress increases.

On the same graph is also plotted the Von Mises yield criterion for an uncracked specimen under plane strain conditions and for biaxial loadings conditions, where $T = S_x - S_y$ and

$K_I = Sy\sqrt{\pi a}$. It was observed that the ellipse which is best fitting the FE results appears to be coincident with the Von Mises yield criterion when K_I is close to zero. This result was expected since a cracked specimen subjected to K_I=0 should behave like an uncracked specimen.

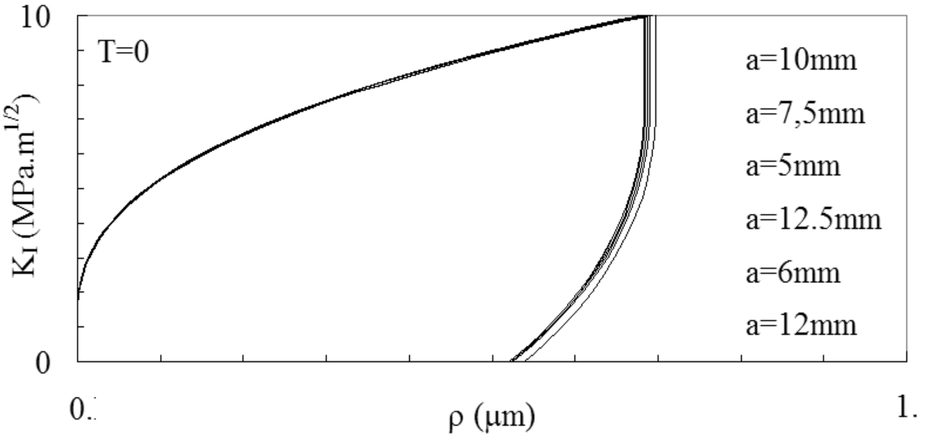

Figure.3. Comparison of the results for various cracks lengths and the same value of T. The discrepancy between each curve (ρ,K_I) obtained for a given value of T but for various crack length and the average one remain always less than 5% of the average one. Material: E=200 GPa, ν=0.3. Re=100 MPa, H=100 MPa

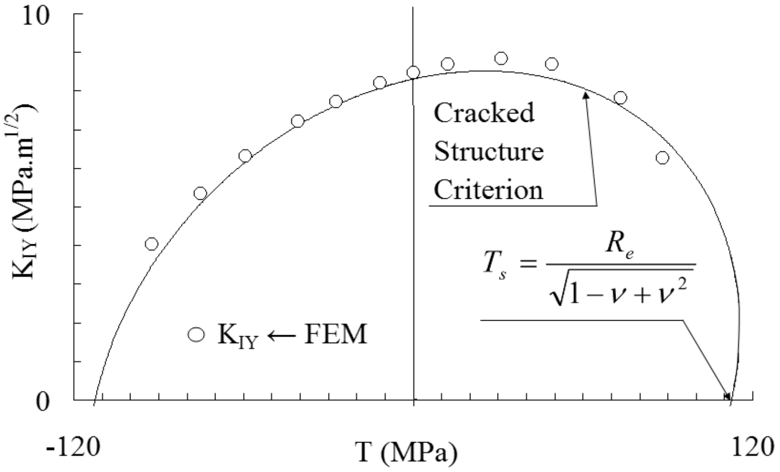

Figure.4. Frontier of the initial elastic domain of the cracked structure, as calculated using the FEM, with a criterion $\Delta\rho$=0.2 μm. The yield criterion for the cracked structure as proposed in the present paper is also added. Material: E=200 GPa, ν=0.3, Re=100 MPa, H=100 MPa

This leads us to propose an extension of the Von Mises yield criterion to the problem of the cracked structure. The Von Mises yield criterion is obtained by assuming that the yield point is reached for a given elastic shear energy. The same approach is employed here, except that the whole K-dominance area is considered. The displacement field of the LEFM in the presence of a T-stress are employed for the calculation of the stresses and strains in the K dominance area, the material is assumed to behave elastically when the cracked structure is in its elastic domain, thus the following equations are employed:

$$u_x(r,\theta)=\frac{K_I}{\mu}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(\frac{\kappa-1}{2}+\sin^2\frac{\theta}{2}\right)+\frac{T}{8\mu}(\kappa+1)r\cos\theta+O(r^{\frac{3}{2}})$$

$$u_y(r,\theta)=\frac{K_I}{\mu}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(\frac{\kappa+1}{2}-\cos^2\frac{\theta}{2}\right)-\frac{T}{8\mu}(3-\kappa)r\sin\theta+O(r^{\frac{3}{2}})$$ and $u_z(r,\theta)=0$

$\varepsilon(K_I,T)=\frac{1}{2}\left(\nabla u+\nabla u^T\right)$ and $\varepsilon^D=\varepsilon-\frac{Tr[\varepsilon]}{3}.I$

$\sigma(K_I,T)=\lambda Tr[\varepsilon]I+2\mu.\varepsilon$ and $\sigma^D=\sigma-\frac{Tr[\sigma]}{3}.I$

The density of elastic shear energy can be written as follows:

$$U(K_I,T)=\frac{1}{2}\varepsilon^D(K_I,T)\sigma^D(K_I,T)$$

And the yield criterion for the cracked structure is therefore as follows:

$$\int_{r=0}^{r=\delta}\int_{\theta=-\pi}^{\theta=\pi}U(K_I,T)dv=\int_{r=0}^{r=\delta}\int_{\theta=-\pi}^{\theta=\pi}U(0,T)dv$$

After simplifications the criterion can be alternatively written as follows:

$\boxed{\left(\frac{K_I}{K_s}\right)^2+f_\nu\left(\frac{K_I}{K_s}\right).\left(\frac{T}{T_s}\right)+\left(\frac{T}{T_s}\right)^2=1}$ where: $T_s=\frac{R_e}{\sqrt{1-\nu+\nu^2}}$, where $K_s=\frac{2\sqrt{2\pi\delta}R_e}{\sqrt{7-16\nu+16\nu^2}}$

where: $f_\nu=\frac{32\left(1-10\nu+10\nu^2\right)}{15\pi\sqrt{\left(1-\nu+\nu^2\right)\left(7-16\nu+16\nu^2\right)}}\approx 0.189-2.10\ \nu$ and where Re is the yield stress in uniaxial tension of the material.

It can be seen in Fig. 4 that this yield criterion for the cracked structure matches almost perfectly the results stemmed from the finite element method. Also, it is coincident with the Von Mises yield criterion (for an uncracked structure) when K_I=0.

Finally, the finite element method was employed in order to study the evolution of the elastic domain of the cracked structure (K_I^-,K_I^+) after various loadings histories. It appears that the elastic domain is not significantly distorted but displaced in a (T,K_I) plane (Fig. 5).

This allows extending the model previously developed by Pommier and Risbet [7] to the problem of a crack subjected to K_I and T, and hence to predict the evolution of crack tip blunting versus time when a T-stress is applied. Besides, since there is a relationship between crack tip blunting and resharpening and fatigue crack growth, [7], the present results allow to predict fatigue crack growth in the presence of a T-stress.

3 CONCLUSIONS

From the definition of crack tip blunting at crack tip, an elastic domain can be defined for the cracked structure, as the domain in a (T,K_I) plane within which the variation of crack tip blunting is below a critical value (here $\Delta\rho$=0.2 µm, for instance). It is shown that the presence of a T-stress modifies significantly the yield point of the cracked structure. A criterion is proposed for the yield point of the cracked structure which is an extension of the Von Mises yield criterion to the problem of the cracked structure. It is found to match almost perfectly the results from the FEM.

Finally the evolution of the elastic domain of the cracked structure in a (T,K_I) plane is studied for complex loadings scheme. It appears that it obeys a simple displacement rule. The present results allow determining crack tip blunting as a function of time, when a T-stress is applied. Since there is a relation between crack tip blunting and re-sharpening and fatigue crack growth, the model allows also taking into account the effect of a T-stress in fatigue crack growth modeling.

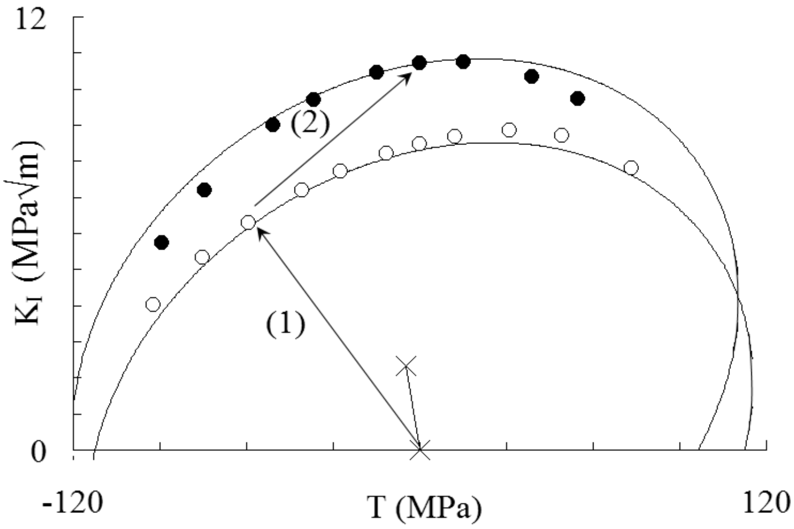

Figure.5. Frontier of the elastic domain of the cracked structure. Empty symbols after loading the structure along the direction (1). Black symbols, after loadings the structure along (1) and (2) subsequently. Material: E=200 GPa, ν=0.3. Re=100 MPa, H=100 MPa

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