3-D EFFECTS ON FATIGUE CRACK CLOSURE PROCESSES IN SMALL-SCALE YIELDING

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ABSTRACT

In ductile metals, plasticity-induced closure of fatigue cracks often retards significantly measured crack growth rates in the Paris regime and contributes strongly to the observed R-ratio effect in experimental data. This work describes a similarity scaling relationship based on the 3D small-scale yielding framework wherein the thickness, B, defines the only geometric length-scale of the model. Dimensional analysis suggests a scaling relationship for the crack opening loads relative to the maximum cyclic loads (K_{op}/K_{max}) governed by the non-dimensional load parameter $\overline{K} = K_{max}/\sigma_0\sqrt{B}$, *i.e.*, a measure of the in-plane plastic zone size normalized by the thickness. Both K_{op} and K_{max} refer to remotely applied values of the mode I stress-intensity factor. Large-scale, 3D finite element analyses described here demonstrate that K_{op}/K_{max} values vary strongly across the crack front in thin sheets but remain unchanged when K_{max}, B, and σ_0 vary to maintain $\overline{K}$ = constant. The paper also includes results to demonstrate that the scaling relationship holds for non-zero values of the T-stress (which affect the K_{op}/K_{max} values). The new similarity scaling relationship makes possible more realistic estimates of crack closure loads for a very wide range of practical conditions from just a few analyses of the type described here.

1. INTRODUCTION

Plasticity induced crack closure (PICC) contributes significantly to the strong effect of loading ratio (R-ratio $= K_{min}/K_{max}$) routinely observed in conventional fatigue testing for crack lengths and ΔK_I levels above those of short-crack and the near threshold behavior. The Paris model for fatigue crack growth rate accommodates the effects of closure through the modified form

$$\frac{da}{dN} = C\left(K_{max} - K_{op}\right)^m = C\Delta K_{eff}^m \tag{1}$$

where K_{op} denotes the load level at which the crack faces become fully "open" upon reloading. Large positive R-ratios (*e.g.*, $R = 0.7$ - 0.9) generally lead to no closure such that $K_{op} = K_{min}$ and $\Delta K_{eff} = \Delta K_I$. At small positive, zero and negative R-ratios, $K_{op} > K_{min}$ and only that part of the load cycle during which the crack faces have no contact contributes to material damage (growth) in this modified model.

This work reviews some results of a systematic study of PICC effects in a practically important subset of 3D configurations characterized by structurally thin, metallic panels containing engineering-scale fatigue cracks growing under cyclic, mode I and SSY conditions. The finite element computations advance initially straight, through-cracks in each load cycle to investigate the interacting effects on PICC of thickness, T-stress and cyclic material flow properties for constant amplitude loading. The finite element results for K_{op}/K_{max} confirm a new, similarity scaling relationship between K_{max}, the uniaxial yield stress (σ_0) and the thickness (B) suggested by dimensional considerations of the SSY framework [1-3].

2. 3-D SMALL-SCALE YIELDING FRAMEWORK

Figure 1 illustrates the model for computational studies that represents a wide-range of practical conditions in a simple framework. The thin metallic panel has a thickness, B, with an initially straight (sharp) through crack. The in-plane dimensions of the panel exceed 25-50 $\times B$ (where B may be only 1-2 mm in actual structures). The combination of peak (mode I) load levels typically experienced in moderate-to-high cycle fatigue crack growth (K_{max}) and the typical values of yield stress (σ_0) for structural metals, leads to crack-front plastic zone sizes of at most equal to a few multiples of B.

The computational model considers a semi-circular disk of thickness B and radius $\overline{R} \gg B$ centered at the crack front. Loading of this 3D model for SSY conditions occurs through cyclically varying displacements applied on the boundary at $\overline{R}$ corresponding to specified levels of ΔK_{max} and ΔT_{max}—here T always varies in proportion to K_I as it does in finite geometries. Symmetry conditions for mode I loading and growth govern on the plane ahead of the crack front while frictionless contact conditions hold behind the advancing front.The thickness B represents the only loading invariant, geometric length-scale in this model.

3. A SIMILARITY SCALING RELATIONSHIP FOR CRACK CLOSURE

The Nakamura and Parks [5] results for a stationary crack subjected to monotonic loading in the same 3D SSY framework adopted here suggests the potential to normalize the near-front fields using $K_I/\sigma_0\sqrt{B}$. For fatigue crack growth, this leads naturally to developing a non-dimensional relationship for the opening load (K_{op}) relative to the maximum load (K_{max}) in each cycle of the form

$$\frac{K_{op}}{K_{max}} = F\left(\frac{K_{max}}{\sigma_0\sqrt{B}}, \frac{T_{max}}{\sigma_0}, R, R_{OL}; \frac{z}{B}, \frac{\Delta a}{B}; \frac{\sigma_0}{E}, \frac{E_T}{E}, \nu\right) \tag{2}$$

where F denotes a non-dimensional function of its non-dimensional parameters. The first loading parameter, $K_{max}/\sigma_0\sqrt{B}$, reflects a measure of the in-plane plastic zone size

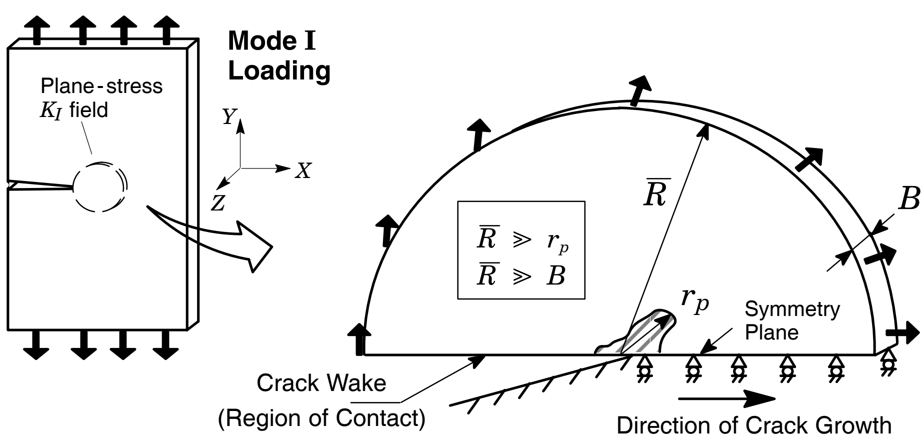

Figure 1: 3-D small-scale yielding framework. Displacements imposed on boundary of the cylindrical disk correspond to those for the linear-elastic, mode I plane-stress solution including a T-stress.

at peak load ($r_{p\text{-}max}$) scaled by the thickness, B (where B represents the only geometric dimension of this SSY framework). The T_{max}/σ_0 term approximates constraint differences at the crack front from variations of sheet geometry (*e.g.*, M(T), SE(B), DE(T), etc.) and remote loading (*e.g.*, bending *vs.* tension).

The z/B term in Eq. (2) reflects the strong variation of opening loads across the crack front that exists over the full loading history. The opening behavior exhibits an initial transient response as the crack grows through the plastic zone created by the first half cycle of loading $0 \rightarrow K_{max}$ from a previously undeformed and stress-free configuration. In the transient period, K_{op}/K_{max} values increase rapidly to reach steady-state levels. Thereafter, the $\Delta a/B$ dependence in Eq. (2) vanishes (see Fig. 2 here and [1]).

The significance of Eq. (2) for applications becomes clear by considering a specific example. Consider the loading $\overline{K} = K_{max}/\sigma_0 \sqrt{B} = 1.0$ and $\overline{T} = T_{max}/\sigma_0 = 0$, which generates a maximum in-plane plastic zone size of $r_p \approx 0.2 \times B$. Then, for constant amplitude cyclic loading and material strain hardening, the opening load levels across the crack front relative to K_{max} remain unchanged during the initial transient response and during steady-state growth as the yield stress (σ_0), thickness (B) and peak loading (K_{max}) all vary to maintain $\overline{K} = 1.0$. Consequently, one numerical solution for a set of non-dimensional parameters becomes scalable to a very wide range of practical configurations.

4. KEY RESULTS AND DISCUSSION

4.1 Verification of Proposed Dimensional Scaling Model

Figure 2 demonstrates the applicability of the proposed non-dimensional scaling relationship for the crack opening loads. This figure shows the evolution of crack opening loads obtained from two analyses of the 3D SSY model using material flow properties representative of a structural aluminum. The baseline solution (solid line) employs a model with thickness $B = \underline{B}$ while the second solution (symbols) uses a model with thickness $B = 2 \times \underline{B}$. Scaling of the peak load levels (K_{max}) for the constant amplitude, $R = 0$ cycling maintains $\overline{K} = K_{max}/\sigma_0 \sqrt{B} = 1.0$ in each case. A value of $\overline{K} = 1.0$ generates an in-plane plastic zone at peak load on the crack plane ($\theta = 0$) of size $r_{p\text{-}max} \approx 0.2 \times B$ for the T-stress $= 0$ loading used here. The two solutions remain identical over the complete crack-growth history to within the load-step size used in the finite element computations, thus validating the non-dimensional scaling for crack opening loads. Similar computations that vary yield stress, the E/σ_0 ratio and R ratio (>0) also demonstrate the applicability of the scaling relationship [1,2].

These results illustrate key features observed in the computed behavior of the crack opening-closing process. Early in the loading history, the crack grows through the initial plastic zone of size $0.2 \times B$ established in the first half-cycle of load ($0 \rightarrow K_{max}$) from an initially unstressed configuration. This initial transient acts to retard the closure process similar to an overload later in the loading. Once the crack front extends through this initial plastic zone, the opening levels stabilize to effectively constant values (here termed steady-state). The opening load levels show a strong variation with position across the crack front from the initial transient to steady-state conditions. At the outside surface ($z/B = 0.48$-0.5), the opening load levels of $K_{op}/K_{max} = 0.4$-0.5 very closely match the expected values given by simple, plane-stress estimates. The opening load levels decrease very sharply at crack front locations only a small distance from the outside surfaces, reaching 0.25 at $z/B = 0.4$. At the centerplane, the opening load slowly decreases, reaching an apparent steady-state value of 0.02, which corresponds to the load-step size used in the analysis. Thus, for $R = 0$ and $\overline{K} = 1.0$ ($r_{p\text{-}max} \approx 0.2 \times B$) with T-stress$\,=0$, these results indicate that the centerplane material experiences little or no closure at steady growth conditions. The plane-strain computations for M(T) and SE(B) specimens described by Fleck [6] show trends very similar to the present centerplane results. At this level of remote loading ($\overline{K} = 1.0$), the opening mode stresses at the centerplane very closely match those for idealized plane-strain conditions [2].

4.2 Effects of *T*-Stress on Closure

Under plane-strain conditions for a stationary crack, the *T*-stress strongly affects the size/shape of the crack front plastic zone [4,8]. Both positive and negative *T*-stress loading increase the size of the plastic zone relative to the neutral configuration ($T = 0$). A negative *T*-stress leads to much lower mean stress and opening mode stresses ahead of the crack plane while a positive *T*-stress leads to marginal increases in opening mode stress. For plane-stress conditions, the *T*-stress has a much less influence on plastic zones and opening mode stresses (the zero out-of-plane stress exerts a dominant effect). These observations for a stationary crack carry over to the crack closure phenomenon studied here for both positive and negative *T*-stress loadings using the 3-D SSY framework. Solanki *et al.* [9] examined *T*-stress effects on closure using 2D model of specific geometries.

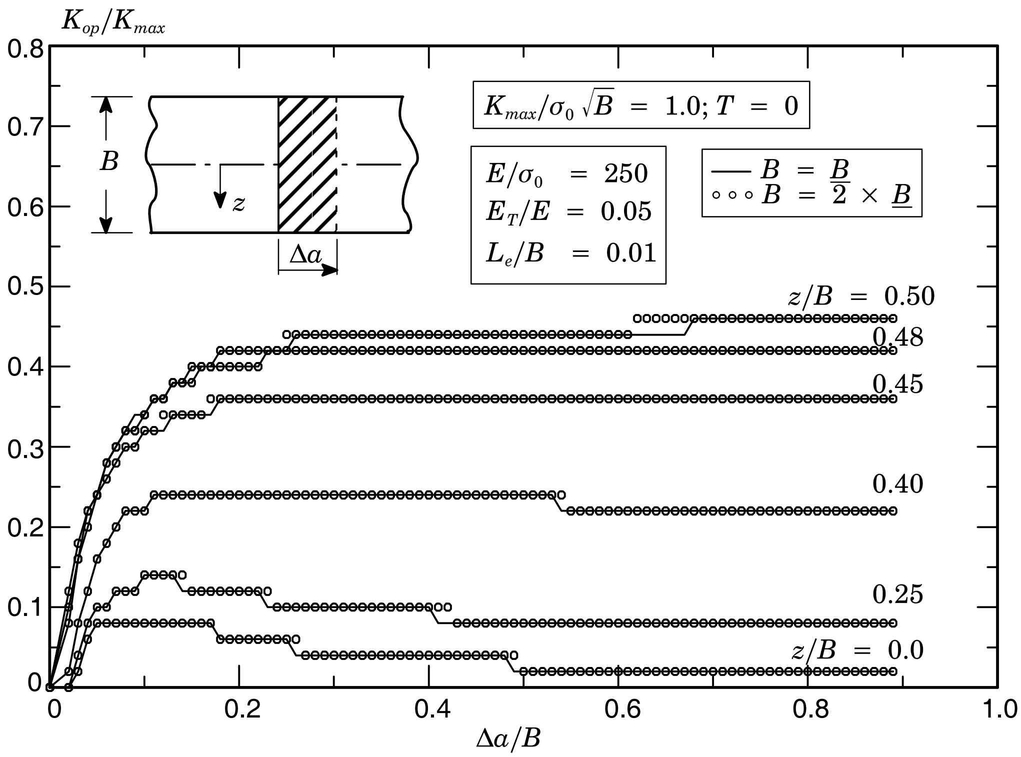

Figure 2: Demonstration of the similarity scaling of normalized opening load at each crack front location when specimens of different thickness are subject to same normalized load.

Figure 3 demonstrates the applicability of the non-dimensional scaling relationship for crack opening loads, Eq. (2), with non-zero T-stress loading. The figure shows the evolution of opening loads with crack extension for models with two different thickness, $B = \underline{B}$ and $B = 2 \times \underline{B}$, for two levels of T-stress ($T_{max}/\sigma_0 = \pm 0.8$) when the remote mode I loading scales with B to maintain $\overline{K} = 1.0$. For both of these (relatively) large values of positive and negative T-stress, the opening loads maintain the non-dimensional scaling over the complete loading history from the initial transient to steady-state conditions at crack extensions approaching the thickness.

A comparison of the crack opening loads in Fig. 3 with those in Fig. 2 (T-stress = 0) readily illustrates the strong effect of T-stress. Both positive and negative T-stress increase the opening loads along the interior of the crack front—negative T-stress has the larger effect (consistent with observations for the stationary crack). The mid-plane portion of the crack front now has a non-ambiguous opening load well above that for the zero T-stress loading. Opening loads near and at the outside surface show only a marginal effect for both the positive and negative T-stress.

5. CONCLUDING REMARKS

Under SSY conditions, the computational results demonstrate that the normalized value of the stress-intensity factor, K_{op}/K_{max}, when the crack opens at each location along the front remains unchanged provided the peak load (K_{max}), thickness (B) and material

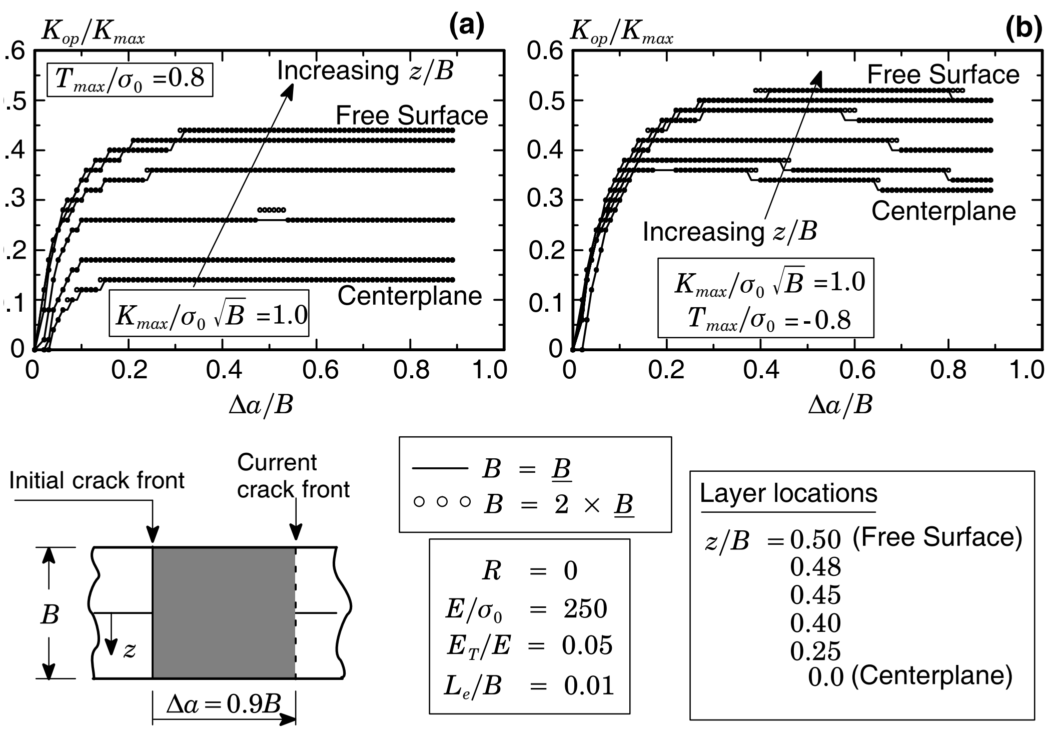

Figure 3: Demonstration of non-dimensional scaling of crack opening loads in the presence of a strong, non-zero T-stress. (a) positive T-stress, (b) negative T-stress.

flow stress (σ_0) all vary to maintain a fixed value of $\overline{K} = K_{max}/\sigma_0\sqrt{B}$. Numerical values of this similarity scaling factor, $\overline{K}$, thus provide a unique description of closure loads across all SSY configurations for a material. This similarity scaling holds both during the initial stages of growth, when opening loads vary with the amount of fatigue crack extension, and during steady-state response when K_{op}/K_{max} values remain constant with further growth.

Under SSY with a non-zero T-stress, a two parameter characterization of crack tip fields in terms of $\overline{K} = K_{max}/\sigma_0\sqrt{B}$ and $\overline{T} = T_{max}/\sigma_0$ correlates successfully the normalized opening load K_{op}/K_{max} across variations of thickness, constraint level and material flow properties. Specifically, the evolution of K_{op}/K_{max} with normalized crack growth $\Delta a/B$, at all locations along the 3-D crack front, remains unchanged when test specimens (and/or structures) experience the same normalized load $\overline{K}$ and the same normalized constraint level $\overline{T}$.

Both positive and negative deviations in T-stress from a zero value increase the crack opening loads along the mid-thickness region and reduce the through-thickness variation of K_{op}/K_{max}. This effect is more pronounced for negative T-stress and at the lower value of $\overline{K} = 1$, where the plastic zone ahead of the crack tip spreads to a distance $\sim 0.2 \times \mathrm{B}$ (under zero T-stress).

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