RECENT RESULTS ON DUCTILE FRACTURE MODELING AT THE MACRO AND MICROSCALES

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ABSTRACT

Ductile fracture modeling still has to be improved at the macro and microscales. New numerical methods are applied to the limit analysis of cells containing one or several voids or void clusters. The results are compared with existing macroscopic models. Voids can also be introduced in analytical polycrystalline models, in order to combine plastic anisotropy and texture evolution with ductile fracture finite element analyses.

1 INTRODUCTION

Macroscopic models developed at the microscale (Gurson [1]) and phenomenologically extended, the so-called Gurson-Tvergaard-Needleman model (GTN), are widely used with the finite element method (FEM) in ductile fracture analyses of specimens and structures. The Gurson problem has been revisited with recent progress in numerical limit analysis (LA) of a cell containing a single void (section 2). Void distributions in real materials are not uniform, and void coalescence can initiate between neighboring voids or in void clusters. The effects of these distributions have been investigated with a new numerical method based on the Fourier transform (section 3). Finally, the combination of a macroscopic ductile fracture model with a polycrystalline model aims at improving the modeling of plastic anisotropy and complex loading paths (section 4).

2 LIMIT ANALYSIS OF A SINGLE VOID

The problem of limit analysis of a cylindrical or spherical cell containing a single void has been studied by Gurson [1]. He used a kinematic approach, with an analytical displacement field depending on 2 parameters. The plastic domain of a cylindrical cell in plane strain is approximated by:

$$\left(1+3f+24f^6\right)^2\frac{\Sigma_{eq}^2}{3k^2}+2f\cosh\left(\frac{\Sigma_h}{k}\right)=1+f^2 \quad , \qquad \Sigma_h=\frac{\Sigma_x+\Sigma_y}{2} \tag{1}$$

where k is the shear flow stress of the rigid perfectly plastic von Mises matrix material.

In order to better approximate the effective plastic domain, it is necessary to combine both *kinematic* (external) and *static* (internal) approaches, and to consider more general displacement and stress fields than the simple analytical field of Gurson. FEM generates such fields, provided the mesh is refined enough. Due to the local yield criterion in plane strain:

$$\left(\sigma_x-\sigma_y\right)^2+4\sigma_{xy}^2=4k^2 \tag{2}$$

both approaches lead to non linear optimization problems. Two recent developments have made these problems numerically tractable. First the use of the recent high performance interior point optimization code XA. Secondly the polyhedral linearization of eqn (2) with the help of the Ben-Tal and Nemirovski algorithm [2], that replaces 2^n inequalities by a system with $3n+1$ rows and $2n$ columns. Otherwise, with $n=10$ in the following, the linearization of eqn (2) with $2^n=1024$ inequalities leads to fatal numerical difficulties.

The boundary conditions of the cylindrical cell are uniform strains: $\underline{u} = \underline{\underline{E}}.\underline{x}$. The void volume fraction is $f = 0.16$. The FEM mesh of a quarter of the cell section is made of 672 linear triangular elements. Any side of the triangles is a potential *discontinuity line* of the displacement or stress fields. Without this possibility, there is no solution in most cases of loading. This is a major difference with the analytical field of Gurson. The results are shown in fig 1 (Pastor [3], [4]). For each value of the loading parameter Σ_h / k , the external and internal approaches give close points. The effective plastic domain lies between the two curves. The Gurson criterion in plane strain (1) is erroneous. Moreover, the effective domain has a finite slope at $\Sigma_{eq} = 0$. The macroscopic fields obey the normality rule in fig 1 (true loading parameters). Consequently, the equivalent strain rate $\dot{E}_{eq}$ is not equal to zero close to the Σ_h axis. This is an important property for a macroscopic yield criterion in the final coalescence stage of ductile fracture, with 2 rigid parts separated by a localization region, hence $\dot{E}^p_{xx} = \dot{E}^p_{zz} = 0$ and $\dot{E}^p_{eq} = 2\dot{E}^p_{yy} / 3 = 2\dot{E}^p_m \neq 0$.

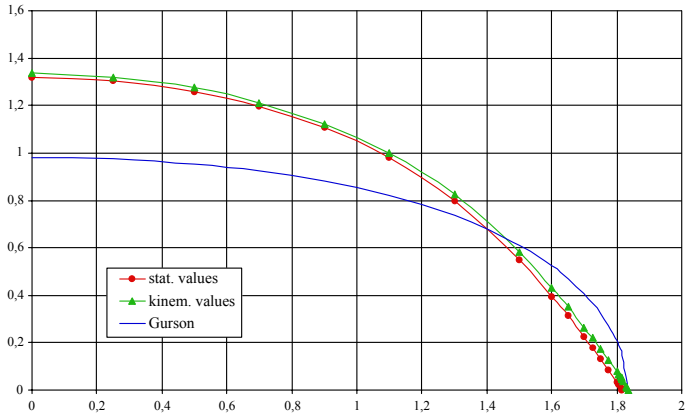

Figure 1: plastic domains in plane $\Sigma_{eq} / k - \Sigma_h / k$ (cylindrical void in plane strain, $f = 0.16$).

The FEM has been applied to a truncated cylindrical cell in generalized plane strain (GPS) (Pastor [4]). In GPS, 3 loading parameters are considered, with $\Sigma^2_{eq} = \Sigma^2_{ps} + \Sigma^2_{gps}$:

$$\Sigma_{gps} = \left(\Sigma_x + \Sigma_y\right)/2 - \Sigma_z \quad , \quad \Sigma_{ps} = \left(\Sigma_x - \Sigma_y\right)\sqrt{3}/2 \quad , \quad \Sigma_h = \left(\Sigma_x + \Sigma_y\right)/2 \qquad (3)$$

and the plastic domain cannot be approximated by a 2-parameters analytical expression like eqn (1), see fig 2. The FEM has also been applied to a spherical void, with axisymmetrical and 3D boundary conditions ("true 3D problem", to be published).

The interest of combined LA and FEM analyses has been demonstrated. Nevertheless, these improved solutions of the Gurson problem are still far from the real ductile fracture problem, regarding the effects of void shape, plastic hardening, plastic anisotropy, void interactions, void coalescence. In the next section, recent results on void interaction are given.

3 FAST FOURIER TRANSFORM ANALYSIS OF INTERACTING VOIDS

FEM analyses of cells containing several voids are prohibitively time-consuming. A numerical method based on fast Fourier transform (FFT) is dramatically more efficient (Michel [5]).

A geometrically periodic cell is considered, with periodic boundary conditions. Unlike FEM, there is no rigidity matrix but a non linear equation to be resolved at each pixel (2D) or voxel (3D) of the cell. Three examples of 2D cells are shown in fig 3 (1215x1215 pixels). They have been calculated with the GPS hypothesis in deformation theory of plasticity (von Mises, no hardening). Similar 3D cells have been calculated with 64x64x64 voxels, but the void distributions are not shown in this short paper (Bilger [6]). Two loading conditions with 2 loading parameters are considered, A: $\Sigma_x = \Sigma_y = \Sigma_z$, Σ_{xy} and B: $\Sigma_x = \Sigma_z$, Σ_y, $\Sigma_{xy} = 0$, z being the direction of the cylindrical voids in 2D. The strain localization bands for $\Sigma_{eq} = 0$ are shown in fig 3. The cells with clusters are softer than the cell without cluster, and the connected clusters are softer than the

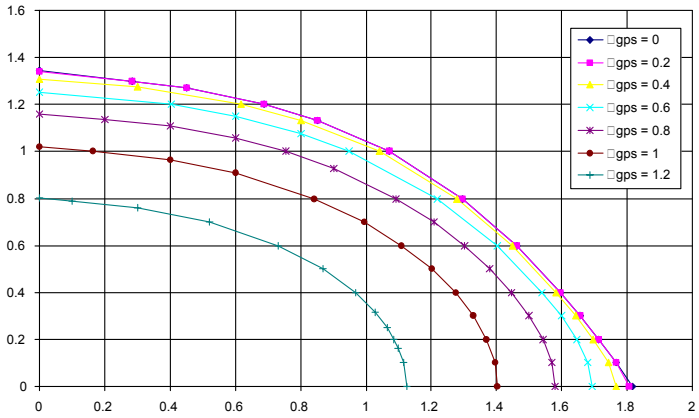

Figure 2: kinematic plastic domains in plane $\Sigma_{eq}/k - \Sigma_h/k$ depending on the loading parameter Σ_{gps}/k (cylindrical void in generalized plane strain, $f = 0.16$).

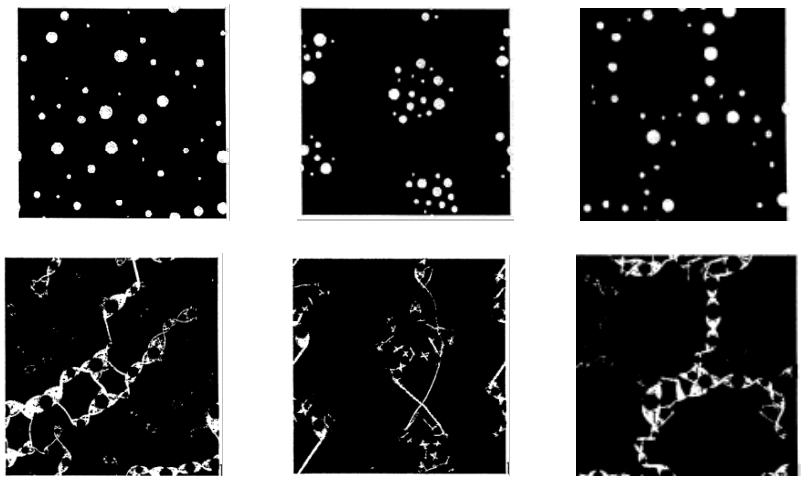

Figure 3: cells with random void distribution, isolated and connected void clusters (*top*), corresponding strain localization bands (*bottom*), $f = 0.046$.

isolated clusters. The plastic domains of fig 4, obtained with voids of identical size, depend on both void distributions and loading conditions. In the 3D case at least, the slope at $\Sigma_{eq} = 0$ is infinite, as predicted by Gurson. The cells are softer than the GTN model with the classical value $q = 1.5$. The differences between loadings A and B demonstrate an effect of the third invariant of the stress tensor, decreasing with increasing stress triaxiality. The main limitation of this constant geometry approach is that void growth and coalescence are not taken into account.

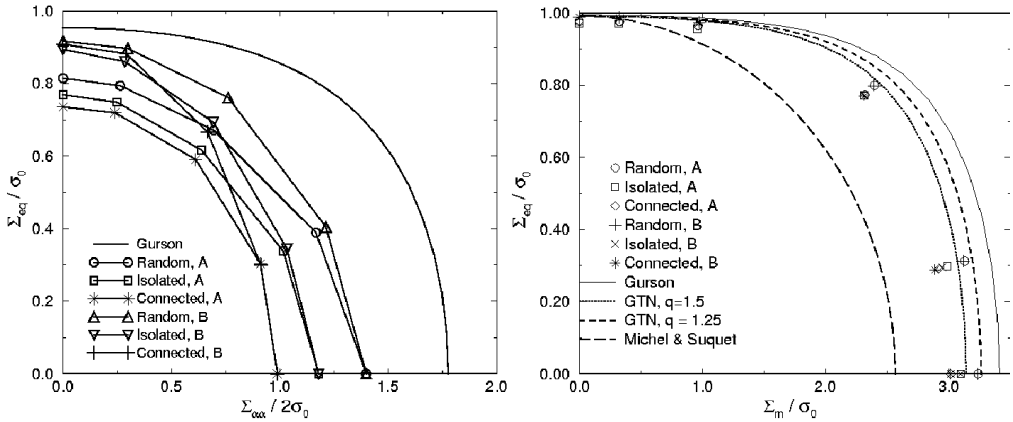

Figure 4: plastic domains of the 2D cells of fig 3 ($f = 0.046$, *left*) and of 3D cells ($f = 0.006$, *right*) for loading conditions A and B.

4 A COMBINED POLYCRYSTALLINE AND DUCTILE FRACTURE MODEL

Until simple homogenization models for porous media are available, it can be useful for FEM analyses to combine existing macroscopic models with classical polycrystalline models. We consider an additional "grain" or "void" (index $g = 0$) with variable volume fraction f and purely volumetric plastic strain $\underline{\underline{\varepsilon}}_0^p = \varepsilon_m^p \underline{\underline{1}}$. It represents the physical voids but it is *not* a void in the sense of homogenization theories. The volume fractions of the classical "grains" $g = 1$ to N, representing the crystallographic orientations, are $(1-f)f_g$. The usual homogenization equations apply:

$$\underline{\underline{\dot{E}}}^p = (1-f)\sum_{g=1}^{N} f_g \underline{\underline{\dot{\varepsilon}}}_g^p + f\underline{\underline{\dot{\varepsilon}}}_0^p = \underline{\underline{\dot{E}}}_{dev}^p + \dot{E}_m^p \underline{\underline{1}} \quad , \quad \underline{\underline{\Sigma}} = (1-f)\sum_{g=1}^{N} f_g \underline{\underline{\sigma}}_g + f\underline{\underline{\sigma}}_0 \qquad (4)$$

The local stress tensor $\underline{\underline{\sigma}}_0$ is *not* equal to 0. It is chosen proportional to $\underline{\underline{\Sigma}}$, in order to obtain the following yield condition in plane $\Sigma_{eq} - \Sigma_m = \Sigma_{ii}/3$:

$$\frac{\Sigma_{eq}}{1-f} - \left(\sum_{g=1}^{N} f_g \underline{\underline{\sigma}}_g \right)_{eq} + D_1 f \sigma_1 \exp\left(\frac{\Sigma_m}{\rho \sigma_1} \right) = 0 \qquad (5)$$

Eqn (5) corresponds to the model of Rousselier [7]. The Gurson model cannot be combined with polycrystalline models in such a simple way, as its yield criterion is not linear in Σ_{eq}. Local stress

tensors $\underline{\underline{\sigma}}_g$ in the matrix "grains" are given by classical "localization equations" (Berveiller [8]), with an additional term for them to be compatible with eqn (4). If we suppose this term to be identical for all matrix "grains", we obtain the generalized localization equation:

$$\underline{\underline{\sigma}}_g = \frac{1}{1-f}\underline{\underline{\Sigma}} + 2\mu(1-\beta)\alpha\left(\sum_{h=1}^{N}\left(f_h \underline{\underline{\varepsilon}}_h^p\right) - \underline{\underline{\varepsilon}}_g^p\right) + D_1 f \sigma_1 \exp\left(\frac{\Sigma_m}{\rho\sigma_1}\right)\frac{\underline{\underline{\Sigma}}}{\Sigma_{eq}} \qquad (6)$$

The additional term represents the overstress in the matrix "grains" due to the "voids". The set of equations is completed as in the Rousselier model:

$$\dot{f} = 3(1-f)\dot{E}_m^p \quad , \quad \dot{E}_m^p = f\dot{\varepsilon}_m^p = \frac{1}{3}\dot{E}_{eq}^p D_1 f \exp\left(\frac{\Sigma_m}{\rho\sigma_1}\right) \qquad (7)$$

Note that for $\Sigma_{eq} = 0$ we have $\dot{E}_{eq}^p = 2\dot{E}_m^p$ if $\sigma_1 = 2\Sigma_{flow}/3$.

The model has been applied to a zirconium alloy. This highly textured material is modeled with 6 "grains" only ($N = 6$), and the hexagonal close-packed crystal mechanical behavior is obtained with 6 slip systems only (Rousselier [9]). Without these simplifications, FEM computation times are prohibitive. The plastic behavior has been calibrated with 3 tests on tubes: tensile, tensile-pressure and torsion. The ductile model parameters are $f_0 = 10^{-4}$, $D_1 = 2$ and $\sigma_1 = 400$ MPa. The material is loaded in the direction θ with $\dot{E}_{\theta\theta} = 6.10^{-4} s^{-1}$, and identical conditions are imposed in the directions r and z of the material in order to obtain various triaxiality ratios Σ_m / Σ_{eq}. The plastic domain is given in fig 5 for $f = 0.01$. The local stress-strain curves in the matrix "grains" are shown in fig 5 for $\Sigma_m / \Sigma_{eq} = 1.5$. The effect of the overstress due to the "voids" is evidenced in the figure for the "grain" with the largest deformation. Note that due to the very large strains at the final stage of the loading, the stress heterogeneity between the matrix "grains" is smaller than 50 MPa.

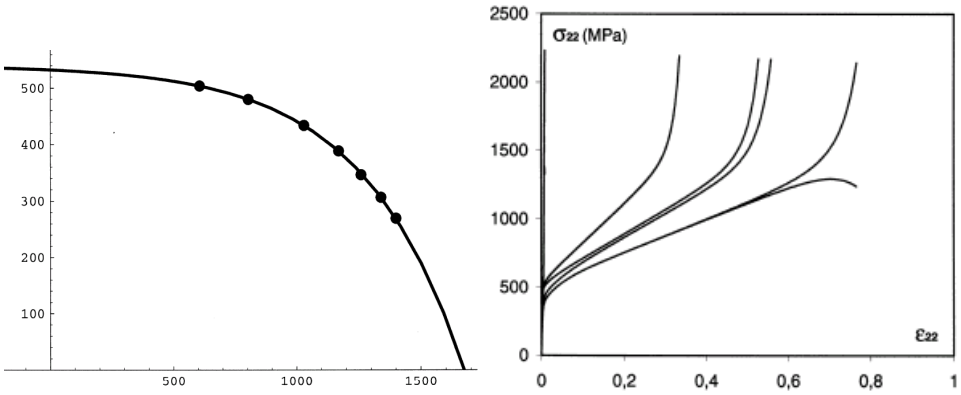

Figure 5: plastic domain in plane $\Sigma_{eq} - \Sigma_m$ ($f = 0.01$, *left*), local stress-strain curves in the 6 matrix "grains" ($\sigma_{\theta\theta} - \varepsilon_{\theta\theta}$, $\Sigma_m / \Sigma_{eq} = 1.5$, 2 curves are identical, *right*); for one "grain", the curve without the additional stress due to the "voids" is given.

5 CONCLUSIONS

Ductile fracture modeling is a difficult task, if we consider real void distributions and shapes, anisotropic plastic hardening, void interactions and plastic instabilities, void initiation, growth and coalescence. Since the pioneering work of Gurson, many authors have improved the micromechanical modeling of media with voids. In this paper, we have presented recent results obtained thanks to new numerical methods:

- high performance optimization codes combined with efficient linearization algorithms and the finite element method,
- fast Fourier transform analyses of simulated void distributions.

The results support the Gurson and GTN models in some situations and give ways of improvement in other cases. More systematic calculations are necessary in order to derive new macroscopic models.

The micromechanical approach of polycrystalline materials has been generalized recently to include void phases (see for example Lebensohn [10]). These models are complicated and time-consuming, and still need a phenomenological calibration. We have proposed in this paper a simple combination of a classical polycrystalline approach with a macroscopic ductile fracture model. With some reductions of the numbers of phases and slip systems, it enables finite element calculations with reasonable computation times while preserving the advantages of polycrystalline models as compared to macroscopic plastic models.

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