OPTIMAL SHAPE DESIGN WITH FATIGUE LIFE AS DESIGN CONSTRAINT USING A 3D BIOLOGICAL METHOD

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ABSTRACT

The damage tolerance design philosophy assumes that cracks are present at all potentially critical locations in a structure. However, the numerical simulation of cracks using FEM requires a fine mesh to model the singularity at crack tip. This makes fracture calculations relatively computationally intensive. As a result, there have been limited applications of durability based optimisation to realistic structures. To overcome this, the paper presents a 3D biological algorithm for the shape optimisation of structures with fatigue life as the design objective. This formulation was used for optimisation of damage tolerant structures with numerous 3D flaws located along the structural surface. A semi-analytical method was employed for computing the stress intensity factors. This method uses the uncracked stress field, and thus does not require explicit modelling of cracks. The stress distribution in the uncracked structure was obtained using the finite element program NE-NASTRAN, and the fatigue life associated with the cracks around the surface being optimised was evaluated using a modified version of the NASA crack growth program FASTRAN. In this work the fatigue life based optimisation is illustrated via the problem of optimal design of 'a through hole in a rectangular block under biaxial loading'. It was found that the optimum hole shapes were approximately elliptical with the aspect ratios being dependent on crack size, structural geometry and boundary conditions. It has been shown that the fatigue life optimised shapes can be different from the corresponding stress optimised solution. This emphasises the need to explicitly consider fatigue life as the design objective. In all cases a significant improvement in the fatigue life was achieved with the generation of a 'near uniform' fracture critical surface. The design space near the 'optimal' region was found to be relatively flat. This is beneficial as a significant structural performance enhancement can be achieved without precise identification of the local/global optimum solution.

1 INTRODUCTION

The damage tolerance design philosophy has become an integral part of the modern design, especially for capital intensive industries such as aerospace, rail, marine, mining etc. On the other hand, structural optimisation is being increasingly applied for shape and topology designs. Recent developments in advanced analysis methods and advances in computer technology have set the scenario for the marriage of optimisation and damage tolerance design. For small initial flaws with large critical crack lengths the stress optimised shapes are similar to the fracture strength or fatigue life optimised shapes. This is often true for aerospace applications. However, for marine, rail, and mining industries, initial flaws can be 5 to 15 mm deep and 10 to 40 mm long. In such cases optimised shapes, or topologies, based on stress and fracture strength can differ considerably [1,2]. Thus, to develop lighter designs,

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that also have increased tonnage capacity, the optimisation methods should incorporate damage tolerance parameters as design goals.

Most of the structural optimisation studies undertaken to date have focused on reducing the peak stress [3]. This does not take into account the effect of initial flaws. Damage tolerance based optimisation is a highly repetitive process with each iteration requiring cracks, with variant geometries, to be modelled at each point around the surface being optimised. This has the potential to make the entire process extremely computationally intensive. In presence of initial flaws the optimal profile (assuming no cracks) may not be optimum due to interactions of initial flaw(s). However, little attention has been paid to optimisation of structures with durability as the design objective [1,2]. This paper will address this issue by developing an approach of optimum shape design using a 3D biological algorithm that will explicitly consider fatigue life as the design goal.

The biological methods mimic '*natural growth*' laws. The simple rule is the addition of material in high stressed zones and the removal from low stressed zones. These natural principle based techniques arose from the observation that biological species, eg. plants, adapt in such a way that local peak stress is avoided by generating a homogeneous surface stress distribution [4]. Schnack [5] was the first to use this method to minimise stress concentrations. Kaye and Heller proposed a mathematical formulation for the stress based optimisation of two dimensional problems [6]. The algorithm used here is based on the 2D approach proposed by Kaye and Heller [6]. In the present work the algorithm has been extended to simple 3D structures with fatigue life as the design objective.

2 3D BIOLOGICAL ALGORITHM FORMULATION

The biological method accomplishes shape change by moving the boundary/surface being optimised. Hence, the representation of geometry or shape is an important step in the optimisation. In this approach the design boundary is specified by a set of design points/nodes. These points/nodes are moved by the optimiser using the 3D biological algorithm. A new design surface/boundary is then constructed from the resultant points by fitting a suitable curve/surface or a new FE model is developed using the displaced/re-positioned nodes. The locations of these design points/nodes constitute the design variables and the movements of these points accomplish the shape change. A moving average technique is employed to ensure a smooth change in the geometry and to prevent mesh distortion. This is done by controlling the movement of a given node in relation to the movements of its neighbours. The displacement of a design node normal to the boundary surface is given by:

$$d_i = \frac{q_{ref} - q_i}{q_{ref}} \times f \tag{1}$$

Here q_i is the local structural parameter at the ith design node, q_{ref} is the reference structural parameter and f is a control factor that depends on the class of problem analysed and is usually determined through numerical tests.

For a general 3D body with its boundary represented by 3D surfaces, the displacement of the ith design node (d_i) can be written as:

$$\vec{d}_i = d_i \hat{n} \tag{2}$$

where $\hat{n}$ is the unit normal to the surface at the location of the ith design node. The software, implementing the 3D biological algorithm, changes the position of each node according to eqn (2). However, the movement is smoothed and also checked for geometrical constraints.

The local structural parameter, i.e. q_i in eqn (1) may be stress, stress intensity factor, fatigue life, or any other design parameter relevant to the design objective. For fatigue life optimisation eqn (1) can be modified as:

$$d_i = \frac{Log(N_i) - Log(N_{ref})}{Log(N_{ref})} \times f \tag{3}$$

where N_i is the fatigue life associated with the surface crack at the ith design node. This life is calculated as the crack grows from an initial semi-elliptical flaw of size (c_i,a_i) to its final size of (c_f,a_f). N_{ref} is the reference fatigue life. In the present study N_{ref} was taken as the minimum fatigue life associated with all of the cracks on the surface being optimised.

3 ILLUSTRATIVE EXAMPLE PROBLEM

To illustrate this approach, let us consider the simple problem shown in Figure 1. It is a 3D block, 320 mm wide, a height of 320 mm, and a thickness of 20 mm with a cylindrical through the thickness hole at its centre. The diameter of the initial hole was 20 mm. The material of the block was assumed to be an aluminium alloy (2219-T851) with a Young's modulus of 71 GPa and a Poisson's ratio of 0.3. The block was subjected to a uniform tensile stress of 50 MPa on the right and left faces (normal to X axis). The top and bottom faces (normal to Y axis) were subjected to a tensile stress of 100 MPa. The problem of the optimum design of the central hole was then studied. It is noteworthy that presence of such 'cutouts' is common in engineering structures such as rail, aerospace, naval, heavy industries. The optimisation example problem considered here is generic in the sense that the section near a cutout, for many structures, can be modelled as a through hole in a block under biaxial load. It should be noted that all the planes (XY, YZ and XZ) mentioned in the rest of the paper refer to Figure 1.

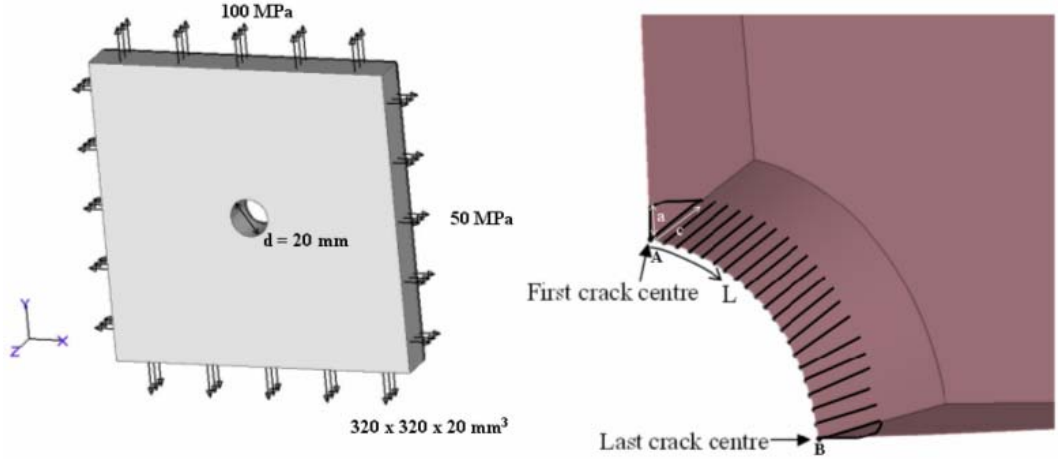

Figure 1: Schematic of a cylindrical hole in a rectangular block under biaxial stress (left) and the locations of the cracks along the hole surface for an one-eighth model (right)

4 RESULTS AND DISCUSSION

Let us now examine the problem of determining the shape that maximises the fatigue life for the specific problem outlined above. As the (uncracked) problem was symmetric, only one-eighth of the structure was analysed. In this study twenty one semi-elliptical cracks were placed along the surface of the cylindrical hole (for one quarter), see Figure 1. The initial spacing between the adjacent cracks was ~0.75 mm. This is less than the smallest initial crack length (1 mm) considered in either direction (major or minor axis). This enabled an effective modelling of the stress intensity factor variation around the boundary surface. Each crack on the structural boundary was allowed to grow in both the directions of major and minor axes from an initial size of (c_i,a_i) to its final size of (c_f,a_f). For each crack

configuration (during the crack growth phase) the semi-analytical method presented in [7] was used to compute the stress intensity factors around the crack front associated with each crack. This method employs the analytical solution for an embedded elliptical flaw to solve for the stress intensity factor for an infinite body and then modifies the resultant solution by appropriate correction factors. As such it avoids the explicit modelling of cracks. The stress intensity factors were then used to compute the fatigue life for each crack present in the optimisation domain using a modified version of the NASA crack growth program FASTRAN.

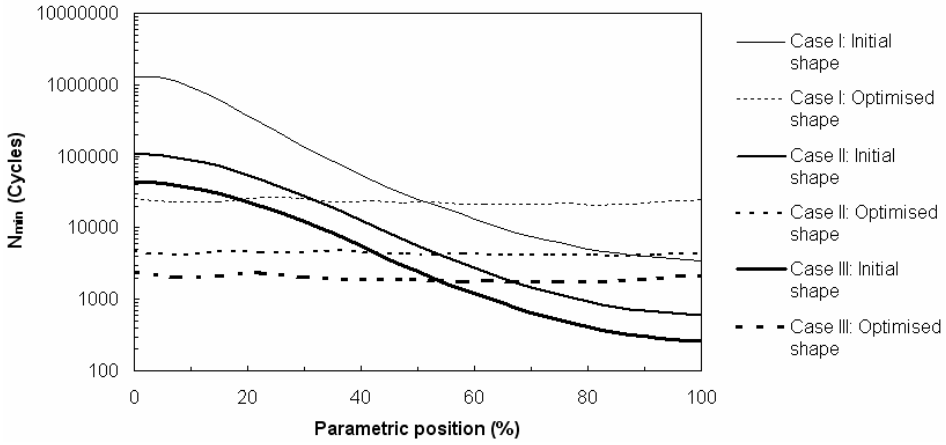

Figure 2: Initial and optimal fatigue life distribution along the hole surface

In the present study our aim was to determine the hole profile that would maximise the minimum fatigue life (N_{min}). We considered three different crack cases, each with varying initial and final flaw sizes, see Table 1. The initial hole shape was assumed to be circular as this is a commonly used shape. Moreover, it was instructive to start from a circular rather than the stress optimised shape (elliptical) as it allowed a wider exploration of the design space. The fatigue life distribution of the semi-elliptical flaws along the hole surface is shown in Figure 2 for both the initial and optimal shapes. For the initial hole shape with circular cross section, it can be observed that for all the cases fatigue lives associated with the cracks were considerably high at the top section of the hole. The fatigue life gradually decreased along the hole boundary towards the bottom portion (from point A to B in Figure 1(right)). For example, for the crack case (i) the fatigue life associated with the crack at point A (in Figure 1) was 12,69,000 cycles, whereas the fatigue life associated with the crack at the bottom of the hole (point B in Figure 1) was 3,429 cycles. This same trend was exhibited by the other crack cases, see Figure 2.

The 3D biological algorithm was then applied to vary the hole surface in order to determine the optimum shape. The hole profile was represented by 'geometric descriptor method' [8] using a set of design points. Material was removed from the region having a high fatigue life, i.e. low fracture criticality and the hole approached a 'near' elliptical shape. The fatigue life improved significantly for all the crack cases. The optimisation results are summarised in Table 1. The objective function (minimum fatigue life) values of the optimal hole shapes were 6.1, 6.8 and 6.6 times higher than those of the initial shapes for the three crack cases respectively. As expected the fatigue life was quite sensitive to the assumed initial and final flaw size (Figure 2).

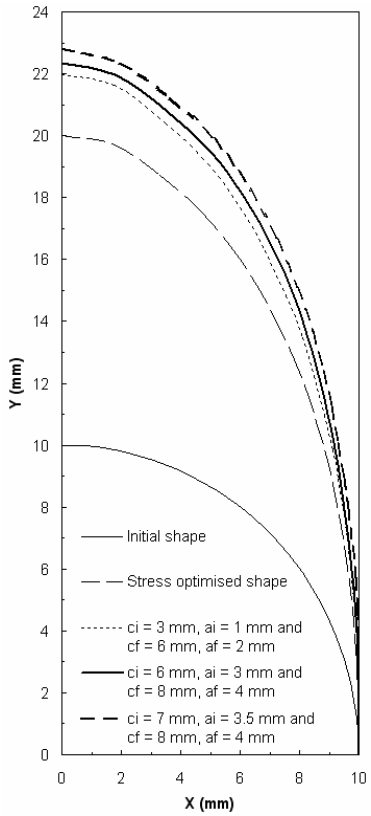

Figure 3: Comparison of the fatigue life and stress optimised hole shapes in the XY plane

The optimal hole shapes in the XY plane are shown in Figure 3. The shapes were found to be 'near' elliptical with the major axes being ~22.0 mm, 22.3 mm and 22.8 mm respectively for the three crack cases considered. The fact that these elliptical shapes have higher aspect ratios than the corresponding stress optimised shape (a 2:1 ellipse) demonstrates that the fatigue life optimised structures can be lighter than the stress optimised structures.

One salient feature of the 'near' optimal shapes produced by the biological method is the generation of a 'near' uniform fracture critical surface. It is evident from Figure 2 that the generated (optimum) hole surfaces were equally fracture critical everywhere, i.e. the fatigue life was essentially uniformly distributed for all the cracks present along the optimisation domain.

It is commonly thought that optimising a structural shape to minimise the maximum stress also optimises the shape for fatigue life. This fact is illustrated by comparing the minimum fatigue life for the stress and fatigue life optimised shapes in Table 1. It is to be noted that in each case the stress optimised shape has a significantly better fatigue life as compared to the initial circular hole. However, using the fatigue life as the optimisation objective led to further improvement of the fatigue life of the resulting shapes by 28.4 %, 35.1 %, and 33.5 % respectively for the three crack cases, see Table 1.

In this study we have also found that the present problem has a flat design space in the 'near optimal' zone, see Table 1. This means that in the near optimal zone the fatigue life will not be greatly degraded even if the actual crack size in the structure is slightly different from that assumed. The same reasoning applies to a structure that contains cracks with slightly varying sizes.

Table 1: Summary of objective functions and optimum geometries for different crack cases

Crack case No.	Crack details		Objective function			Increase in N_{min} from initial shape (%)	Increase in N_{min} from stress optimised shape (%)	Optimal hole size (mm)
	Initial crack size	Final crack size	Initial shape N_{min} (Cycles)	Stress optimised shape N_{min} (Cycles)	Life optimised shape N_{min} (Cycles)			
1	c_i = 3 mm, a_i = 1 mm	c_f = 6 mm, a_f = 2 mm	3429	16329	20964	511.4	28.4	21.97
2	c_i = 6 mm, a_i = 3 mm	c_f = 8 mm, a_f = 4 mm	596	3012	4069	582.7	35.1	22.34
3	c_i = 7 mm, a_i = 3.5 mm	c_f = 8 mm, a_f = 4 mm	268	1315	1756	555.2	33.5	22.80

5 CONCLUSIONS

This paper has presented a 3D biological algorithm for structural optimisation with fatigue life as the design objective. It has been shown that the fatigue life optimised shape can differ from the corresponding stress optimised solution and that the fatigue life optimised structures can be lighter. This emphasises the need to explicitly consider fatigue life, as distinct from stress, as the design objective. In all cases a significant improvement in fatigue life was achieved in the 'near' optimal shape. Furthermore, all points around the surface were essentially equally fracture critical, i.e. the fatigue life distribution was approximately uniform around the design surface. The design space near the 'optimal' region was found to be relatively flat. This is beneficial in that we can achieve a significant enhancement in fatigue life without the need to precisely identify the local/global optimum. Alternatively, we can lighten the structure without significantly reducing its fatigue life.

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