CALCULATION OF STRESS INTENSITY FACTORS FOR A CRACK ALONG THE ±45° INTERFACE OF A FIBER REINFORCED MATERIAL

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ABSTRACT

The three-dimensional $M-$integral is presented for calculating stress intensity factors for an interface crack between two fiber reinforced materials which are treated effectively as transversely isotropic materials. The material in the upper half-space is in the $+45°$-direction, whereas the material in the lower half-space is in the $-45°$-direction. An interface crack in this material possesses oscillatory and square-root singularities.

A test case of a three-dimensional bimaterial slab containing an edge interface crack is described. The first term of the asymptotic displacement field is applied with the complex stress intensity factor $K = 0$ and mode II stress intensity factor $K_{II} = 1$ on the outer boundaries of the body. The crack faces are traction free. Solution to this problem, should be precisely the imposed stress intensity factor. Excellent results are obtained. Other test problems, not presented here, also produce excellent results.

1 INTRODUCTION

The three-dimensional $M-$integral is extended here for an interface between two fiber reinforced materials which are treated effectively to be transversely isotropic. The material in the upper half-space has fibers in the $+45°$-direction; whereas in the lower material, they are in the $-45°$-direction (see Fig. 1). It is assumed that the interface crack is along the x_1-axis, the crack front is along the x_3-axis and the materials are symmetric with respect to the $x_2 = 0$ plane.

The two-dimensional $M-$integral for obtaining stress intensity factors for an interface crack between two isotropic materials was first presented by Wang and Yau [1]. It was extended to three-dimensions by Nakamura and Parks [2]. Of course, for that derivation, they required an extension of the $J-$integral for three dimensions which may be found in Li, et al. [3] and Shih, et al. [4]. Other applications of the $M-$integral were presented by Charalambides and Zhang [5] for an interface crack between two orthotropic materials when crack and material coordinates coincide and Banks-Sills and Boniface [6] for two transversely isotropic materials similar to the material treated in this study, but when the crack is along a $0°/90°$ interface. Both of these cases may be approached as two-dimensional problems.

In Section 2, the $M-$integral is extended from the three-dimensional $J-$integral for a crack along the $\pm 45°$ interface. Stress intensity factors are calculated for a test case and presented in Section 3.

2. THREE-DIMENSIONAL M-INTEGRAL

For the $M-$integral, auxiliary solutions are required. These are the asymptotic stress and displacement fields which were obtained by Banks-Sills, et al. [7]. The Stroh [8] formalism was employed to determine these fields. Three singularities were determined: two complex conjugate ones, $-1/2 \pm i\varepsilon$, which lead to oscillatory stress behavior and a real one equal to

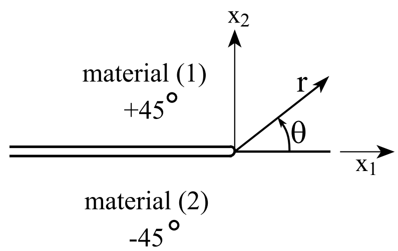

Figure 1: Crack tip coordinates.

$-1/2$. The variable ε is the oscillatory parameter and depends upon material properties; $i = \sqrt{-1}$. The complex stress intensity factor $K = K_1 + iK_3$ is the amplitude of the complex singularity and K_{II} is the amplitude of the square-root singularity.

To proceed, Griffith's energy may be derived for this material pair from the crack closure integral as

$$\mathcal{G} = \frac{\operatorname{sgn}(E_{22})\,|E_{22}|}{2\cosh^2 \pi\varepsilon}\left(K_1^2 + K_3^2\right) + \frac{\operatorname{sgn}(E_{11})\,|E_{11}|}{2} K_{II}^2 \tag{1}$$

where E_{11} and E_{22} depend upon mechanical properties of the material; expressions for these quantities are given in Banks-Sills, et al. [7] and sgn represents the sign of the variable in parentheses.

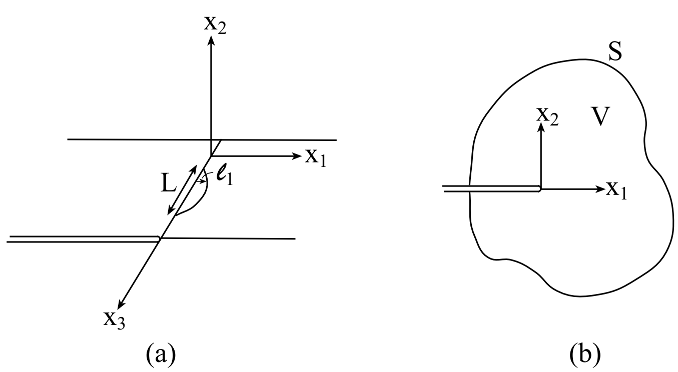

Figure 2: (a) Virtual crack extension ℓ_1 for a through crack. (b) In-plane, cross-section of volume V and outer surface S.

For a straight through crack which is treated in this study, the three-dimensional volume J−integral is given by

$$\int_0^L \mathcal{G}(x_3)\,\ell_1(x_3)\,dx_3 = \int_V \left[\sigma_{ij}\frac{\partial u_i}{\partial x_1} - W\delta_{1j}\right]\frac{\partial q_1}{\partial x_j}\,dV \tag{2}$$

where L is the length of the normalized virtual crack extension $\ell_1(x_3)$ along the crack front (see Fig. 2a), σ_{ij}, ϵ_{ij} and u_i are the stress, strain and displacement components. The strain energy density is $W = 1/2\sigma_{ij}\epsilon_{ij}$ and δ_{ij} is the Kronecker delta. The volume V reaches from the crack tip to an outer surface S. Taking the integral from the crack tip ensures path independence. On S, q_1 is zero; it takes on the value $\ell_1(x_3)$ along the crack front; it is continuously differentiable in V. The requirements for q_1 guarantee the validity of the right hand side of eqn (2). Since the integrand of the right hand side of eqn (2) is reminiscent of the $J-$integral, it is denoted as such. Details in choosing q_1 for obtaining accurate results may be found in Banks-Sills and Sherman [9].

The right hand side of eqn (2) may be obtained accurately along the crack front. By assuming that $\mathcal{G}$ is a constant for length L as illustrated in Fig. 2, this allows one to determine the combined value of the stress intensity factors given in eqn (1). In order to obtain the individual stress intensity factors, the $M-$integral is extended here for the $\pm 45°$ pair.

To this end, the method of superposition is employed to propose two solutions, namely,

$$K_1 = K_1^{(1)} + K_1^{(2)} , \tag{3}$$
$$K_{II} = K_{II}^{(1)} + K_{II}^{(2)} , \tag{4}$$
$$K_3 = K_3^{(1)} + K_3^{(2)} , \tag{5}$$
$$u_i = u_i^{(1)} + u_i^{(2)} , \tag{6}$$
$$\epsilon_{ij} = \epsilon_{ij}^{(1)} + \epsilon_{ij}^{(2)} , \tag{7}$$
$$\sigma_{ij} = \sigma_{ij}^{(1)} + \sigma_{ij}^{(2)} . \tag{8}$$

Solution (1) is the sought after solution; the fields are obtained by means of a finite element calculation. Solution (2) consists of three auxiliary solutions which are derived from the first term of the asymptotic solution for the displacements which are given by Banks-Sills, et al. [7]. The stress intensity factors of solution ($2a$) are given by

$$K_1^{(2a)} = 1 , \qquad K_{II}^{(2a)} = 0 , \qquad K_3^{(2a)} = 0 . \tag{9}$$

The relations in (9) are substituted into the expresssions for the displacements. These are differentiated to obtain the strains and Hooke's law is employed to determine the stresses. For solution ($2b$)

$$K_1^{(2b)} = 0 , \qquad K_{II}^{(2b)} = 1 , \qquad K_3^{(2b)} = 0 . \tag{10}$$

In a similar manner the fields for solution ($2b$) are found. Finally,

$$K_1^{(2c)} = 0 , \qquad K_{II}^{(2c)} = 0 , \qquad K_3^{(2c)} = 1 . \tag{11}$$

Substitution of eqns (6) through (8) into the right hand side of eqn (2) and manipulation of the left hand side leads to

$$\int_0^L M^{(1,2)}(x_3)\,\ell_1(x_3)\,dx_3 = \int_V \left[\sigma_{ij}^{(1)}\frac{\partial u_i^{(2)}}{\partial x_1} + \sigma_{ij}^{(2)}\frac{\partial u_i^{(1)}}{\partial x_1} - W^{(1,2)}\delta_{1j}\right]\frac{\partial q_1}{\partial x_j}\,dV \tag{12}$$

where V is the volume in Fig. 2b, (2) takes on the values ($2a$), ($2b$) and ($2c$) in succession, $i = 1, 2, 3$, $j = 1, 3$ and the interaction energy density

$$W^{(1,2)} = \sigma_{ij}^{(1)}\epsilon_{ij}^{(2)} = \sigma_{ij}^{(2)}\epsilon_{ij}^{(1)} . \tag{13}$$

It is assumed that $M^{(1,2)}(x_3)$ is a (different) constant along the crack front within each element. For a sufficiently refined mesh, this assumption should provide a good approximation to the continuous function $M^{(1,2)}(x_3)$. The left hand side of eqn (12) is written within a particular element as

$$\int_0^L M^{(1,2)}(x_3)\,\ell_1(x_3)\,dx_3 = M^{(1,2)} \int_0^L \ell_1(x_3)\,dx_3\ . \tag{14}$$

The virtual crack extension is assumed to be parabolic such that $0 \le \ell_1(x_3) \le 1$ within each element. So that, eqn (12) may be written as

$$M^{(1,2)} = \frac{\displaystyle\int_V \left[\sigma_{ij}^{(1)} \frac{\partial u_i^{(2)}}{\partial x_1} + \sigma_{ij}^{(2)} \frac{\partial u_i^{(1)}}{\partial x_1} - W^{(1,2)}\delta_{1j}\right] \frac{\partial q_1}{\partial x_j}\,dV}{\displaystyle\int_0^L \ell_1(x_3)\,dx_3}\ . \tag{15}$$

The value of $M^{(1,2)}$ is an average value of the exact values along the crack front calculated in a particular element.

On the other hand, substitution of eqns (3) through (5) into eqn (1) leads to

$$M^{(1,2)} = \frac{1}{H_1}\left(K_1^{(1)}K_1^{(2)} + K_3^{(1)}K_3^{(2)}\right) + \frac{1}{H_2}\,K_{II}^{(1)}K_{II}^{(2)}\ , \tag{16}$$

where

$$\frac{1}{H_1} = \frac{\mathrm{sgn}\,(E_{22})\ |E_{22}|}{\cosh^2 \pi\varepsilon}\ , \tag{17}$$

$$\frac{1}{H_2} = \mathrm{sgn}\,(E_{11})\ |E_{11}|\ . \tag{18}$$

Using solution $(2a)$ in eqns (15) and (16) leads to

$$K_1^{(1)} = H_1\,\frac{\displaystyle\int_V \left[\sigma_{ij}^{(1)} \frac{\partial u_i^{(2a)}}{\partial x_1} + \sigma_{ij}^{(2a)} \frac{\partial u_i^{(1)}}{\partial x_1} - W^{(1,2a)}\delta_{1j}\right] \frac{\partial q_1}{\partial x_j}\,dV}{\displaystyle\int_0^L \ell_1(x_3)\,dx_3}\ ; \tag{19}$$

using solution $(2b)$ in eqns (15) and (16) leads to

$$K_{II}^{(1)} = H_2\,\frac{\displaystyle\int_V \left[\sigma_{ij}^{(1)} \frac{\partial u_i^{(2b)}}{\partial x_1} + \sigma_{ij}^{(2b)} \frac{\partial u_i^{(1)}}{\partial x_1} - W^{(1,2b)}\delta_{1j}\right] \frac{\partial q_1}{\partial x_j}\,dV}{\displaystyle\int_0^L \ell_1(x_3)\,dx_3}\ ; \tag{20}$$

using solution $(2c)$ in eqns (15) and (16) leads to

$$K_3^{(1)} = H_1\,\frac{\displaystyle\int_V \left[\sigma_{ij}^{(1)} \frac{\partial u_i^{(2c)}}{\partial x_1} + \sigma_{ij}^{(2c)} \frac{\partial u_i^{(1)}}{\partial x_1} - W^{(1,2c)}\delta_{1j}\right] \frac{\partial q_1}{\partial x_j}\,dV}{\displaystyle\int_0^L \ell_1(x_3)\,dx_3}\ . \tag{21}$$

In eqns(19) through (21), $K_1^{(1)}$, $K_{II}^{(1)}$ and $K_3^{(1)}$ are average local stress intensity factors along the crack front.

3 TEST PROBLEM

An edge crack in a finite thickness, bimaterial slab was considered. The first term of the asymptotic displacement field given in Banks-Sills, et al. [7] was prescribed on the outer boundaries of the body with $K_1 = K_3 = 0$ and $K_{II} = 1$. Along the crack faces, traction free conditions were assumed. Fiber reinforced carbon/epoxy material (AS4-3502) was analyzed. Some material properties may be found in [6].

The program ADINA [10] was employed to carry out the finite element analyses. Twenty noded isoparametric brick elements were used; at the crack tip, they were distorted to quarter-point elements leading to a square-root singularity. It should be noted that the mode II stress intensity factor is the amplitude of the square-root singularity, whereas the complex stress intensity factor is the amplitude of the square-root oscillatory singularity. Thus, this element does not completely model the stress behavior. Two meshes were employed: one containing 34,641 nodal points and the other 55,181 nodal points. Both meshes are focused at the crack front. Through the thickness, both meshes contain 15 elements.

Integration of the $M-$integral is carried out in volumes which are orthogonal to the crack front and are an element thickness in that dimension. The volume extends away from the crack tip, but always includes the crack tip elements.

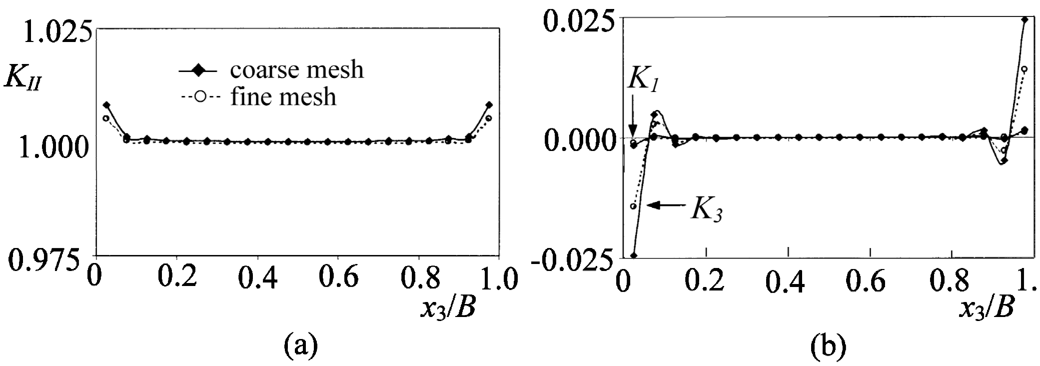

Figure 3: Stress intensity factors (a) K_{II}, (b) K_1 and K_3.

Results for K_1, K_{II} and K_3 obtained by means of both coarse and fine meshes are exhibited in Fig. 3. It may be noted that K_{II} differs by 0.01% from its expected value in the central part of the body for both meshes rising to 0.8% and 0.5% at the edges for the coarse and fine meshes, respectively. The modes 1 and 3 stress intensity factors are zero in the central part of the body with K_1 rising to 0.001 and K_3 oscillating and rising to 0.014 at the edges for the fine mesh.

These results were obtained with the volume V in Fig. 2b including three rows of elements in the x_1 and x_2-directions. The distance to the boundary S along the x_1-axis is 3.75% and 2.1% of the crack length a for the coarse and fine meshes, respectively. Although the $M-$integral is theoretically path independent, the region adjacent to the crack front which includes only the crack tip elements is not as accurate as other domains. This is attributed to the near tip elements which do not model correctly the oscillatory stress behavior. Moreover, it appears that the error is concentrated within these elements. Unlike the two-dimensional $M-$integral, even a domain removed from the crack tip region, as in Fig. 2b, includes inaccurate results from the near tip elements.

4. SUMMARY AND CONCLUSION

A crack along the $\pm 45°$ interface of two fiber reinforced materials has been considered. The effective mechanical properties are used so that the materials are treated as being transversely isotropic. The $M-$integral is presented and employed to obtain stress intensity factors in one test case. Excellent results are obtained. Elsewhere, other results will be presented.

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