# WAVE SCATTERING AND THERMAL EFFECT OF CRACKED PIEZOELECTRIC MATERIALS

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## ABSTRACT

The scattering problems of plane wave by the interfacial crack between the piezoelectric layer and elastic half-space are studied. The dynamic stress intensity factors (DSIF) of the left and the right crack tip are derived, and the effects of the dimension of crack, the material combinations and the incident direction of waves are discussed. Then two kinds of thermal effects are investigated: due to the electrical saturation and electrical and/or mechanical impact loading. Based on the solutions of the dynamic impact of the piezoelectric materials, the thermal effects near the crack tip region are calculated under the assumption of decoupling approach between the thermal effect and mechanical-electrical coupling. Temperature fields of those two kinds of thermal effects are derived both analytically and numerically. Significant temperature rise in a small region near crack tip is deduced under electrical loading.

## 1. INTRODUCTION

Recently, the dynamic fracture of piezoelectric materials has attracted more and more attention because of the wide application, the demand of design and the loading environment of piezoelectric structures. As to scattering problems of wave, Narita and Shindo [1] have analyzed the scattering of Love wave to the edge crack in the piezoelectric layer bonded with infinite homogeneous medium. Wang and Yu [2], Gu and Yu [3] have investigated wave scattering of the cracked piezoelectric materials. In the present paper, the scattering problems of plane wave by the interfacial crack between the piezoelectric layer and elastic half-space are investigated. The dynamic stress intensity factors (DSIF) of the left and the right crack tip are derived, and the effects of the dimension of crack, the material combinations and the incident direction of wave are discussed.

In the second part of this paper, two kinds of thermal effects are investigated: due to electrical and/or mechanical impact loading and due to the electrical saturation. Fu Qiang and Zhang [5] found from the experiment on the fracture of piezoelectric material that the fracture toughness of piezoelectric medium under electrical loading for conductive crack is much higher than mechanical loading, and that the high temperature effect can be explicitly observed near crack tip region in the case of electrical loading. Du et al. [6] discovered that the heat generation rate in piezoelectric material is higher at higher electrical frequencies.

When a cracked piezoelectric medium is loaded electrically or mechanically or both, notable temperature rise is observed around the crack tip. In this paper, two important topics of the performance and mechanics for the piezoelectric materials are investigated. Firstly, using the integral transform method, the scattering problem of plane wave to the interfacial crack between a piezoelectric layer and a homogeneouse substract is investigated . The dynamic stress intensity factors under mode I, II of the left and the right crack tip are deduced. Based on the solutions of the dynamic impact of the piezoelectric materials, the thermal effects near the crack tip region are calculated under the assumption of decoupling approach between the thermal effect and mechanical-electrical coupling.

## 2. WAVE SCATTERING TO THE CRACK BETWEEN PIEZOELECTRIC MEDIUM AND SUBSTRATE OR MATRIX

### *2.1 Basic Equations*

For transversely isotropic piezoelectric medium and elastic medium, the linear constitutive equations read [6]:

$$\underline{\mathbf{s}} = \underline{\mathbf{c}}\cdot\underline{\mathbf{e}} - \underline{\mathbf{e}}\cdot\underline{\mathbf{E}}, \quad \underline{\mathbf{D}} = \underline{\mathbf{e}}^{\mathrm{T}}\cdot\underline{\mathbf{e}} + \underline{k}\cdot\underline{\mathbf{E}}, \quad \underline{\mathbf{s}}^{\mathrm{e}} = \underline{\mathbf{c}}^{\mathrm{e}}\cdot\underline{\mathbf{e}}^{\mathrm{e}}. \tag{1}$$

Then, the governing equation can be expressed as:

$$\nabla\cdot\underline{\mathbf{s}} = r\underline{\ddot{\mathbf{u}}}, \quad \nabla\cdot\underline{\mathbf{D}} = 0, \quad \nabla\cdot\underline{\mathbf{s}}^{\mathrm{e}} = r^{\mathrm{e}}\underline{\ddot{\mathbf{u}}}^{\mathrm{e}}. \tag{2}$$

where the superscript e denotes the quantities of elastic medium. $\underline{\mathbf{u}}$, $\underline{\mathbf{s}}$, $\underline{\mathbf{e}}$, $\underline{\mathbf{D}}$ and $\underline{\mathbf{E}}$ stand for the displacement, the stress, the strain, the electrical displacement and electrical intensity, respectively. $\underline{\mathbf{C}}$, $\underline{\mathbf{e}}$ and $\underline{k}$ are the elastic moduli, the piezoelectric and dielectric constants. $r$ is the mass density.

### *2.1 Scattering of Plane Wave*

Consider the configuration as shown in Fig. 1 and assume that a wave propagates from the elastic substrate with an angle $q$ of incidence with respect to the z-axis. As the same as the problem of Love wave, the stresses induced by the incident wave on the crack surfaces are obtained,

$$s^{c}_{xz}(x,t) = t_0(w,k)e^{i(kx-wt)}, \; s^{c}_{zz}(x,t) = s_0(w,k)e^{i(kx-wt)}, \quad (-c<x<c) \tag{3}$$

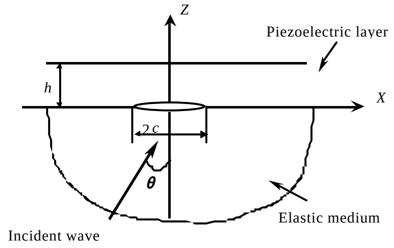

Figure 1: Crack configuration of the plane wave scattering.

Introducing the displacement potential $j$ and $y$, and then utilizing Fourier integral transform, the solutions of the scattering fields are expressed under electrical isolation boundary conditions. The Cauchy singular integral equations of the second kind can be derived from the isolated electric boundary conditions

$$\underline{\mathbf{A}}\,\underline{\mathbf{f}}(x) + \frac{1}{p}\int_{-c}^{c}\underline{\mathbf{B}}\frac{\underline{\mathbf{f}}(t)}{t-x}\mathrm{d}t + \frac{1}{p}\int_{-c}^{c}\underline{\mathbf{Q}}(x,t)\,\underline{\mathbf{f}}(t)\mathrm{d}t = \underline{\mathbf{L}}(x). \tag{4}$$

with the single value conditions $\int_{-c}^{c}\underline{\mathbf{f}}(t)\mathrm{d}t = 0$.

Finally, the DSIF of mode-I and mode-II at the left and right crack tips $K^L_I, K^L_{II}, K^R_I, K^R_{II}$ can be deduced. To illustrate the basic features of the solutions, numerical calculations have been carried out for two different material pairs, PZT-5H/Al and $BaTiO_3$/Al. Fig. 2 indicates that with the increase in the value of $c/h$, the maximal value of the mode-I DSIF increases for both the material combinations of $BaTiO_3$/Al and PZT-5H/Al when a P-wave is incident. It can also be seen from Fig. 2 that for $BaTiO_3$/Al and PZT-5H/Al, the maximal values and the resonant circular frequency of the DSIF are different though they exhibit the same changing tendency. This means that the DSIF may be impeded or accelerated by specifying different material combinations.

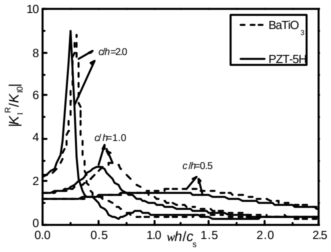

Figure 2: Normalized DSIF versus normalized $w$ for different material combinations under an incident P-wave ($BaTiO_3$/Al).

## 3. THERMAL EFFECTS IN CRACKED PIEZOELECTRIC MEDIUM CONSIDERING THE ELECTRICAL SATURATION OR ELECTRICAL IMPACT LOADING

### *3.1 Problem I: energy dissipation mechanism under strip-saturation model and an analytical solution of the temperature near crack tip region*

The *problem I* aim to investigate the thermal effect caused by nonlinear behavior of piezoelectric materials. To explain the experiments of effects of electrical field on indentation fracture, Gao, Zhang, and Tong [7] presented a strip saturation model which assumed an electrical yielding strip in front of the crack tip. In this paper we try to figure out the generation and dissipation mechanism of this part of energy and calculate the temperature field.

First consider an infinite piezoelectric plate with a crack of length $2a$. The polarization direction is parallel with $y$-axis. In the far field we load an electrical displacement $D_\infty$ along $y$-axis. There is an electrically yielding strip ahead of the crack tip suggested by Gao, Zhang, and Tong [7].

As a consequence of employing the electrical saturation model, the polarization energy stored in this electrical saturation zone will not increase after electrical displacement saturation. Hence, while the far field loading is increasing, some of the energy is dissipated in this strip to transform as thermal source in the thermal balance equation. This part of energy, on which our attention has focused, has raised the temperature around the crack tip and been very likely to influence on the fracture process. While the far field loading is subjected, we assume there is a distribution of heat source in the saturation strip, and then calculate the temperature field.

Referring to dielectric physics and dissipation theories, we introduce a distributed heat power along the saturation strip:

$$\dot{\Phi}=D_S\dot{E}_{c-a} \tag{5}$$

where $\dot{E}_{c-a}$ is the loading velocity of electrical filed in the saturation strip. We assume the electrical field in the strip is uniform along the y direction so it can be expressed as:

$$E_{c-a}=\frac{(f^{+}-f^{-})}{B} \tag{6}$$

where $f^{+}$ and $f^{-}$ are the electrical potential of the upper and lower face of saturation strip. $B$ is the width of the electrical saturation. Compared with Dugdale model, as a simple assumption, $B$ is equal to the thickness of the plate. Employing Green's Function the analytical expression of the temperature field can be derived

$$\begin{aligned} T(x,y,t) &= \frac{1}{4pl}\int_0^t \frac{1}{t-t}\mathrm{d}t \iint_\Omega \dot{\Phi}\exp[\frac{-rc[(a^2-x^2)+(b^2-y^2)]}{4l(t-t)}]\mathrm{d}a\,\mathrm{d}b \\ &+\frac{1}{4plt}\iint_\Omega T_0\exp[\frac{-rc[(a^2-x^2)+(b^2-y^2)]}{4l(t-t)}]\mathrm{d}a\,\mathrm{d}b \end{aligned} \tag{7}$$

In this part, the piezoelectric material PZT-5 as a sample is taken to perform some numerical calculations. The parameters of PZT-5 are adopted from Dunn [10]. In the numerical calculation, we take the density $r=7500kg/m^3$, heat capacity $c=100W/(m\cdot K)$, the thermal conductivity coefficient $l=300J/(kg\,K)$ and the initial temperature $T_0=300K$.

For expressing the intensity of heat resource we use the saturated electrical displacement $D_S$. The typical D-E curve of ferroelectric material can be simplified to a rectangle. This simplicity is reasonable because though the shape of the curve changes much, the area of the closed curve which indicates the total amount of dispassion energy changes very little. Then the Fig. 4(a) can be simplified to Fig.4 (b) and furthermore to Fig.4 (c). Thus the value of $D_S$ is determined from Fig. 4(c). This kind of simplicity is reasonable in expressing the dispassion energy.

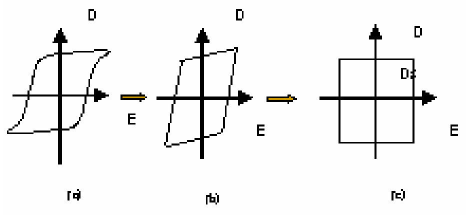

Figure 4: A simplification of electrical displacement versus electrical field.

Some researchers indicate that it is the electrical polarization $P$ but not the electrical displacement $D$ that reaches a saturated value (McMeeking [8]). Same reason as above, whether $P$ or $D$ saturates, the total amount of dispassion is almost similar in this quasi-quantitative investigation for the temperature rising near the crack tip region.

Fig.5 shows a sample of temperature distribution under electrical loading. The maximum temperature rising is 69 K.

### *3.2 Problem II: Temperature rising under impact loading*

The temperature rise is likely caused by many factors besides the nonlinear behavior of materials. Impact load is another reason induced the temperature rising. The thermal effects induced by both of electrical and mechanical impact are taken into account. It can be found in Gu and Yu [9].

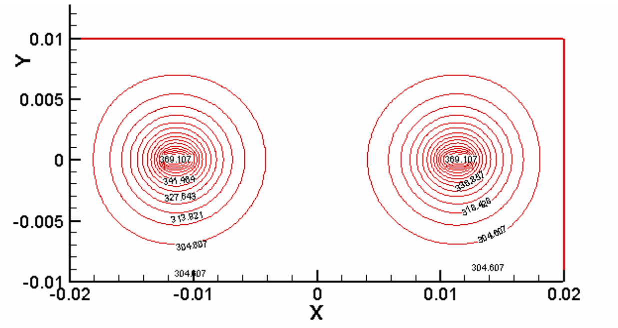

Figure 5: Temperature field with $a = 0.01m, B = 0.001m, D_y^\infty = 0.08C/m^2, \dot{D}_y^\infty = 8C/m^2s, t = 0.01s$.

To illustrate the basic features of the solution, numerical calculations for piezoelectric layer of $BaTiO_3$ have been carried out. The following material parameters were taken as [9]: $C_{44} = 4.4\times10^{10}\,\mathrm{N/m^2}$, $e_{15} = 11.4\,\mathrm{C/m^2}$, $k_{44} = 128.3\times10^{-10}\,\mathrm{C/Vm}$, $r = 5700\,\mathrm{kg/m^3}$, $z = 1$. Furthermore, the non-dimensional time $tv/c$ is used. It is noticed in the Fig. 6 that the distances in the $x$-axis direction origin from the crack tip. It is also found from Fig. 6 that the region of high temperature is quite small because the temperature rapidly decreases with the increasing distance in the $x$-axis.

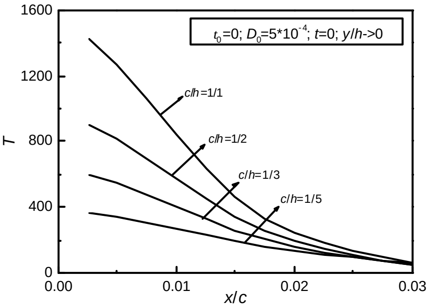

Figure 6: Temperature versus the distance along the $x$-axis for different value of $c/h$. ($c$ : crack length; $h$ : the thickness the piezoelectric material layer. )

## 4 CONCLUSION

The scattering problems of plane wave to an interfacial crack between a piezoelectric layer and homogeneous substrate are solved by means of the integral transform technique and the singular integral equations method. The DSIF of the left and the right crack tip are deduced.

The thermal effects and energy dissipation mechanisms in cracked piezoelectric medium are investigated. In problem I, the electrical saturation model of strip-style heat source is presented and the distribution of it in the electrical saturation strip is given. In problem II the anti-plane problem of thermal e?ect of piezoelectric material with a crack subjected to the electrical and mechanical impact loading is solved. Furthermore the temperature fields due to the energy dissipation mechanisms of electrical saturation and of impact loading are calculated.

## ACKNOWLEDGEMENTS

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