DAMAGE MODEL FOR FAILURE ANALYSIS WITH A VIEW TO HYDROMECHANICAL PROBLEMS

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ABSTRACT

In some cases, the evaluation of the leaking rate through a concrete structure is a crucial step. For nuclear power plants for example, the hydraulic integrity of the containment building is a point of concern for energy producers like EDF in France. The sealing of the barrier between the inner nuclear part and the outer environmental one has indeed to be ensured even in cases of strong accidents.

To estimate the durability of the structure in time (ageing process), the effect of a mechanical load on the transport properties, and especially on the hydraulic conductivity, has thus to be investigated. The main question is to quantify the evolution of the permeability with load and to determine which mechanical properties affect it.

From a combined mechanical – hydraulic discrete approach, based on lattice models, it is shown how, for pre – peak situations, the value of the damage is the most appropriate variable to quantify the evolution of permeability. It also proves, for unloaded specimens, the independence of the law on the material properties of concrete, proposing a theoretical explanation to some former experimental observations (Picandet *et* al. [1]).

From this study, the description of the damage evolution during loading becomes a crucial point. A misevaluation of this variable would indeed directly induce an error on the coupled permeability. The choice of the associated model has thus to be done carefully. For pre-peak simulations, continuum mechanics is the most adapted framework. Nevertheless, elastic damage models or elastic plastic constitutive laws are not totally sufficient to describe the concrete behaviour. They indeed fail to reproduce the unloading slopes during cyclic loads which define experimentally the value of the damage in the material. A combined elastic plastic damage model is thus proposed : damage is responsible for the decrease in the unloading slope (cracking) while plasticity reproduces the evolution of the irreversible strains. The constitutive relation is validated on a cyclic compression test. It gives the opportunity, in a view to hydromechanical problems, to highlight the interest of including plasticity if a misevaluation of the permeability needs to be avoided.

1 INTRODUCTION

Generally, the concrete structures designed for safety purposes, are monitored carefully to investigate the evolution of their integrity in time. In some particular cases (sensitive environments for example), the experimental investigation is difficult and the numerical simulation remains one of the only choices to evaluate the ageing process. For nuclear power plants for example, one has to guarantee the sealing of the containment building, even in cases of strong accidents. The question is thus to know which mechanical parameters may influence the global hydraulic problem and especially the value of the permeability. To answer this question, a discrete study, based on a lattice model, is first considered to specify the key parameter. Once it has been defined, an elastic plastic damage constitutive law is then proposed in a continuum mechanics approach. The aim is to develop an appropriate tool to determine accurately the experimental value of the damage in the material. It is applied to a cyclic compression test which gives the opportunity to discuss the necessity to include plasticity if a misevaluation of the permeability needs to be avoided.

2 LATTICE MODEL

A discrete approach is introduced to determine which properties affect the hydromechanical behaviour of concrete. It is based on the following hypothesis: a point of the continuum medium can be qualitatively represented by a lattice model of infinite size (see Delaplace *et* al. [2] for details).

The mechanical lattice is composed of different bonds which behave as a brittle material (figure 1a). An electrical analogous is used to solve the equations of the problem with the substitution of the strain ε by the voltage U_m, the stress σ by the current I_m and the Young modulus E by the conductance G_m. The bonds have a brittle behaviour with an initial conductance equal to one then equal to zero when the current in the bond reaches a critical value i_c (figure 2a). i_c is different from bond to bond and randomly distributed from 0 to 1.

When a mechanical bond is broken (conductance equal to one), it creates a flow path in the perpendicular direction. For this reason, the hydraulic problem is represented with a second lattice perpendicular to the mechanical one (figure 1b). A second analogous is considered where the voltage U_h (in the *h*ydraulic network) replaces the drop in pressure, the current I_h the flow rate Q and the conductance G_h the permeability K. When a given bond fails, its "permeability" increases from its initial value (equal to 1) to a final one (equal to 10^6) (figure 2b).

The boundary conditions of both problems are periodic to attain the creation of an infinite system and to partly avoid boundary effects. The calculation is done with the following sequence which is repeated until the failure of the complete lattice :

1. Application of a unit tension on the mechanical lattice
2. Determination of the weakest mechanical bond and decrease in its conductance
3. Increase in the permeability of the perpendicular hydraulic bond
4. Application of a unit tension on the hydraulic lattice

At the end of each step, mechanical and hydraulic moments of different orders are calculated with $M_{kx} = \int i_x^k N_x(i) di$ where *x* is *m* or *h*, *i* is the current in each bond, *N(i)* is the number of bonds whose current is in the range of [*i*, *i*+*di*] and *k* is the order of the moment.

The moments with an important meaning are of order up to 4. The zero order moment is the number of unbroken bonds. The first order is proportional to the average value of the current (analogous to the average stress or average flow rate). The second order is proportional to the overall conductance (analogous to the overall Young modulus or permeability). Finally, the fourth order is a measure of the dispersion of conductance. Details of the numerical resolution are provided in (Delaplace *et* al. [2] and Chatzigeorgiou [3]).

Using this approach, figure (3a) gives the permeability – damage evolution (computed from the overall conductance of the two lattices) for three sizes of the problem. As the relation is independent on the lattice size for pre – peak situations ($\frac{E_0 - E}{E_0}$ ranging from 0 to 0.25), it can represent the behaviour of the lattice of infinite size. The hydro – mechanical behaviour of a point of the continuum medium can thus be defined by a unique relation between damage and permeability.

Nevertheless, the lattice does not represent the real material. It gives qualitative information that has to be corroborate by some experiments. In our case, it gives a theoretical explanation for the experimental approach proposed in (Picandet *et* al [1]) in which the permeability of three concrete are related to their damage by a unique expression (figure 3b).

As a conclusion, for pre – peak simulations and unloaded material, damage is the most adapted variable to evaluate the load influence on the permeability.

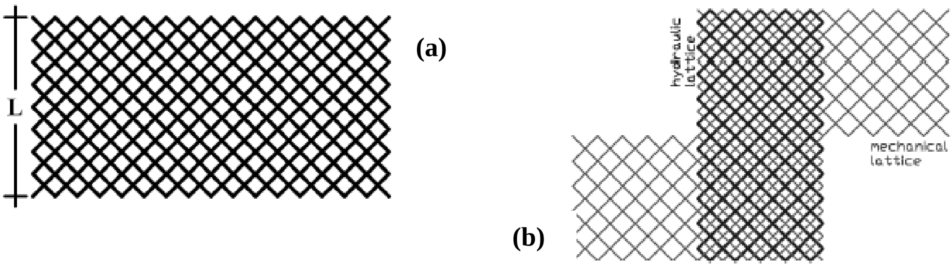

Figure 1: Mechanical (a) and hydraulic (b) lattices

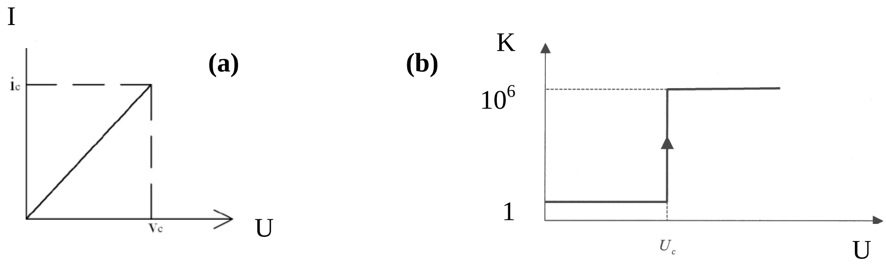

Figure 2: Brittle behaviour for the mechanical (a) and hydraulic (b) bond

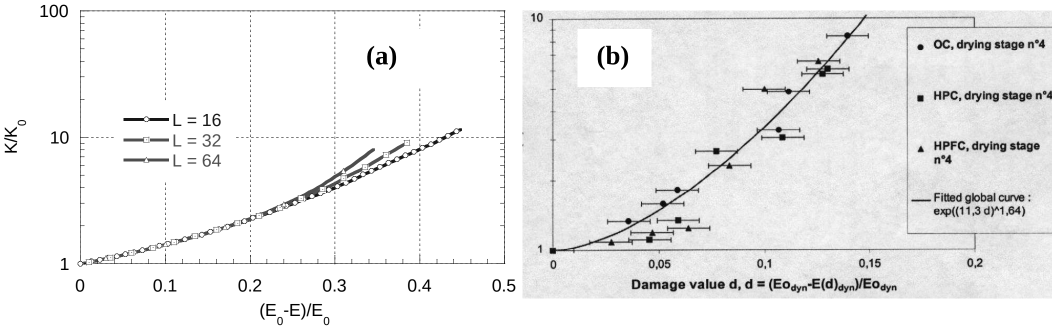

Figure 3: Damage – permeability law from lattice (a) and from experiment [1] (b)

3 ELASTIC PLASTIC DAMAGE FORMULATION

3.1. Needs for a combined approach

From the discrete study, the description of the damage evolution is a crucial step in hydro-mechanical problem. The constitutive law for the mechanical part has thus to be chosen carefully if a misevaluation of the permeability needs to be avoided. When continuum mechanics is considered, elastic damage models or elastic plastic laws are not totally sufficient to correctly capture the constitutive behaviour of concrete. They indeed fail to reproduce the unloading behaviour from which the experimental value of the damage is usually determined. An elastic damage model is not appropriate as irreversible strains cannot be captured. A zero stress corresponds to a zero strain and the value of the damage is thus overestimated. An elastic plastic relation is not adapted (even with softening) as the unloading curve follows the elastic slope. Another alternative consists thus in combining these two approaches to propose an elastic plastic damage law. The softening behavior and the decrease in the elastic modulus are so reproduced by the damage part while the plasticity effect accounts for the irreversible strains. With this formulation, experimental unloading can be simulated correctly. It is such a model which is presented in this contribution.

3.2. Model formulation

Plasticity effects and damage are both described by the formulation. Nevertheless, they are not entirely coupled. From the total strain tensor ε, an effective stress σ' is computed from plasticity equations. Then, with the elastic – plastic strain decomposition ($\varepsilon = \varepsilon^e + \varepsilon^p$), the damage variable D and the real stress σ are calculated.

3.2.1. Plastic yield surface

The plastic yield surface has been chosen to fulfil three main objectives. First, irreversible effects have to develop during loading. Then, the volumetric behaviour in compression has to be reproduced, especially the change from a contractant to a dilatant evolution. Finally, the brittle – ductile transition has to be simulated in confinement. For high hydrostatic pressures, plastic effects appear experimentally.

With these three conditions, the chosen yield surface depends on the three normalised stress invariant ($\overline{\rho}, \overline{\xi}, \theta$) and on one internal variable k_h ranging from 0 to 1 (definition of a limit surface for k_h equal to one).

$$\overline{\xi} = \frac{\sigma'_{ii}}{\sqrt{3} r_c} \quad \overline{\rho} = \frac{\sqrt{s'_{ij} s'_{ji}}}{r_c} \quad \theta = \frac{1}{3} \arcsin\left(-\frac{\sqrt{3}}{2} \frac{s'_{ij} s'_{jk} s'_{ki}}{(s'_{ij} s'_{ji})^{3/2}}\right) \tag{1}$$

with σ'_{ij} and s'_{ij} the effective and deviatoric stress components respectively. r_c is a parameter of the model. F is defined with three main functions $\hat{k}$ (hardening function), $\overline{\rho}_c$ (deviatoric invariant) and r (deviatoric shape function)

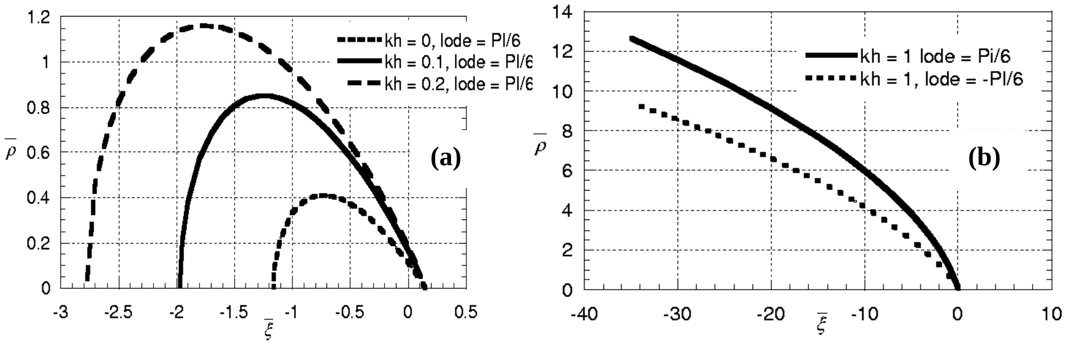

Figure 5: plastic yield surface with increasing hardening (a) and limit surfaces (b)

$$F = \overline{\rho}^2(\boldsymbol{\sigma}') - \frac{\hat{k}(\boldsymbol{\sigma}', k_h)\overline{\rho}_c^{\,2}(\boldsymbol{\sigma}')}{r^2(\boldsymbol{\sigma}')} \tag{2}$$

The classical equations of plasticity models are solved using an iterative algorithm based on a Newton Raphson scheme.

Figure (4a) shows the evolution of the yield surface with the hardening parameter in simple compression. Figure (4b) illustrates the limit surfaces for two values of θ. When the hardening parameter k_h reaches its critical level (equal to one), the yield surface becomes a failure one and does not evolve any more.

3.2.2. Damage model

The damage model used in this contribution was initially developed in (Mazars [4]). It describes the constitutive behavior of concrete by introducing a scalar variable D which quantifies the influence of microcracking. To describe the evolution of damage, an equivalent strain ε_{eq} is computed from the elastic strain tensor $\boldsymbol{\varepsilon}^e$.

$$\boldsymbol{\varepsilon}^e = E^{-1}\boldsymbol{\sigma}' \tag{3}$$

with E^{-1} the inverse of the elastic stiffness

$$\varepsilon_{eq} = \sqrt{\sum_{i=1}^{3}(<\varepsilon^e{}_i>_+)^2} \tag{4}$$

where $<\boldsymbol{\varepsilon}^e{}_i>$ are the positive principal values of the elastic strains.

The damage loading surface g is defined by :

$$g(\boldsymbol{\varepsilon}^e, D) = \tilde{d}(\boldsymbol{\varepsilon}^e) - D \tag{5}$$

where the damage D takes the maximum value reached by $\tilde{d}$ during the history of loading $D = Max_{/t}(\tilde{d}, 0)$. $\tilde{d}$ is computed from an evolution law that distinguishes tensile and compressive behaviors through two couples of scalars (α_t, D_t) for tension and (α_c, D_c) for compression.

$$\tilde{d}(\boldsymbol{\varepsilon}^e) = \alpha_t(\boldsymbol{\varepsilon}^e)D_t(\varepsilon_{eq}) + \alpha_c(\boldsymbol{\varepsilon}^e)D_c(\varepsilon_{eq}) \tag{6}$$

The definition of the different parameters can be found in [4]. The damage evolution conditions are finally given by the Kuhn – Tucker expression:

$$g \le 0, \quad \dot{D} \ge 0, \quad g\,\dot{D} = 0 \tag{7}$$

Once the damage has been computed, the “real” stress is determined using the equation :

$$\boldsymbol{\sigma} = (1-D)\boldsymbol{\sigma}' \tag{8}$$

3.3. Model validation

Cyclic compression is used to validate the model. Experimental results are taken from (Sinha *et* al., [5]).The numerical axial response is given in figure (6a). Damage induces the global softening behaviour while the plastic part reproduces quantitatively the evolution of the irreversible strains. Experimental and numerical unloading slopes are thus similar and the constitutive law provides appropriate values of the experimental damage. Figure (6b) illustrates the volumetric response. The introduction of plasticity induces a change from a contractant (negative volumetric strains) to a dilatant evolution, a phenomenon which is experimentally observed.

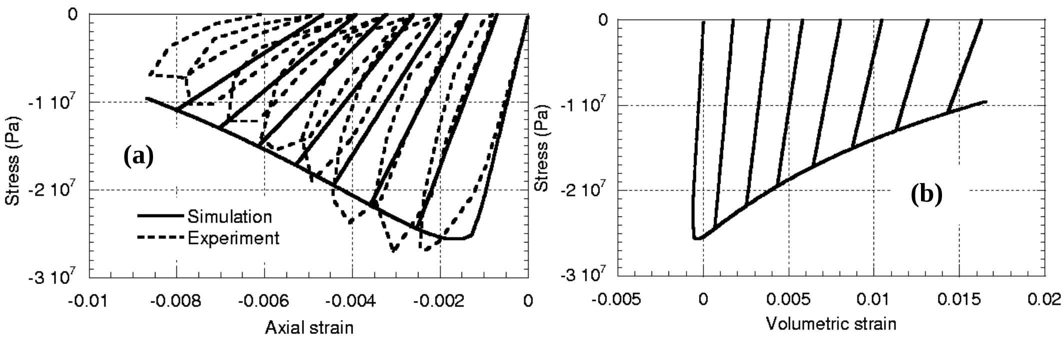

Figure 6:Cyclic compression test. Axial (a) and volumetric (b) responses.

The introduction of plasticity in the damage model plays thus a key role. Table 1 highlights the differences between the standard damage model (without plasticity) and the proposed constitutive law in term of damage and permeability estimations. Even if the global mechanical behaviour is the same (same stress for a given strain), the damage is different (see 3.1.) and may cause a strong misevaluation of the hydraulic conductivity (by a factor of 200).

	Stress (Pa)	Strain	Damage	**Permeability***
Standard damage	$-2.64\ 10^7$	-0.002	0.3641	**26000 K_0**
Damage + plast.	$-2.54\ 10^7$	-0.002	0.2327	**130 K_0**

Table 1: Comparison between the standard damage model and the elastic plastic damage law
* permeability calculation from the equation proposed in [1]

4 CONCLUSIONS

From a discrete lattice approach, it has been proven that the evolution of the experimental damage is one of the most significant parameter to take into account to evaluate the permeability in hydromechanical problems. To capture an appropriate value of this variable, an elastic plastic damage model has been proposed. Based on an isotropic damage model to describe the softening behavior and the decrease in the elastic slope, combined with a plastic yield surface for irreversible effects, it has been successfully applied on a cyclic compression application. Both axial and volumetric responses have been correctly simulated. Moreover, the importance of using a plastic – damage approach has been highlighted in a view to hydro mechanical problems : the evaluation of the permeability is strongly influenced by the value of the simulated damage and by the choice of the constitutive law. As a final conclusion, this elastic plastic damage approach may represent an appropriate tool to simulate the constitutive behaviour of concrete and may be used for coupled problems (see Jason *et* al. [6] for complementary applications)

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