PARTIAL CLOSURE ANALYSIS OF A SURFACE CRACK IN A GRADED MEDIUM LOADED BY A FRICTIONAL FLAT STAMP

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ABSTRACT

In this study the behavior of a surface crack in a graded isotropic elastic medium is examined under the effect of loading due to a frictional rigid flat stamp. The contact between the graded medium and the rigid stamp is assumed to transfer both normal and tangential forces which are related through Coulomb's friction law. The elastic modulus of the graded medium is assumed to increase exponentially in depth direction. The surface crack is oriented parallel to the material property gradation. The coupled crack and contact problem is formulated using transform techniques and reduced to a system of singular integral equations. The main emphasis is on the partial closure of the crack surfaces for small values of the friction coefficient at the material surface. The surfaces of the crack are assumed to be in smooth contact. This leads to a boundary value problem which is highly nonlinear in terms of the unknown length of the closed portion of the crack. An expansion – collocation technique is used to convert the integral equations to a system of linear algebraic equations and an iterative solution algorithm is developed to compute the length of the closed portion of the crack and the modified mode II stress intensity factors. It is found that for frictionless contact at the material surface, the surface crack is completely closed in both homogeneous and graded media. Some other sample results are also provided to present the effects of the material nonhomogeneity parameter and friction coefficient on the contact stresses, stress intensity factors and length of the closed portion of the crack.

1 INTRODUCTION

The contact of machine elements or structures may in certain cases lead to the initiation and further propagation of surface cracks in the contact zone. Hertzian fracture of brittle surfaces during indentation by a spherical indentor and formation of partial cone cracks due to sliding contact are two basic examples of fracture failures due to contact stresses (Lawn [1]). Fretting fatigue is another example of surface cracking failure resulting from the contact between a pair of machine elements, with high normal stresses and cyclic frictional forces. This failure is commonly observed in engineering applications like bolted or riveted joints, shrink – fitted shafts and blade dovetail regions of turbomachinery (Hattori et al. [2]). As a result, from the engineering point of view it is important to examine the behavior of surface cracks under the effect of contact stresses. In some previous studies, it is shown that, especially when the surface crack is at the edge of the contact area, the contact stress field is affected by the presence of the crack. In this type of problems, the presence of the crack also alters the singular behavior of the contact stresses (Dag and Erdogan [3]). Hence, in some certain cases a coupled formulation of the crack and contact problems is required in the computation of the mixed mode stress intensity factors. This study primarily deals with the coupled crack and contact problem in graded materials by considering the partial closure of the crack surfaces.

Graded materials, also known as functionally graded materials (FGMs) are generally metal/ceramic composites synthesized in such a way that the volume fractions of the constituents vary continuously in thickness direction to give a predetermined composition profile. FGMs proved to be highly effective when utilized as protective layers against tribological damage as shown by Suresh et al. [4]. Scratch tests performed on a graded alumina – glass FGM with 100%

glass on the surface and increasing stiffness in depth direction show that in the graded medium surface cracking may occur only under very high loads compared to homogeneous surfaces (Suresh and Mortensen [5]). Contact mechanics of graded or nonhomogeneous materials has been studied by various researchers in the past. Indentation of a graded half – space by a point force is examined by Giannakopoulos and Suresh [6]. A review of the analytical, computational and experimental results on the spherical indentation of a graded elastic half – space is given by Giannakopoulos [7]. The analysis of cracks under frictional contact stress fields in graded media is carried out by Dag and Erdogan [3].

The present study is an extension of the study by Dag and Erdogan [3]. It is shown by Dag and Erdogan [3] that, faces of a surface crack will generally be in contact for small values of the frictional tangential force acting at the material surface. In the present study, the formulation given by Dag and Erdogan [3] is extended in order to be able to compute the length of the portion of the crack that is under closure and the modified mode II stress intensity factors by assuming smooth contact of the crack surfaces. Once the contact of the crack surfaces is considered, the problem becomes highly nonlinear in terms of the unknown length of the closed portion of the crack. This problem is treated using an iterative solution algorithm. The problem is formulated using transform techniques and reduced to a system of coupled singular integral equations. The integral equations are solved numerically using an expansion – collocation technique. Numerical results are presented to examine the effects of the material nonhomogeneity parameter and the friction coefficient on the modified mode II stress intensity factors, length of the closed portion of the crack and the contact stresses.

2 FORMULATION AND SOLUTION OF THE COUPLED PROBLEM

The coupled crack and contact problem considered in this study is shown in Figure 1. The elastic medium is assumed to be isotropic and graded in the thickness direction and it is in frictional contact with a rigid flat stamp. The elastic parameters of the medium are expressed as:

$$\mu(x) = \mu_0 \exp(\gamma x), \ \kappa = \text{constant}, \tag{1}$$

where μ is the shear modulus, γ is a nonhomogeneity parameter, $\kappa = 3 - 4\nu$ for plane strain and $\kappa = (3-\nu)/(1+\nu)$ for generalized plane stress, ν being the Poisson's ratio. It is assumed that the dimesions of the graded medium are large in comparison with the dimensions of the surface crack and the contact area. Thus, the graded medium is approximated as a semi – infinite elastic solid. Coulomb's friction law is assumed to be valid. Hence, the normal and tangential forces transfered by the contact are given as P and ηP, respectively where η is the coefficient of friction. The contact area is assumed to extend from $y = 0$ to $y = b.$ The graded solid contains a surface crack of length d at the edge of the contact area at position $y = 0.$ In the study by Dag and Erdogan [3], the mode I stress intensity factors at the crack tip $x = d$ are shown to be negative especially for small values of the friction coefficient η and the normalized nonhomogeneity parameter γd . The implication of this finding is that, the surface crack is partially closed for small values of the nonhomogeneity parameter and the friction coefficient. Therefore, in the present study, the partial closure of the crack surfaces is taken into account. The crack surfaces are assumed to merge at the point of closure in the form of a cusp as shown in Figure 1. The contact between the crack surfaces is assumed to be frictionless. As a result, mode II deformation can freely occur in the closed portion of the crack.

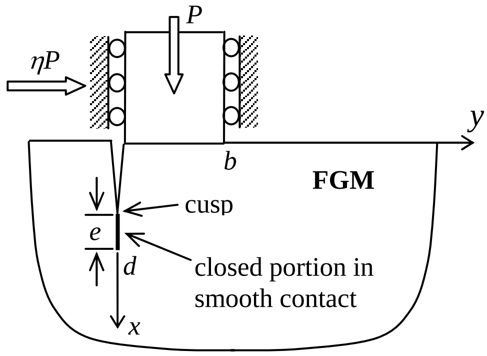

Figure 1: Partially closed surface crack in a graded medium loaded by a frictional rigid flat stamp

In Figure 1, e denotes the length of the portion of the crack that is closed. The surfaces of the crack between $x = d - e$ and $x = d$ are in smooth contact, hence $x = d - e$ is the point of closure. In the formulation of the problem, the following functions are used as the primary unknowns:

$$\frac{2\mu_0}{\kappa+1}\frac{\partial}{\partial x}\left[v\left(x,0^+\right)-v\left(x,0^-\right)\right]= f_1(x), \quad 0 < x < (d-e), \tag{2}$$

$$\frac{2\mu_0}{\kappa+1}\frac{\partial}{\partial x}\left[u\left(x,0^+\right)-u\left(x,0^-\right)\right]= f_2(x), \quad 0 < x < d , \tag{3}$$

$$\sigma_{xx}(0,y)= f_3(y), \quad 0 < y < b , \tag{4}$$

where u and v are displacement components in x and y directions, respectively. Note that normal relative displacements are zero in the closed portion of the crack. Relative tangential displacements on the other hand are nonzero for $0 < x < d$. The input functions of the problem are the crack surface tractions, stamp profile and the applied contact force which are given by

$$\sigma_{yy}(x,0)=0, \quad 0 < x < (d-e), \quad \sigma_{xy}(x,0)=0, \quad 0 < x < d , \tag{5}$$

$$\frac{\partial}{\partial y}u(0,y)=0, \quad 0 < y < b , \quad \int_0^b f_3(y)dy = -P. \tag{6}$$

Using the equations of elasticity and Fourier transform techniques, the problem is reduced to a system of three singular integral equations of the form

$$\int_0^{d-e}\left\{\frac{1}{\pi}\frac{1}{t-x}+h_{11}(x,t)\right\}f_1(t)dt+\int_0^b h_{13}(x,t)f_3(t)dt=0, \quad 0 < x < (d-e), \tag{7}$$

$$\int_0^{d}\left\{\frac{1}{\pi}\frac{1}{t-x}+h_{22}(x,t)\right\}f_2(t)dt+\int_0^b h_{23}(x,t)f_3(t)dt=0, \quad 0 < x < d , \tag{8}$$

$$\int_0^{d-e} h_{31}(y,t) f_1(t) dt + \int_0^{d} h_{32}(y,t) f_2(t) dt - \frac{1}{\pi}\int_0^{b} \frac{f_3(t)}{t-y} dt - \eta \frac{\kappa-1}{\kappa+1} f_3(y) + \int_0^{b} h_{33}(y,t) f_3(t) dt = 0 ,$$

$$0 < y < b . \qquad (9)$$

The details of the derivations and expressions of the kernels are given by Dag and Erdogan [3]. The terms h_{ij} contain both the generalized Cauchy and the Fredholm kernels. The singular behaviors of the unknown functions f_i, $(i = 1,2,3)$ at the end points of their respective integrals can be expressed as

$$f_1(x) = \phi_1(x)((d-e)-x)^{-1/2} x^{\alpha} , \quad 0 < x < (d-e) , \qquad (10)$$

$$f_2(x) = \phi_2(x)(d-x)^{-1/2} x^{\alpha} , \quad 0 < x < d , \qquad (11)$$

$$f_3(y) = \phi_3(y)(b-y)^{\beta} y^{\alpha} , \quad 0 < y < b , \qquad (12)$$

where $\phi_i(x)$, $(i = 1,2,3)$ are unknown bounded functions. The exponents α and β are functions of the Poisson's ratio and friction coefficient and are given by Dag and Erdogan [3]. Note that due to the smooth contact of the crack surfaces, the crack faces merge near $x = (d-e)$ and they take the shape of a cusp. Thus, $f_1(x)$ is equal to zero at this point. In the numerical solution however, first the square – root singularity is retained for $f_1(x)$ near $x = (d-e)$ as given by eqn (10). Then, the value of e is adjusted iteratively during the numerical computations until the mode I stress intensity factor at $x = (d-e)$ is equated to zero. Mode I stress intensity factor at $x = (d-e)$ and mode II stress intensity factor at $x = d$ are defined as

$$k_1(d-e) = \lim_{x \to (d-e)^+} \sqrt{2(x-(d-e))}\,\sigma_{yy}(x,0), \quad k_2(d) = \lim_{x \to d^+} \sqrt{2(x-d)}\,\sigma_{xy}(x,0). \qquad (13)$$

The integral equations given by (7) – (9) are solved by expanding the unknown functions into series of Jacobi polynomials. Using suitable collocation points, the singular integral equations are then converted to a system of linear algebraic equations which are solved for the unknown constants of the series expansions. The mixed mode stress intensity factors given by eqn (13) and contact stress distribution at the material surface are calculated using the computed values of the unknown constants. For each data point, multiple iterations are required to adjust the value of e such that the cusp condition, $k_1(d-e) = 0$ is satisfied. This iterative solution technique makes it possible to apply the numerical solution method developed by Dag and Erdogan [3] to the present partial crack closure problem in the graded medium.

3 RESULTS AND DISCUSSION

The primary results of this study are the contact stress distribution beneath the flat stamp, normalized length of the closed portion of the crack e/d and the mode II stress intensity factors at the crack tip $x = d$ in the graded medium. The results are provided in Figures 2 – 4. In all the numerical analyses conducted κ is assumed to be equal to 2. Figure 2a shows the contact stress distributions beneath the flat stamp calculated by taking into account the crack closure effects and calculated by assuming the crack as fully open. It is seen that closure of the crack surfaces affects the pressure distribution beneath the stamp. The normalized contact stress distribution for a friction coefficient of $\eta = 0.2$ is depicted in Figure 2b for various values of the normalized nonhomogeneity constant γd. It can be seen that the contact stresses are singular near the end points of the contact area. The effects of the material nonhomogeneity and friction coefficient on the length of the closed portion of the crack are shown in Figure 3. Examining the figures we first observe that e/d is independent of the magnitude of the force P applied to the contact. It depends on the geometry, friction coefficient and the nonhomogeneity constant γd.

In Figure 3a, e/d is plotted with respect to normalized contact length b/d for various values of the nonhomogeneity constant γd and for a friction coefficient of $\eta = 0.2$. We observe that in the homogeneous medium $(\gamma d = 0)$ the crack is almost completely closed. As the medium becomes slightly nonhomogeneous, that is for $\gamma d = 0.1$ and 0.5, the crack becomes partially open. It can be seen that for $\gamma d = 0.5$ and b/d > 5.7 the crack is fully open. Further increase in the material nonhomogeneity causes an increase in e/d. Hence, the degree of partial closure is strongly affected by the level of material nonhomogeneity in the graded medium as shown in Figure 3a.

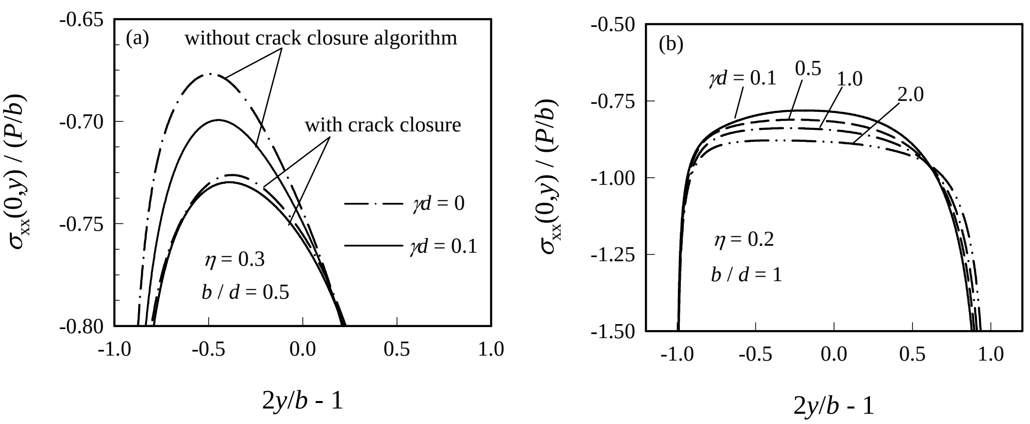

Figure 2 (a,b): Contact stress distribution at the surface of the graded medium

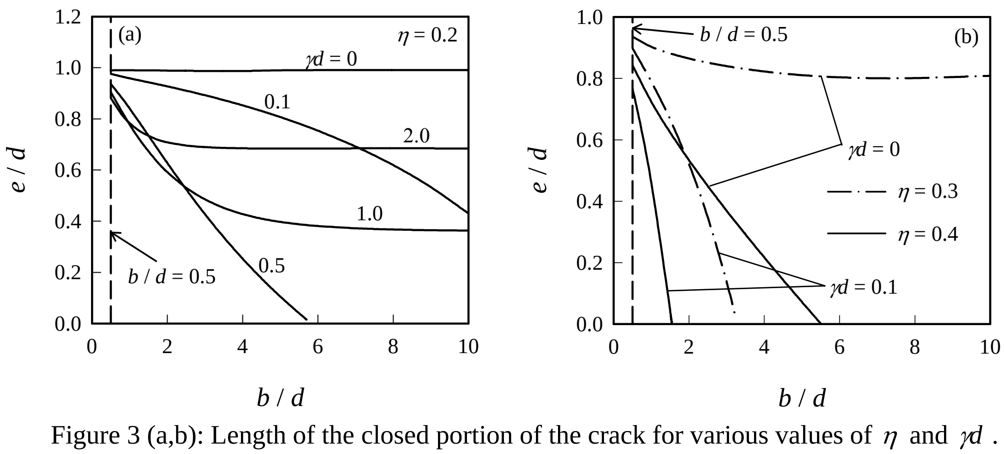

Figure 3 (a,b): Length of the closed portion of the crack for various values of η and γd.

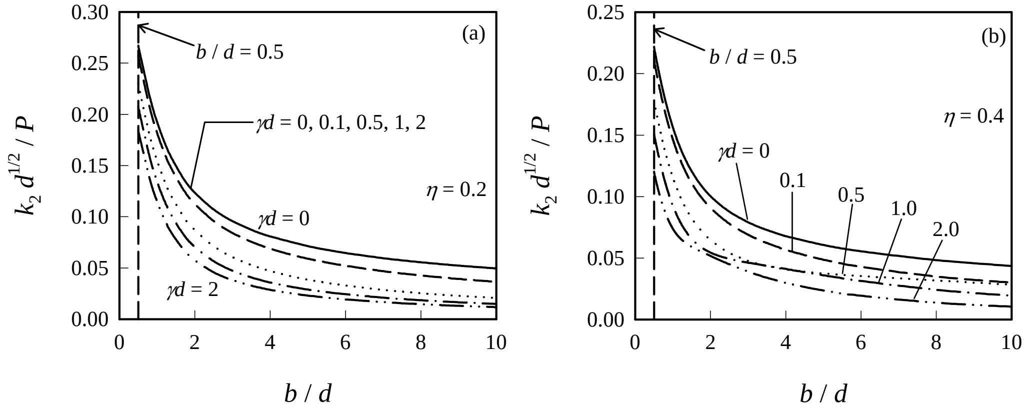

Figure 4 (a,b): Normalized mode II stress intensity factors at the crack tip $x = d$.

Figure 3b shows the variation of e/d for $\eta = 0.3$ and 0.4. The length of the closed portion of the crack decreases as the friction coefficient is increased. The normalized mode II stress intensity factors at the crack tip $x = d$ are given in Figures 4a and 4b for $\eta = 0.2$ and $\eta = 0.4$, respectively. The comparison of these results to the results provided by Dag and Erdogan [3] shows that the results for the mode II stress intensity factors computed by taking into account the crack closure effects are very close to the results computed by assuming the crack as fully open.

In this study, a method is developed to be able to investigate the closure of the surface cracks subjected to stamp loading in graded materials. Numerical analyses which are not presented in this study also showed that in frictionless contact conditions $(\eta = 0)$ the surface crack is completely closed in both homogeneous and graded media. For sufficiently large values of η, the crack is found to be fully open. The crack is only partially open for moderate values of the coefficient of friction $(0 < \eta < 0.4)$. For this range of the friction coefficient, sample results are provided in the present study to examine the behavior of the surface crack in the graded medium.

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