MODELING THE EVOLUTION OF SLOPE FAILURE AS A CRACK PROPOGATION PROBLEM

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ABSTRACT

A comprehensive modeling of the evolution of a slope failure has to address two important issues. One is how the slope becomes unstable; the other how the unstable mass separates itself. This study tackles the first issue considering that the formation the unstable mass to be a mixed mode crack propagation problem. A mesh based partition of unity method, also known as the manifold method, is employed in this undertaking. Because of the method's root in the discrete element method, it is also capable of modeling discrete-continuum interaction problem. The use of this method, therefore, also takes care of the second issue. Sample problems are solved as an illustration. The problem configuration consists of a simple slope that has a pre-existing tensile crack along its crest. The slope failure is triggered by rainfall which raises water pressure in the crack. As the stress around the crack tip increases, an existing crack grows and a failure surface is eventually developed. The maximum stress criterion is adopted in determining the crack growth and growth direction. After a failure surface is formed, the analysis switches from a static to a dynamic formulation. The mechanical interaction between the unstable soil mass and the remaining stable slope dictates how the unstable slope separates itself.

1 INTRODUCTION

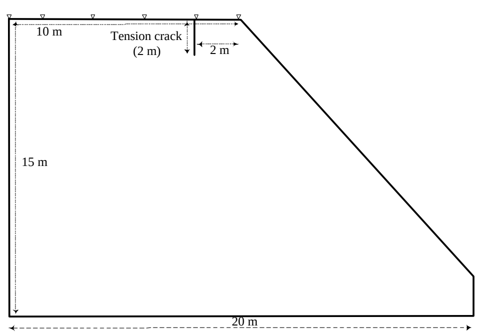
Modeling the whole process of a slope failure is a challenging task. First, there is the question of as to how a failure surface is formed. Then, as a failure surface is developed and the unstable mass becomes a separate entity, the problem domain no longer remains an integral continuum. Furthermore, as the unstable mass separates itself such as by sliding or by falling, an initial static problem becomes a dynamic one. This study presents a unified approach that encompasses all essential phases of a slope failure within one framework. Specifically, this study models the formation of failure surface as a crack propagation problem within the context of linear fracture mechanics.

There is a significant advance in the computational methods in the area of modeling crack propagation. Of particular interest is the progress emerged from a new class of formulation that shares the common base of the partition of unity concept (Duart and Oden, [3]). This includes the mesh free Galerkin method (Belytschko et al, [1]**)** and the extended finite element (Daux et al, [2]). For the present study, a mesh based partition of unity method, also knows as the manifold method, is employed. The basic idea of the method was proposed Shi [4] as an extension to an implicit discrete element method (Shi, [5]) so that each discrete element can have many more degrees of freedom. A geometric interpretation of the method can be found in Lin [6]. Because of its discrete element root, the manifold method can combine the continuum and the discrete analysis within a single framework (Lin, [7]). This, in turn, makes it possible for tackling all the different phases of a slope failure within one computational method.

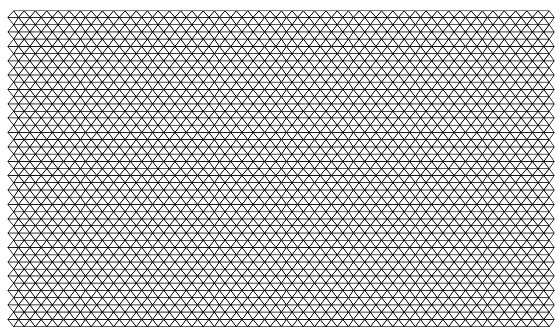
2 A MESH BASED PARTITION OF UNITY METHOD

This method employs two sets of meshes in modeling a problem: the physical mesh and the mathematical mesh. The former defines the integration domain for each element, the latter the nodes of interpolation. To begin with, a portrait of a problem geometry including discontinuities, cracks in the present case, is drawn. Then a pattern of grid defining potential nodes is selected. By superimposing these two sets of drawings, the nodes and elements for an analysis are obtained.

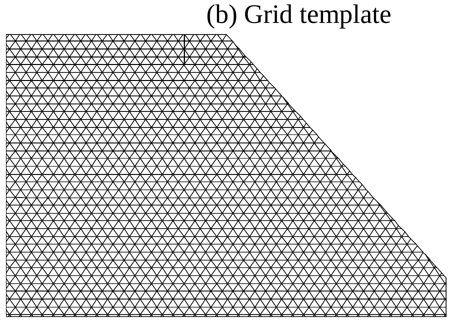
For the sample problem solved in this study, the layout of the slope, an existing crack, the left and the bottom fixed boundaries constitute a portrait of the problem which is depicted in Figure 1(a). A triangular mesh grid is used to build the two sets of meshes. The mesh grid, depicted in Figure 1(b), is only required to be large enough to cover every point of a problem domain. These two meshes are then superimposed and nodes for the triangles that do not intersect the problem domain are trimmed. Figure 1 (c) is the resulting physical mesh. To avoid cluttering a drawing, nodes that are kept but lie outside a problem domain are not shown. A mathematical mesh is a mesh containing those nodes not shown in Figure 1 (c). In an analysis, two lists of nodes are maintained: one defines the integration area, the other the interpolation. Each node is updates as geometry changes.



(a) Problem portrait



(b) Grid template



(c) Physical mesh

Figure 1: Construction of analysis meshes

The manifold method uses nodes that lie outside the problem domain. Following Lin [6], a local cover, ω_α, associated with a node, α, is formed by the union of all the triangular areas, in the case of triangular mesh, share this common node. The collection $\bigcup_{\alpha=1}^{N} \omega_\alpha$ on a mathematical mesh with N nodes provides a finite covering of the domain of solution.

The manifold method also uses the partition of unity function although it was originally called the weighting function. Following Duarte and Oden [3], a partition of unity function, $\varphi_\alpha(\mathbf{x})$, subordinates to each cover satisfies the following,

$$\sum_{\alpha=1}^{N} \varphi_\alpha(\mathbf{x}) = 1 \quad \mathit{for}\ \mathbf{x} \in \Omega \tag{1}$$

$$\varphi_\alpha(\mathbf{x}) \in C_0^S(\omega_\alpha) \quad \mathit{for}\ 1 \le \alpha \le N;\ S \ge 0 \tag{2}$$

To simplify contact detection and contact modeling for unstable mass separation, this study uses linear shape function as the partition of unity function. Furthermore, a constant approximate function over each cover is employed as follows,

$$\hat{\mathbf{u}}_\alpha(\mathbf{x}) = u_\alpha \tag{3}$$

As such, an element is defined as an overlapped area of three covers. The solution approximation within an element, $\mathbf{u}^{\mathbf{h}}(\mathbf{x})$, becomes

$$\mathbf{u}^{\mathbf{h}}(\mathbf{x}) = \sum_{i=1}^{3} \varphi_i(\mathbf{x}) \cdot \hat{\mathbf{u}}_i(\mathbf{x}) \tag{4}$$

When a cover becomes disjoint due to discontinuity, a cover is partitioned which is equivalent to have more than one nodes at a point. Thus, in a crack propagation problem, the manifold method just adds additional covers, or nodes, to follow the growth of a crack. This requires that each involved disjoint cover be modified so that it only covers a partial area:

$$\varphi_i^{\ j}(\mathbf{x}) = \varphi_i(\mathbf{x}) \cdot \delta_i^j(\mathbf{x}) \tag{5}$$

where $\delta_i^j(\mathbf{x})$ is 1 on ϖ_i^j, and 0 elsewhere, and ϖ_i^j represents the jth division of a cover at node i. As a crack grows and if the new crack surface cuts through a cover, a new cover is added. In other words, new nodes are introduced as a crack propagates. Together with introduction of $\delta_i^j(\mathbf{x})$, the approach can handle the introduced crack surface without remeshing. $\delta_i^j(\mathbf{x})$ of all the involved covers defines the integration area.

3 CRACK PROPAGATION CRITERION

For mixed mode crack propagation, the interaction integral is often used to find the stress intensity factors, which then gives the stress field around the crack tip. This approach can readily be incorporated in the manifold method but it requires the use of higher order approximation then presented here. This also requires a reformulation of the contact detection and modeling for the continuum-discrete interaction problem. To accommodate a simple low order approximation, this study adopts the maximum stress criterion by Erdogan and Sih [8]. It is used in determining initiation of crack growth and the direction of a crack propagation:

1) Crack initiation takes place when the maximum tangential stress reaches the tensile strength of the material.
2) Crack initial extension occurs along an orientation in which the tangential stress is at a maximum.
3) The orientation in which the crack extends is perpendicular to the direction of the maximum tangential stress.

In the present study, the material is considered linear elastic. The maximum stress criterion is applied by sampling the stress at a distance r_o away from the crack tip (Williams and Ewing, [9]). The r_o used in this study is set to be about 0.1a, where a is half length of the crack. The material strength at r_o is determined from fracture toughness. The criterion has been used successfully with the manifold method in modeling crack propagation that involves interaction across crack surfaces (Ku, [10]).

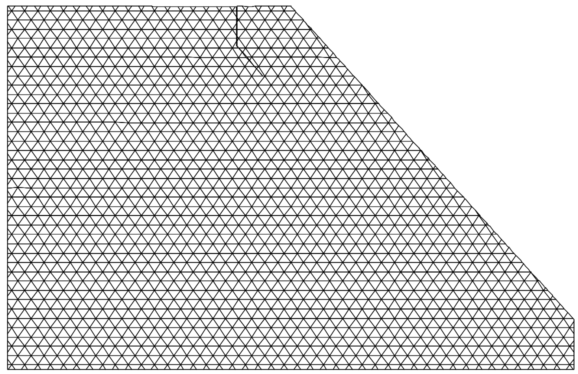
4 MODELING CONSIDERATION

The example problem under study is a slope 15 m high with a 2-m deep initial tension crack located on the top of the slope. The soil is considered to be homogeneous and has a unit weight of 23.6 kN/m^3. The elastic modulus for the material is 71,820 kPa, and the Poisson ratio is 0.3. A material shear strength of 11.97 KPa is considered. Any newly fractured failure surface will be frictional and has a cohesive strength of 3.59 kPa, and a friction angle of 28 degrees.

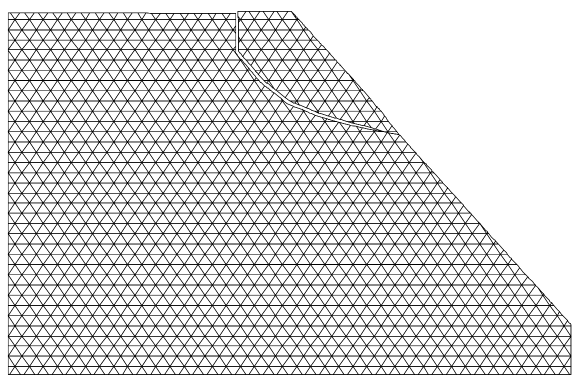
The failure event studied is one that causes by a rise of water pressure in the crack. Due to rainfall, the water fills the crack to its top and exerts pressure on the crack surface. Each fresh crack growth length is fixed at a size three times that of an element, or about 1.2 m in this study. When a crack opens up, the pressure is derived by considering that the water to be at the crest level.

The penalty method is used in imposing the no penetration constrains across the block boundary. A contact point can be in stick or slip state depending upon whether the strength has been exceeded using Mohr-Coulomb criterion.

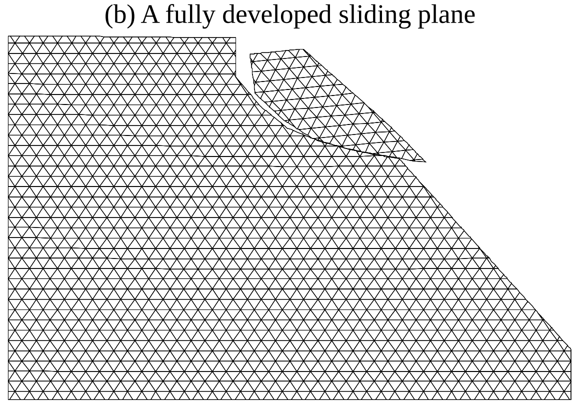
Evolution of slope failure is initially computed using static analysis. When a failure surface extends the boundary of a slope and forms a separate body, the dynamic analysis is kicked in. Figure 2 presents a snapshot of the evolving stages in the progression of the slope failure. Another case with an initially slanted crack is partially depicted in Figure 3.



(a) Initial crack growth



(b) A fully developed sliding plane



(c) A snapshot during sliding

Figure 2: Evolution of a Slope Failure

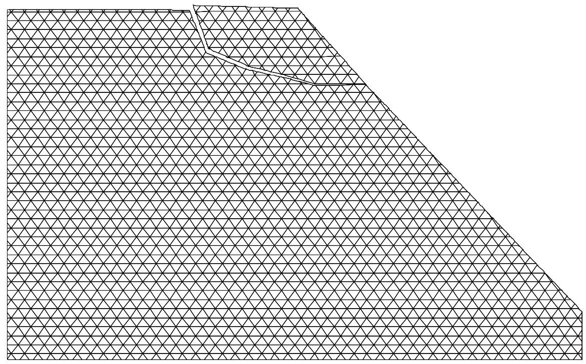


Figure 3: A snapshot of an initially slant crack just grows into a failure surface

5 CONCLUSIONS

Even though the use of the maximum stress criterion is simplistic and needs to be improved, the study does show that the present method is possible to simulate as complicated scenario as the evolution of a slope failure. The advantages of an integrated discrete-continuum approach are clearly demonstrated in that an analysis can continue even after a continuum split into discrete objects.

6 REFERENCES

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