A MECHANICS BASED STUDY OF THE FATIGUE CRACK GROWTH THRESHOLD SIZE EFFECT

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ABSTRACT

A size effect has recently been reported in the literature, such that the use of larger C(T) specimens resulted in the measurement of larger fatigue crack growth thresholds. In an effort to understand the mechanics of this phenomenon, fatigue crack growth in C(T) specimens under a fixed *R* load reduction is simulated using 2D plane-stress elastic-plastic finite element analyses. A new method for predicting the fatigue crack growth threshold from these analyses is presented and utilized. The method uses the plastic strain hysteresis energy dissipated at the crack tip. The finite element simulations were able to replicate the experimental observations, and a larger C(T) specimen was predicted to produce a larger fatigue crack growth threshold. This was found to be a consequence of a more rapidly diminishing crack tip plastic strain hysteresis energy for the larger specimens.

1 INTRODUCTION

Experimental measurement of the fatigue crack growth threshold ΔK_{th} requires that a gradual reduction in the stress intensity factor range be applied during a fatigue crack growth test. While numerous types of load reduction are employed, a load reduction that holds the load ratio R fixed is commonly used. Recently, Garr and Hresko [1] reported a size effect when conducting threshold measurements for Inconel 718 Alloy using C(T) specimens and a fixed R load reduction with R = 0.10. Using specimen widths of 25, 50, and 125 mm, they reported threshold values of 6.5, 9.7, and 15.5 MPa√m respectively.

In an effort to understand the mechanics of this phenomenon, fatigue crack growth in a C(T) specimen under a fixed R load reduction was simulated using plane-stress elastic-plastic finite element analyses. A method for predicting the fatigue crack growth threshold is described and utilized, which uses the plastic strain hysteresis energy dissipated at the crack tip.

2 SIMULATING FATIGUE CRACK GROWTH

Fatigue crack growth was simulated by the repeated loading and unloading of a finite element model. The model was incrementally loaded to the maximum load P_{max}, at which time the crack tip node was released, allowing the crack to advance one element length L per load cycle. The applied load was then incrementally lowered until the minimum load P_{min} was attained. This load cycle was repeated as needed to simulate the desired amount of fatigue crack growth. Crack surface closure was introduced by changing the boundary conditions on the crack surface nodes. During each increment of unloading, the crack surface nodal displacements were monitored. Between any two increments, if the nodal displacement became negative, the node was closed and a node fixity was applied to prevent crack surface penetration during further unloading. During incremental loading, the reaction forces on the closed nodes were monitored, and when the reaction force became positive the nodal fixity was removed. The crack opening load P_o was found as the applied load which first fully opened the crack.

The success of this methodology depends heavily on the use of a suitably small loading and unloading increment. It has been found that an increment of 0.0125 P_{max} or smaller results in negligible variation of the contact forces along the closed crack surface [2]. For constant amplitude loading, perhaps 5 load cycles are required to achieve an approximate steady state condition in which the crack opening stress remains relatively constant [2]. For a detailed discussion of fatigue crack growth simulation, the reader is referred to reference [2].

Both the model generation and solution were performed using the commercial finite element analysis program ANSYS. The C(T) model used consisted of 12,250 nodes and 12,150 elements. Three separate C(T) specimens were modeled, with widths W of 25, 50, and 125 mm. These widths were chosen to simulate the tests performed by Garr and Hresko. A thickness B = 7.6 mm and an initial crack length a/W = 0.25 were used. The element size L along the crack path was proportional to W. For the three widths modeled, the element sizes along the crack path were 580, 1160, and 2900 μm.

The material was assumed to be an elastic-perfectly plastic Nickel-based alloy with modulus E = 207 GPa and flow stress σ_o = 1200 MPa. The von Mises yield criterion and the associated flow rule were used. Small deformation theory was employed. Following a pre-cracking with 10 load cycles, a total of 37 load cycles were used for the load reduction with fixed R. The crack front was advanced one element length during each load cycle with $da = L$.

The load reduction used was defined by the following relationship:

$$\Delta K = \Delta K_o e^{C\Delta a} \qquad (1)$$

where Δa is the amount of crack growth following pre-cracking, ΔK_o is the initial stress intensity factor range at the start of load reduction, and C is a constant. While the ASTM standard test method E647 recommends a maximum value for C of – 0.08/mm, values of –650/mm, –325/mm, and –130/mm were employed respectively for the specimen widths of W = 25, 50, and 125 mm. These large values were chosen because small values of C result in the need for large amounts of crack growth before appreciable reductions in the stress intensity factor range are produced. The magnitude of C has been shown to exhibit negligible influence on the opening stresses predicted from finite element analyses when increasing C an order of magnitude above the ASTM value [3]. However, the C increase utilized in this study was four orders of magnitude larger. In the opinion of the authors, this was considered acceptable given the comparative nature of the research effort.

Pre-cracking was performed using constant amplitude loading with $\Delta K = \Delta K_o$ = 13 MPa $\sqrt{\text{m}}$ and R = 0.10 for 10 load cycles. After pre-cracking, the crack was grown and the applied stresses were changed according to Eqn. 1. Crack growth under this load reduction was continued until the crack tip plastic strain hysteresis energy was negligibly small. This corresponded to an applied stress intensity factor range of approximately 7–9 MPa $\sqrt{\text{m}}$. Previous finite element studies [3,4,5] employed much larger applied ΔK.

Achieving a small ΔK in finite element analyses is a difficult task, due to the highly refined meshes required to properly discretize the plastic zone at the crack tip. The computationally intensive nature of plasticity-induced closure simulation is further aggravated when a variable amplitude load reduction such as that used for threshold testing is considered. Large amounts of crack growth are required to generate meaningful results in which the stress intensity factor range undergoes a significant reduction. To simulate large amounts of crack growth under the cyclic loading, a large number of load cycles are required, which will generally exceed the number needed for a constant amplitude simulation.

3 PREDICTION OF FATIGUE CRACK GROWTH THRESHOLD

The plastic strain hysteresis energy dissipated in laboratory specimens under cyclic loading is a function of the number of cycles to failure as shown by Halford [6]. Halford compiled a large amount of fatigue data for a wide variety of metals and alloys under various types of loading, and demonstrated that the (stable) plastic strain hysteresis energy dissipated per cycle ΔW can be related to the fatigue life N as follows:

$$\Delta W N = A N^{B} \tag{2}$$

where A and B are constants with $A = 0.0042E$, $B = 1/3$, and E is the modulus of elasticity. Solving this expression for N we have:

$$N = \left(\frac{\Delta W}{A}\right)^{\frac{1}{B-1}} \tag{3}$$

ΔW can be correlated with the fatigue crack growth rate da/dN as discussed extensively by Ellyin [7]. When simulating fatigue crack growth with the finite element method, the incremental crack extension da is the element size L at the crack tip. Envisioning the crack growth rate to represent a fatigue failure of the material over a length L after N cycles, we may write:

$$\frac{da}{dN} = \frac{L}{N} = \frac{L}{(\Delta W / A)^{1/(B-1)}} \tag{4}$$

Eqn. 4 was used to predict the fatigue crack growth rate using ΔW values from the finite element analyses.

4 RESULTS

Figure 1 illustrates the local stress σ_{yy} as a function of total strain near the crack tip during loading and unloading for the three C(T) specimens. These hysteresis loops were computed in the plastic zone for a single node located at a distance 2L from the crack tip. This node was selected because it was possibly far enough from the crack tip such that errors induced from the small strain solution and the attendant zero crack-tip radius could be avoided, and also close enough to the crack tip to be located within the reversed plastic zone. While perhaps an average plastic strain hysteresis energy near the crack tip would be preferred, the single node value was chosen for simplicity. This was considered adequate given the comparative nature of the research effort.

From Figure 1, the larger the specimen width, the more rapidly the crack tip hysteresis loop diminished with the load reduction. This is also illustrated in Figure 2, where the area within the hysteresis loop ΔW is plotted as a function of crack growth. Given that da/dN = da/dN(ΔW), the results presented in Figure 2 suggest that a larger specimen reaches a given value of da/dN (such as that defining a threshold) after a smaller amount of crack growth Δa. This smaller Δa corresponds to larger ΔK values.

Every effort was made to insure that the finite element mesh used for the specimen with W = 25 mm was adequately refined such that the reversed plastic zone at the crack tip was properly discretized. This mesh was then used to model the specimens with W = 50 and 125 mm by increasing the element sizes proportionally. Given that each of the three specimens was loaded with the same initial ΔK, the level of mesh refinement used for the larger specimens is suspect. A mesh refinement study will be necessary to insure that the results presented in Figure 2 have not been influenced by the element size used.

Using eqn. (4), the plastic strain hysteresis energy ΔW can be used to compute a crack growth rate. The computed rates are shown in Figure 3, where they are compared to the measured values reported by Garr and Hresko. The experimental and predicted crack growth rates compare favorably for the specimens with W = 25 and 50 mm. For the specimen with W = 125 mm, the crack was predicted to grow at a faster rate than that observed.

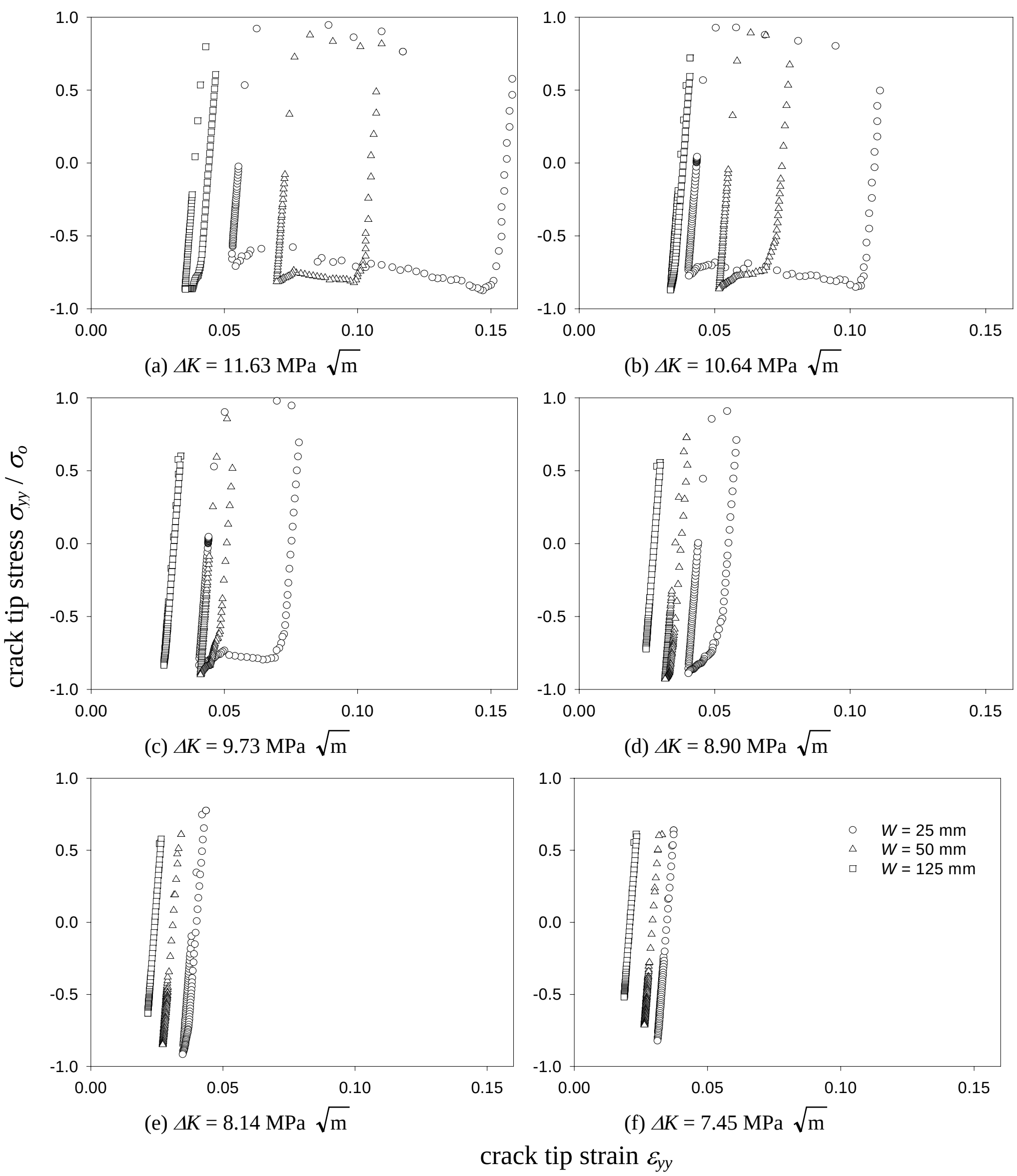

Figure 1: Crack tip hysteresis loops.

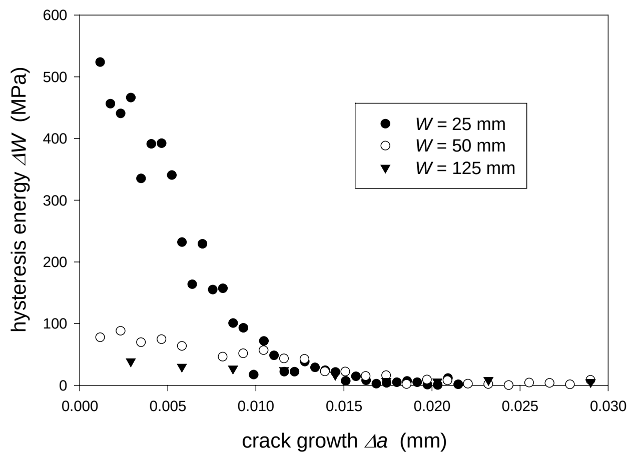

Figure 2: Variation of crack tip plastic strain hysteresis energy.

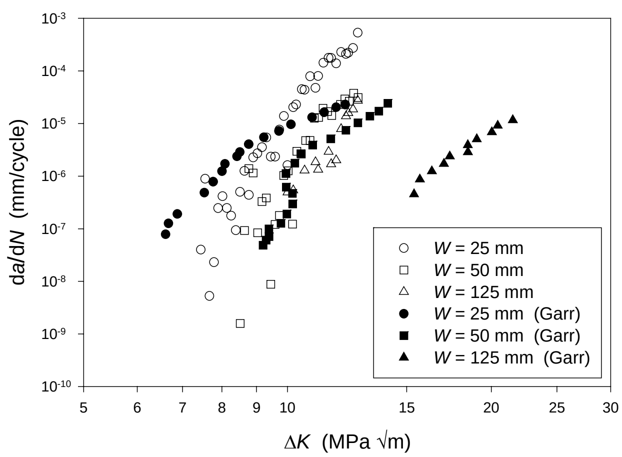

Figure 3: Crack growth rate data and predictions

5 SUMMARY AND CONCLUSIONS

A size effect has been reported in the literature, such that the use of larger C(T) specimens results in the measurement of larger fatigue crack growth thresholds. The primary objective of this research was to perform a limited number of exploratory elastic-plastic finite element fatigue crack growth simulations to determine if this trend could be replicated, and to elucidate the origin of this phenomenon from a mechanics perspective. A method of estimating the fatigue crack growth threshold using finite element analysis was presented.

The finite element simulations were able to replicate the experimental observation that under a fixed R load reduction, a larger C(T) specimen will give a larger fatigue crack growth threshold. The larger threshold was a consequence of a more rapidly diminishing crack tip plastic strain hysteresis energy ΔW as shown in Figure 2. A mesh refinement study will be necessary to insure this observation is not an artifact of the proportional element sizes used for the three models. It is speculated that the rapid decrease in ΔW is the result of crack closure remote to the crack tip as this has observed by Daniewicz and Skinner [5] for load reductions under fixed R. A alternate possible explanation is a reduction in ΔW due to T stress effects. More research will be necessary to clarify this matter.

It is noteworthy that the experimental results of Garr and Hresko [1] were reasonably well simulated using a modeling methodology which considered plasticity-induced crack closure only. Many researchers have suggested that roughness induced closure becomes the dominant closure mechanism in the near-threshold regime.

In this investigation, an approximate measure of the crack tip plastic strain hysteresis was utilized. This method needs to be extended, from the single point value used here, to an average over some portion of the crack tip plastic zone. While additional work is needed, the authors believe that the use of hysteresis energy to define the fatigue crack growth threshold from numerical analyses is a methodology with both merit and promise.

6 REFERENCES

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