ON THE USE OF PLATE AND INTERFACE VARIABLES FOR DELAMINATION IN COMPOSITES

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ABSTRACT

In this study an investigation on the utilization of delamination models based on plate theories and interface technique for analyzing 3D delamination problems is presented. The proposed method analyses the laminated structures as composed by first-order shear deformable plate elements interconnected by interfaces, whose constitutive relationships are based on fracture and contact mechanics. Delamination is simulated by reducing to zero interface stiffnesses, which otherwise perfectly connect the plate models by considering them as penalty parameters. Lagrange and penalty methods are adopted in order to simulate interactions between layers. The influence of plate and interface quantities on the interface fracture problem is investigated by means of closed form expressions for energy release rates, developed in terms of interface strains and of plate stress resultant discontinuities. At first, a simple two-dimensional delamination problem is considered in order to highlight the main characteristics of the model. Then, numerical results for the energy release rate distributions are given for typical three-dimensional mixed mode delamination problems by implementing the method in a 2D finite element analysis. Comparisons with 3D finite element models show the accuracy and the computational efficiency of the proposed procedure. Some applications are proposed to point out the convergence of the mode partition procedure as the delamination front element size decreases, also when oscillatory singularities exist.

1 INTRODUCTION

According to the fracture mechanics approach, the propagation of an existing delamination is analyzed by comparing the amount of energy release rate with interface toughness. When mixed mode conditions are involved, such as in practical delamination problems for composite structures, the decomposition of the total energy release rate G into mode I (G_I), mode II (G_{II}) and mode III (G_{III}) components, becomes a necessary task due to the mode-mix dependency of interface toughness. The problem of computation of the total energy release rate and of its mode decomposition for delamination in composite plates has been usually analyzed in the framework of the elasticity theory by applying the virtual crack closure technique (Kanninen [1]) to results obtained from continuum (2D) or solid (3D) finite element analyses. Due to the high gradient of stress and strain states in the neighborhood of the delamination front, a very accurate solution is indispensable which can be obtained by adopting an appropriate mesh of solid finite elements in the neighborhood of the delamination front. Illustrations of this methodology can be found both for two-dimensional (see for instance, Raju [2]; Beuth [3]) and three-dimensional delamination problems (Whitcomb [4]; Davidson et al. [5]). However, the computational cost of the continuum model is usually very high, due to the notable number of solid elements required, and additional complications arise due to the oscillatory behavior of energy release rates predicted by the three-dimensional elasticity theory when the delamination is placed between two dissimilar materials (see Raju [6]). In order to reduce the cost of delamination analysis the “global/local analysis” concept has been proposed (Suo and Hutchinson [7], Davidson et al. [8]). In this method the classical plate theory is adopted to predict G, whereas mode decomposition into individual energy release rates is completed by means of a separate local continuum problem with reference to a small element containing the delamination front. As a consequence, transverse shear deformation is not completely accounted for in the analysis and when the influence of shear effects is notable as in the case of composite laminate, one must resort to the expensive continuum model.

On the basis of a previous authors' model, developed in the context of 2D delamination by using refined plate theories and interface approach (Bruno & Greco [3], Bruno et al. [4]), in the present study a simple but appropriate method for the analysis of 3D delamination problems is presented for laminated composite structures, based on the multi-layer shear deformable plate modeling and interface technique. The model takes advantage of the computational efficiency of plate-based models and provides a reasonable accuracy in the determination of both the total and the individual energy release rates in comparison with 3D models. Applications, carried out by implementing the model by means of an FE formulation, demonstrate the accuracy of the proposed method in comparison with results obtained by using 3D solid finite elements available in the literature, and the avoidance of the non convergence behavior of individual energy release rates related to the oscillatory singularities.

2 A TWO-DIMENSIONAL EXAMPLE

Consider the interface fracture problem shown in Figure 1, where a delamination is located between the upper plate and the lower plate made of dissimilar homogeneous isotropic materials with equal thicknesses h. Plane strain assumptions are used along the width direction of the plate and two opening moment resultants per unit width M are applied at the delaminated portions of the structure. The longitudinal modulus and the Poisson ratio of the i-th plate are E_i and ν_i, respectively. The total and individual components of the energy release rate for this bi-layer configuration can be obtained by solving the boundary value problem governing the edge delamination, using a continuum 2D finite element analysis. Due to the applied loads, each of the two plates bend differently and shear stresses σ_{zx} arise along the interface to maintain adhesion at the interface ($z=0$). As a consequence, a mixed mode condition occurs. It is well known that when the Dundurs' elastic mismatch parameter β does not vanish (see Suo and Hutchinson [7]), the so-called oscillatory singularity develops for stresses and displacements near the delamination tip, of the form $r^{-1/2+i\varepsilon}$ where ε is the oscillatory power. Note that the oscillatory power vanishes when β also vanishes for certain combinations of material properties that satisfy $G_1(1\text{-}2\nu_2)=G_2(1\text{-}2\nu_1)$. The components of the energy release rates were calculated by applying the virtual crack closure technique (VCCT) with nodal forces and displacements near the delamination tip evaluated by using 8-noded plane strain continuum elements. Nodal forces near the delamination tip are recovered as the Lagrange's multipliers associated with the constraint equations imposed to ensure interfacial displacement continuity. The commercial package LUSAS (FEA Ltd) was chosen as solver. The material parameters used are: a=10mm, h=1mm, ℓ=15mm, E_1=100,000 N/mm^2, E_2= E_1/10, ν_1=ν_2=ν=0.3.

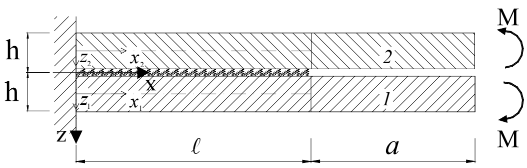

Figure 1: Bilayer plate under bending loading.

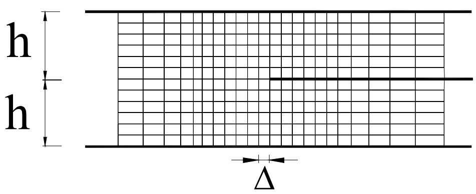

Figure 2: Enlarged view of the delamination tip Mesh.

Several finite element analyses were carried out with delamination tip element sizes equal to Δ/h=0.5,0.25,0.167,0.125,0.0625,0.050,0.033,0.0167,0.01,0.067,0.005 (see Figure 2). Figure 3 shows the behaviour of individual and total energy release rates obtained from the VCCT, as the delamination tip element size decreases. It can be observed that the total energy release rate is practically constant, whereas the individual components show non-convergent behaviours: the mode I energy release rate component decreases whereas the mode II energy release rate component increases as Δ/h becomes smaller. As shown in Raju [6] the reason for the non-convergent behaviour of energy release rate mode components is the non-zero oscillatory part of singularity. A matter of fact, for the two considered materials β=-0.2338 and ε=0.0758.

On the other hand, the energy release rates can be evaluated by modelling the bilayer structure as an assembly of first order shear deformable plate elements connected through interfaces, in terms of plate or of interface quantities (see Bruno et al. [10]). The use of beam/plate elements instead of solid elements is preferable not only for computational convenience, but since the non-convergent behaviour of energy release rate mode components is avoided. As a matter of fact, instead of oscillatory singular stresses, delamination tip forces exist which cause stress resultant discontinuities across the delamination tip. Moreover, a good approximation of the total energy release rate can be obtained if only one plate element is used to model each layer, whereas a more refined subdivision is necessary to obtain an accurate mode partition. When a two-plate model is used energy release rates can be obtained in a closed-form, once the governing equations of the bilayer structure are solved, by using the formulas obtained in (Bruno et al. [10]) for energy release rate mode components in terms of plate variables:

$$G_I = \sum_{i=1}^{2}\left(\frac{1}{2}[\![T_i\gamma_i]\!] - [\![T_i]\!]\psi_i\right),\quad G_{II} = \sum_{i=1}^{2}\left(\frac{1}{2}[\![N_i\varepsilon_i + M_i\kappa_i]\!]\right),\quad G = G_I + G_{II}, \tag{1}$$

where, with reference to the plate i, ε_i, κ_i, γ_i denote the membrane strain, the curvature and transverse shear strain, respectively, ψ_i denotes rotation of transverse sections, N_i is the membrane force resultant, M_i the moment resultant and T_i the transverse shear force resultant. Moreover, the double bracket denotes the jump in the enclosed quantity. Eqn (1) leads to:

$$G_I = \frac{3(c_1-1)^2(1-\nu^2)\left(E_1^2+14E_1E_2+E_2^2\right)}{2(1+c_1)^2 E_1E_2(E_1+E_2)h^3}M^2,\quad G_{II} = \frac{9(E_1-E_2)^2(1-\nu^2)}{2E_1E_2(E_1+E_2)h^3}M^2,\quad c_1 = e^{\frac{\ell\sqrt{5(E_1^2+14E_1E_2+E_2^2)h^2(1-\nu^2)}}{(E_1+E_2)h^2}}. \tag{2}$$

Eqns (2) show that mode II energy release rate arises from mismatch in longitudinal moduli. Moreover, from eqns (2) it can be noted that individual energy release rates are well defined despite the non-convergent behaviour predicted by the 2D elastic theory.

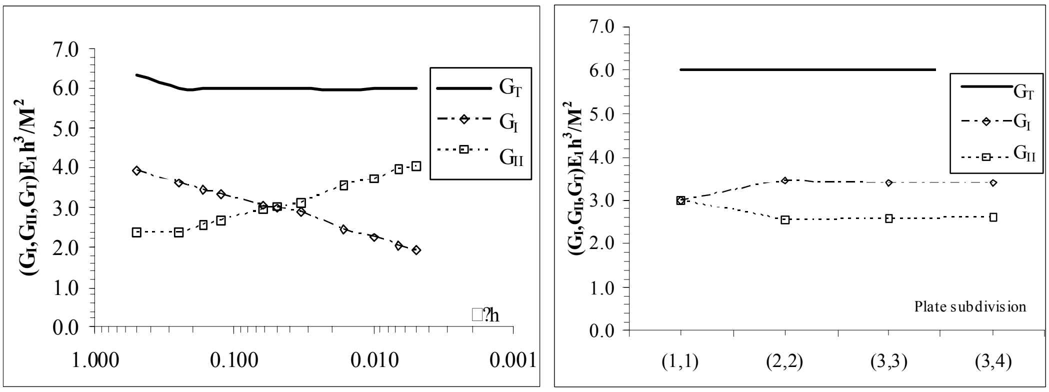

Figure 3: Energy release rates calculated from the VCCT technique (left) and from the coupled multilayer-interface approach (right).

When a refined plate elements subdivision is considered, a numerical integration procedure must be adopted to solve the governing differential problem. In this case it is more convenient to extract energy release rates from interface stresses and strains evaluated in the linear elastic interface located between the two layers by using interface stiffnesses k_{xy} and k_z as penalty parameters:

$$G_I = \lim_{k_{yx}, k_z \to \infty} \frac{1}{2} k_z \Delta w^2, \qquad G_{II} = \lim_{k_{yx}, k_z \to \infty} \frac{1}{2} k_{yx} \Delta u^2, \tag{3}$$

where Δw and Δu are the opening and sliding relative displacements at the delamination tip, respectively. The results for the present model, illustrated in Figure 3 (right), show that the total energy release rate and its individual components converge to definite values as interface stiffnesses approach infinity and the accuracy of the mode partition can be refined by increasing the number of plate elements in the structure. The previous considerations are based on a plane problem, on the other hand in the subsequent sections, where the analysis will be devoted to the 3D delamination problem, similar conclusions will be established.

3 MECHANICS OF A DELAMINATED STRUCTURE

Consider the delamination problem of Figure 4, where a laminate structure contains an in-plane delamination crack of area Ω_D and arbitrary front Γ_D which divides the laminate into two sub-laminates of respective thickness h_1, h_2. Each sublaminate is schematized by an assemblage of first order shear deformable plate bonded by zero-thickness interfaces in the transverse direction: the upper one is subdivided into n_u plates whereas the lower one into n_l. A perfect adhesion is ensured in the undelaminated region Ω-Ω_D by means of a linear interface model, whose constitutive law involves two stiffness parameters, k_z, k_{xy}, imposing displacement continuity in the *z,* and *x-y* directions, respectively, by considering them as penalty parameters. In the delaminated region Ω_D sublaminates are free to deflect but no to penetrate. Consequently, if a damage variable *d* is introduced, taking the value *1* value for no adhesion and the value *0* for perfect adhesion, the following constitutive law is introduced:

$$\sigma_{zz} = \left[1 - \frac{1}{2} d\left(1 + sign\left(\Delta w\right)\right)\right] k_z \Delta w, \ \sigma_{zx} = \left(1-d\right) k_{xy} \Delta u, \ \sigma_{zy} = \left(1-d\right) k_{xy} \Delta v, \tag{4}$$

where σ_{zz} and σ_{zy}, σ_{zx}, are the interlaminar normal and shear stresses and Δw, Δu and Δv are the corresponding interface relative displacements. The displacement continuity conditions between any two adjacent plates, *i* and *i+1*, into each sublaminate, i.e. $\Delta u_i = \Delta v_i = \Delta w_i = 0$, are ensured by the Lagrange's method.

By using fracture mechanics procedures expressions similar to eqns (1) and (3) can be found for the point-wise energy release rate and its mode I, II and III components along the delamination front, in terms of plate and interface quantities. In particular, the one expressed in terms of interface variables, which is more convenient for numerical applications, can be written as

$$G(s) = G_I(s) + G_{II}(s) + G_{III}(s)$$

$$G_I(s) = \begin{cases} \lim\limits_{k_z, k_{xy} \to \infty} \frac{1}{2} k_z \Delta w^2(s) & \Delta w(s) \geq 0 \\ 0 & \Delta w(s) < 0 \end{cases}, \ G_{II} = \lim_{k_z, k_{xy} \to \infty} \frac{1}{2} k_{xy} \Delta u_n^2(s), \ G_{III} = \lim_{k_z, k_{xy} \to \infty} \frac{1}{2} k_{xy} \Delta u_s^2(s), \tag{5}$$

where Δu_n and Δu_t are the relative interface displacements in the normal (***n***) and tangential (***t***) directions at the delamination front, respectively.

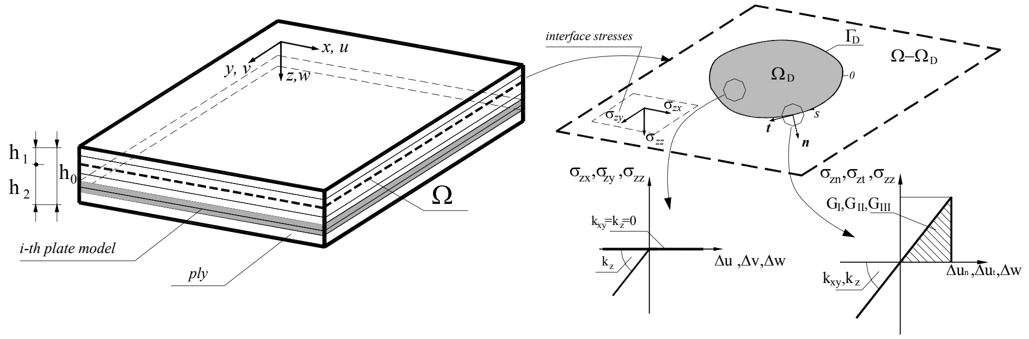

Figure 4: Mechanics of the delaminated composite structure.

4 NUMERICAL APPLICATIONS

In order to analyze applications to three-dimensional delamination geometries, the proposed theoretical method has been implemented in a 2D finite element model by using the commercial FE code LUSAS, licensed by FEA Ltd. For plate models four-noded isoparametric thick shell elements are used, whereas eight-noded interfaces are simulated by using joint elements composed by three translational springs. These springs have high stiffnesses and connect node pairs belonging to upper and lower plate elements. The same mesh is adopted for all the plate models in the thickness direction. Rigid links are adopted in order to simulate the thickness dimension of the two plate, n_l and n_{l+1}, sharing the delamination plane. As a consequence offset node pairs are generated with respect to the midplanes of the n_l and n_{l+1} plate models, belonging to the delamination plane. Interface elements are connected to these offset nodes. Interfacial displacement continuity in each sublaminate, is ensured by using constraint equations. Eqns (5) are applied in a modified version in order to avoid an excessive fine meshing at the delamination front, by using reactions from interface spring elements at the delamination front and relative displacements of the nodes ahead the delamination front collocated along the normal direction.

A normal axial loading condition is now considered for a three-dimensional symmetric delaminated plate geometry. The plate is isotropic and its properties are that used by (Davidson et al., [8]) and are as follows: a=256, h_1=h_2=h=16, B=400, L=512, E=80,000, ν=0.3, N=6.25. The data are in nondimensional units. The mesh for plate models is the same of that utilized in the x-y plane for the 3D solid FE model of Davidson et al. [8]. The analysis shows that when mixed mode conditions are involved, a double plate model is able to capture accurately mode decomposition in the region near the midpoint of the delamination front. On the other hand, an accurate mode decomposition near the free edges of the delamination front, where 3D effects are more complex, necessitates of more than one plate element in each sublaminate along the thickness direction, due to the large gradients in in energy release rates. However, the solution converges rapidly since a small number of plates is needed to obtain a reasonable approximation, as shown in Figure 5 where G_{II} and G_{III} distributions, normalized with respect to the classical beam theory results, are plotted. On the contrary, the use of solid finite elements leads to an increase of the number of degrees of freedom with respect to the present model. Additional investigations, not shown here, have highlighted that both the individual components and the total energy release rate converge as the delamination front elements are made smaller when delamination is placed between two dissimilar sublaminates.

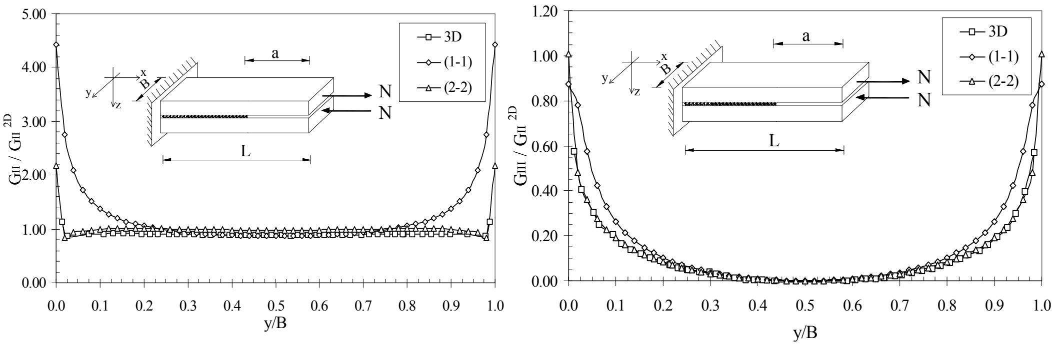

Figure 5: Convergence to 3D FE results with increasing number of plate elements.

5 CONCLUSIONS

A computationally efficient and accurate delamination model for laminated composite plates under loading conditions involving a combination of the three fracture modes, has been presented by considering the plate as a sequence of shear deformable plates connected by interface layers in the thickness direction. Adopting a relatively small number of plates in the thickness direction leads to an accurate prediction of the total and individual energy release rates, which also includes the effects of shear deformation and of delamination faces interaction. The good agreement with results from highly refined numerical solutions based on a 3D continuum description, have shown that, despite the good degree of accuracy, the proposed model involves a lower computational cost than full three-dimensional FE models, which may require a large number of solid finite elements to describe the delamination front region. Finally, our investigations have shown that the use of plate variables avoids the numerical complications in energy release rates convergence due to the oscillatory singularity behavior of stresses and strains at the delamination front predicted by the elasticity theory.

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